

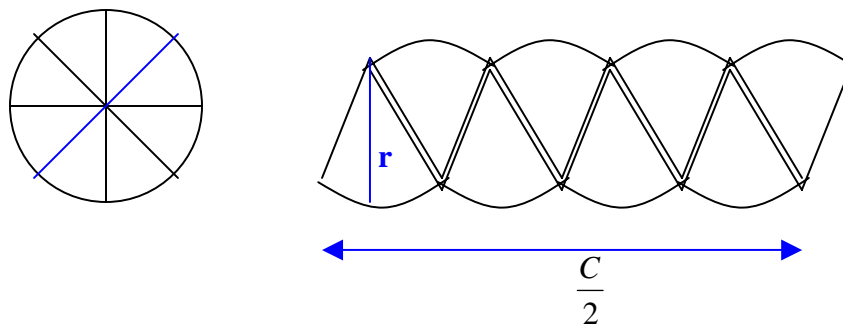
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Circles: Circumference and Area

Geometry Key Topics: Circles, circumference of circle, diameter, pi, perimeter of a regular polygon, area of a circle, area of a regular polygon.

Materials: TI-89. Flash application: Cabri Geometry. (available for download from www.eduation.ti.com)

Focus: The idea of perimeter can be applied not only to polygons but to circles as well. The same idea can be said for the area of a circle. In trying to see the relationship that exists between the area of a polygon and a circle imagine a circle that has been cut into triangles and placed in a regular fashion, one next to the other.



If you can imagine the radius being the equivalent of height in parallelogram then the circumference would be half the perimeter of the circle or half the circumference.

What happens as we divide the circle into more and more pieces? How will the parallelogram we form look? _____

What is the formula for the area of a parallelogram? _____

If you were to replace $\frac{C}{2}$ in place of length and r in place of height what will your equation look like for the area of a circle? _____

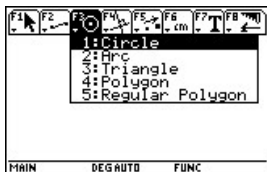
Introduction: In your study of the circle, you will be looking at its circumference as well as the derivation of its area formula.

Procedure: You will be using Cabri Geometry to investigate the relationship that exists between circles and regular polygons. As you do the activities, try to see how the formulas that we use to find the circumference and the area of a circle is simply the same equations we use on regular polygons only written in a different way.

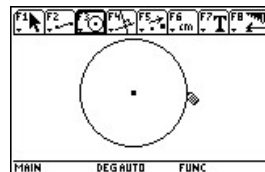
Circumference of a Circle:



Start Cabri Geometry from APPS.



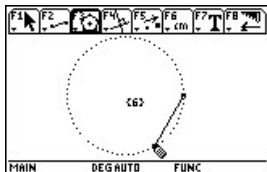
Press **[F3]** and select 1:Circle to draw a CIRCLE.



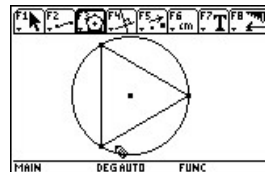
Complete drawing the Circle.



Press **[F3]** and choose #5 to construct a regular polygon.



Position the center of Your polygon to coincide With the center of your Circle. Press **[ENTER]**. The default number of sides is 6. Press **⬇** or **⬆** to increase or decrease the number.

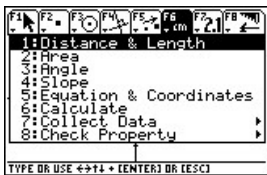


Start with a three sided polygon. Continue by drawing another with 4 sides. Draw another with 5 sides.

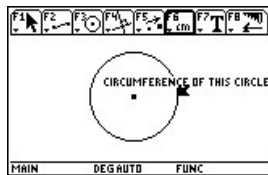
1. As the sides of your polygons increase what do you notice? _____
 2. What happens to the perimeter of the polygons? _____
 3. Looking at the perimeter of your polygons what is the limit that it can reach? Why do you think this will happen? _____
- _____
- _____

Pi (π)

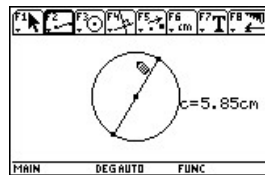
Press **[2nd] [F3]** and select #8 so you can clear all objects on your drawing board. Create a new circle. In the following investigation you will be looking at the relationship that exists between the circumference and the diameter of a circle.



Press **[F6]** to enter the measuring menu and select #1 to calculate the distance and length.



Position your cursor On the circle until you see the message “Circumference of this Circle.” Press **[ENTER]** and you will have the circumference.



Press **[F2]** and select #5 to draw a segment. Create a segment that spans the distance of the circle and passes through the center of the circle.

1. Complete the following chart. Use your pointer tool to increase or decrease the size of your circle. Note the measurements as they change and compute $\frac{C}{d}$:

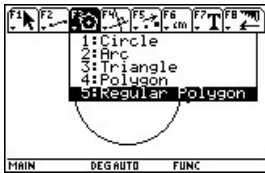
| Circle | Circumference Of Circle | Diameter Of Circle | C/d |
|--------|-------------------------|--------------------|-----|
| 1 | | | |
| 2 | | | |
| 3 | | | |

2. What pattern do you observe? _____
3. What would the ratio of $\frac{C}{d}$ be for a circle with a circumference of 90.5 cm. and 29 cm. across? _____. Do your finding agree with those you have been finding above? _____.
4. Using this ratio, how would you solve for C if you were given the diameter? _____.
5. What is another way of symbolizing the diameter of a circle? _____

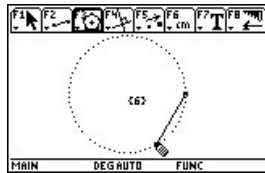
Area:

Press **[2nd]** **[F3]** and select #8 so you can clear all objects on your drawing board. Create a new circle. In the following investigation you will be investigating the area of a circle and how the formula is derived.

