TECHNIQUES FOR EVALUATING LIMITS

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(Please feel free to email me questions and /or comments.)

Key Topic: Limits and Continuity

Abstract:

In this activity students will be given the techniques for evaluating limits with and without using a TI-89. Both limits as $x \to a$ and as $x \to \pm\infty$ will be given. Examples of each technique will be furnished, and students will be asked to hone their skills by attacking a set of exercises.

Prerequisite Skills:

- Basic understanding of the concept of a limit
- Ability to reduce rational expressions and multiply polynomials
- Understanding of the basic features of the TI-89
- Ability to graph functions on the TI-89.

Degree of Difficulty: Easy to moderate

Needed Materials: TI-89

NCTM Principles and Standards:

- Content Standards – Algebra
  - Represent and analyze mathematical situations and structures using algebraic symbols
  - Use mathematical models to represent and understand quantitative relationships
  - Draw a reasonable conclusion about situation being modeled

- Process Standards
  - Representation
  - Connections
  - Problem Solving

TECHNIQUES FOR EVALUATING LIMITS

We will look at techniques for evaluating limits by hand and by using the CAS (Computer Algebra System) found on the TI-89.

Limits as $x \to a$

1. Plug in: If plugging in a for x does not result in division by zero, then you have found the limit.

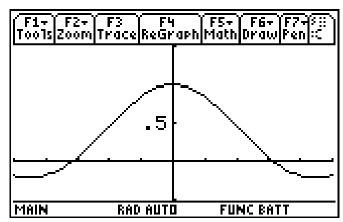
Example: $\lim\limits_{x\to 1} \dfrac{x^2-1}{x+1} = \dfrac{1^2-1}{1+1} = \dfrac{0}{2} = 0$

2. Algebraic simplification: If "plug in" results in division by zero, algebraically simplify the expression (if possible), and then "plug in" will give you the limit if you do not get division by zero.

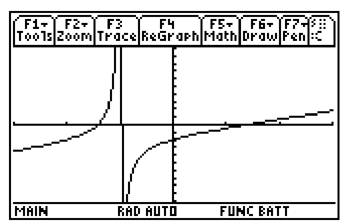
Example: $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1} = \lim\limits_{x\to -1} \dfrac{(x-1)(x+1)}{x+1} = \lim\limits_{x\to -1}(x-1) = -2$

3. Graph it: If the methods above do not produce the desired limit, as a last resort, look at the graph or a table of values.

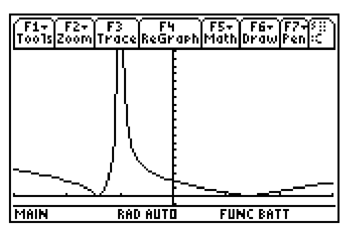
Example: $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$



Example: $\lim\limits_{x\to -1} \dfrac{x^2-2}{x+1}$ does not exist



Example: $\lim\limits_{x\to -1} \left|\dfrac{x^2-2}{x+1}\right| = \infty$



Limits as $x \to \pm\infty$

1. Rational functions: Divide the numerator and denominator by the highest power of x in the denominator, and then use the fact that $\lim\limits_{x\to\pm\infty} \dfrac{1}{x} = 0$.

Examples:

$$\lim_{x\to\infty} \frac{x-1}{x^2+2} = \lim_{x\to\infty} \frac{\frac{1}{x}-\frac{1}{x^2}}{1+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}-\left(\frac{1}{x}\right)^2}{1+2\left(\frac{1}{x}\right)^2} = \frac{0-0}{1+0} = 0$$

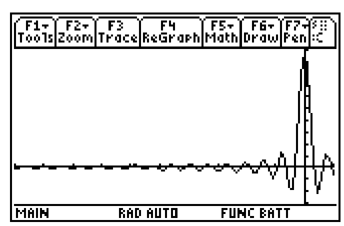
$$\lim_{x\to\infty} \frac{x^2-1}{x^2+2} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1+\frac{2}{x^2}} = \frac{1-0}{1+0} = 1$$

$$\lim_{x\to-\infty} \frac{x^2-1}{x+2} = \lim_{x\to\infty} \frac{x-\frac{1}{x}}{1+\frac{2}{x}} = \lim_{x\to-\infty} x = -\infty$$

$$\lim_{x\to-\infty} \frac{1-x^2}{x+2} = \lim_{x\to\infty} \frac{\frac{1}{x}-x}{1+\frac{2}{x}} = \lim_{x\to-\infty} -x = -(-\infty) = \infty$$

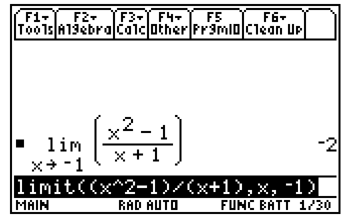
2. Non-rational functions: Look at the graph or a table of values.

Example: $\lim\limits_{x\to-\infty} \dfrac{\sin x}{x} = 0$



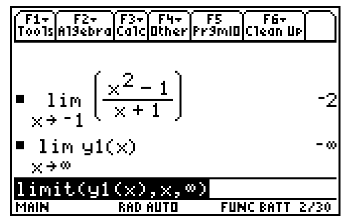
Using the CAS features of the TI-89

Example: Evaluate $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x + 1}$



Keystrokes: [F3] [3] [(] [X] [^] [2] [−] [1] [)] [÷] [(] [X] [+] [1] [)] [,] [X] [,] [(-)] [1] [)] [ENTER]

Example: Evaluate $\lim\limits_{x \to \infty} \dfrac{2 - x^2}{x + 1}$



Note: If the function $y1(x) = \dfrac{2 - x^2}{x + 1}$ has been defined using the [Y=] editor, then $y1(x)$ can be used in place of entering the function. To enter the symbol $y1$, simply press [Y] [1]. If you are hunting for the ∞ symbol, it is above the [CATALOG] key which is in the middle of the fourth row from the top.

EXERCISES:

1. Find the *exact* value of the limit of each of the following without using the CAS features of the TI-89. You may use the graphing features of the TI-89.
   Caution: Using **Trace** or a table will not always yield an *exact* value.

   a. $\lim\limits_{x \to 5} \dfrac{x - 5}{x^2 - 25}$ b. $\lim\limits_{x \to -5} \dfrac{x - 5}{x^2 - 25}$ c. $\lim\limits_{x \to 0} \dfrac{1 - e^{-x}}{e^x - 1}$

   d. $\lim\limits_{x \to 1} \dfrac{\ln x^2}{\ln x^7}$ e. $\lim\limits_{x \to 2} \dfrac{3}{x - 2}$ f. $\lim\limits_{x \to \infty} \dfrac{x + 1}{9x - 1}$

   g. $\lim\limits_{x \to -\infty} \dfrac{1}{1 + e^{-x}}$ h. $\lim\limits_{x \to -\infty} \dfrac{5 - 2x}{2 - 5x^2}$ i. $\lim\limits_{x \to -\infty} \dfrac{5 - 2x^2}{2 - 5x}$

2. Check your answers to #1 using the CAS features of the TI-89. That is, on the [HOME] screen, use [F3] [3].

ANSWERS:

1. a. $\frac{1}{10}$ b. $-\frac{2}{15}$ c. $\frac{1}{2}$ d. $\frac{2}{7}$ e. does not exist

   f. $\frac{1}{9}$ g. 1 h. 0 i. $-\infty$