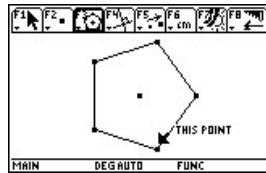
Step 1



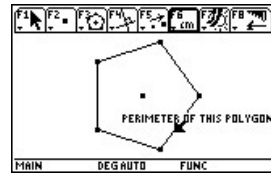
Press **[F3]** and select Regular Polygon.



Press your left arrow and change the number of sides from 6 to 5.



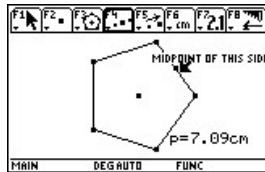
Press **ENTER** to create your polygon



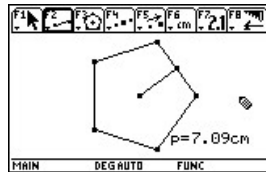
Find the perimeter of the polygon.



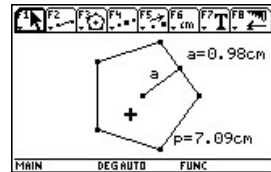
Press **[F4]** and choose the Midpoint option.



Find the midpoint of the sides.



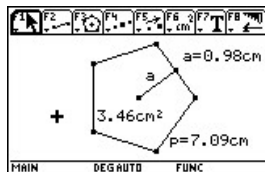
Draw a segment joining the midpoint to the center of the polygon.



Label everything.



Press **[F6]** and choose the Area option.



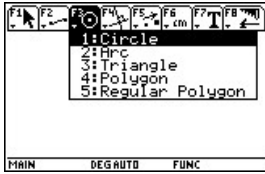
Find the area of the polygon.

1. In the above polygon what do you notice if you implement the following formula:

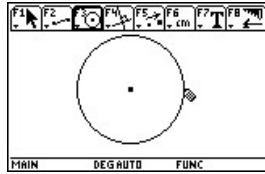
$$\frac{1}{2} aP \underline{\hspace{2cm}}$$

2. Draw your own polygon with 8 sides. Follow the same steps as above; use the same formula as in question 1 to solve for the area. Verify your answer by using the AREA option in Cabri. What do you notice:

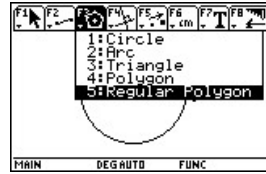
Step 2



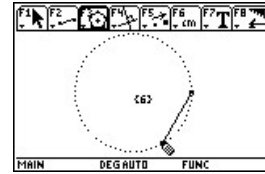
Create a circle with a Size of your choosing



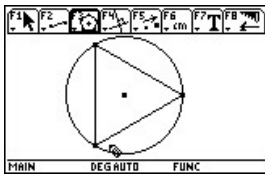
Press ENTER to fix the size



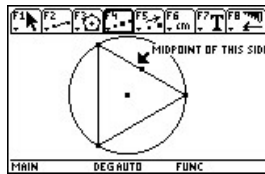
Press **F3** and choose REGULAR POLYGON



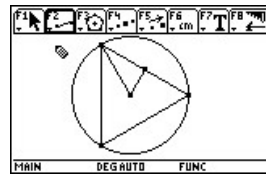
Create an inscribed polygon whose center is the same as your circle. Start by Drawing a three sided Polygon



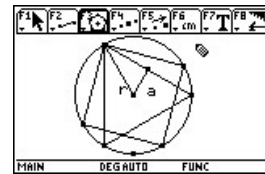
Press **ENTER** to fix the size of your 3 sided polygon



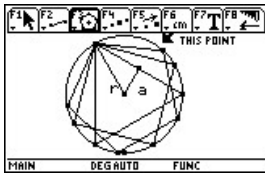
Locate the midpoint of one of the sides of the triangle.



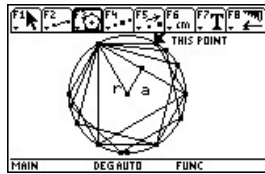
Draw a segment from the center of the circle/polygon to the midpoint. Draw another segment from the center to one of the points of the inscribed polygon.



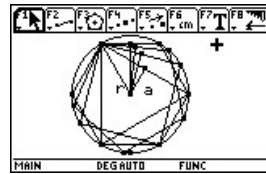
Repeat the previous steps and create a 4 sided polygon.



Once again repeat the previous steps and create a 5 sided Polygon.



Create a 6 sided polygon.



In each case draw the perpendicular segment joining the center of the polygon/circle to the midpoint of each Polygon.

3. What appears to happen if you were to create other polygons with n sides?

4. What is the limit that the perimeter of this P_n would approach? _____
5. What is the limit that a would approach? _____
6. If we are looking at a circle (a polygon with n sides) what can you replace in the above formula you looked at earlier ($\frac{1}{2}aP$) to solve for the area of the circle?
