5-5 Quadratic Equations

What You'll Learn

- To solve quadratic equations by factoring and by finding square roots
- To solve quadratic equations by graphing

... And Why

To solve equations involving art, as in Example 5

Check Skills You'll Need

GO for Help Lessons 5-2 and 5-4

Factor each expression.

1. $x^2 + 5x - 14$ **2.** $4x^2 - 12x$ **3.** $9x^2 - 16$

Graph each function.

4. $y = x^2 - 2x - 5$ **5.** $y = x^2 - 4x + 4$ **6.** $y = x^2 - 4x$

New Vocabulary

- standard form of a quadratic equation
- Zero-Product Property
- zero of a function

1 Solving by Factoring and Finding Square Roots

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$. You can solve some quadratic equations in standard form by factoring the quadratic expression and then using the Zero-Product Property.

Key Concepts

Property **Zero-Product Property**

If $ab = 0$, then $a = 0$ or $b = 0$.

Example If $(x + 3)(x - 7) = 0$, then $(x + 3) = 0$ or $(x - 7) = 0$.

1 EXAMPLE Solving by Factoring

Solve $2x^2 - 11x = -15$.

$2x^2 - 11x + 15 = 0$ **Write in standard form.**

$(x - 3)(2x - 5) = 0$ **Factor the quadratic expression.**

$x - 3 = 0$ or $2x - 5 = 0$ **Use the Zero-Product Property.**

$x = 3$ or $x = \frac{5}{2}$ **Solve for x.**

The solutions are 3 and $\frac{5}{2}$.

Check

$$2x^2 - 11x = -15$$
$$2(3)^2 - 11(3) \stackrel{?}{=} -15$$
$$18 - 33 \stackrel{?}{=} -15$$
$$-15 = -15 \checkmark$$

$$2x^2 - 11x = -15$$
$$2\left(\frac{5}{2}\right)^2 - 11\left(\frac{5}{2}\right) \stackrel{?}{=} -15$$
$$\frac{25}{2} - \frac{55}{2} \stackrel{?}{=} -15$$
$$-15 = -15 \checkmark$$

Quick Check

1 Solve each equation by factoring. Check your answers.

a. $x^2 + 7x = 18$ **b.** $2x^2 + 4x = 6$ **c.** $16x^2 = 8x$

You can solve an equation in the form $ax^2 = c$ by finding square roots.

2 EXAMPLE Solving by Finding Square Roots

Solve $5x^2 - 180 = 0$.

$5x^2 - 180 = 0$

$5x^2 = 180$	**Rewrite in the form $ax^2 = c$.**
$\frac{5x^2}{5} = \frac{180}{5}$	**Isolate x^2.**
$x^2 = 36$	**Simplify.**
$x = \pm 6$	**Find square roots.**

Quick Check 2 Solve each equation by finding square roots.

a. $4x^2 - 25 = 0$ **b.** $3x^2 = 24$ **c.** $x^2 - \frac{1}{4} = 0$

Real-World Connection

Careers Smoke jumpers are firefighters who parachute into areas near forest fires.

3 EXAMPLE Real-World Connection

Firefighting Smoke jumpers are in free fall from the time they jump out of a plane until they open their parachutes. The function $y = -16t^2 + 1600$ models a jumper's height y in feet at t seconds for a jump from 1600 ft. How long is a jumper in free fall if the parachute opens at 1000 ft?

$y = -16t^2 + 1600$	
$1000 = -16t^2 + 1600$	**Substitute 1000 for y.**
$-600 = -16t^2$	**Isolate t^2.**
$37.5 = t^2$	
$\pm 6.1 \approx t$	**Find square roots.**

The jumper is in free fall for about 6.1 seconds.

Check Is the answer reasonable? The negative number -6.1 is also a solution to the equation. However, since a negative value for time has no meaning in this case, only the positive solution is reasonable.

Quick Check 3 **a.** A smoke jumper jumps from 1400 ft. The function describing the height is $y = -16t^2 + 1400$. Using square roots, find the time during which the jumper is in free fall if the parachute opens at 1000 ft.

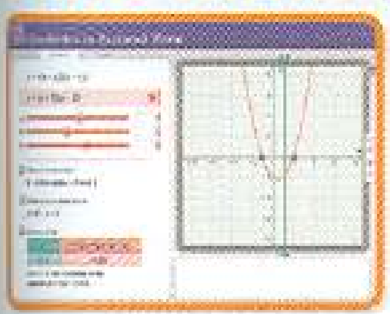
b. Solve the equation in part (a) by factoring. Which method do you prefer—using square roots or factoring? Explain.

2 Solving by Graphing

Not every quadratic equation can be solved by factoring or by finding square roots. You can solve $ax^2 + bx + c = 0$ by graphing $y_1 = ax^2 + bx + c$—its related quadratic function. The value of y_1 is 0 where the graph intersects the x-axis. Each x-intercept is a **zero of the function** and a root of the equation.

You can also solve $ax^2 + bx + c = 0$ by displaying values of $y_1 = ax^2 + bx + c$ in a table. Scroll through the table to find where y_1 changes sign, effectively where the graph crosses the x-axis. Then "zoom-in" on the y_1 values by adjusting TblStart and ΔTbl.

Online active math



For: Factored-Form Activity
Use: Interactive Textbook, 5-5

4 EXAMPLE Solving by Tables

Solve $x^2 - 5x + 2 = 0$.

Enter $y_1 = x^2 - 5x + 2$ in your graphing calculator. Use the TABLE window to see where y_1 changes sign. Change ΔTbl to 0.1 and look again for the sign change. Continue this tabular zoom-in and find one solution to be $x \approx 0.44$.

X	Y1
0	2
1	-2
2	-4
3	-4
4	-2
5	2
6	8

Y1 =X2-5X-2

X	Y1
.4	.16
.41	.1181
.42	.0764
.43	.0349
.44	-.0064
.45	-.0475
.46	-.0884

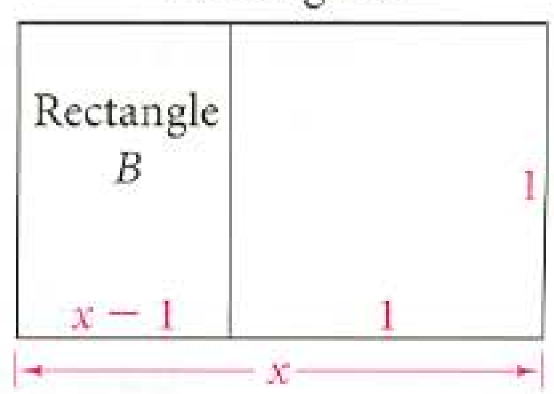
Y1 =X2-5X-2

Quick Check 4 In the first window above, the second sign change occurs between $x = 4$ and $x = 5$. Use tabular zoom-in to find the other solution to two decimal places.

5 EXAMPLE Solving by Graphing

Art Artists often use a golden rectangle in their work because forms based on it are visually pleasing. You can divide a golden rectangle into a square of side length one and a smaller rectangle that is similar to the original one. The ratio of the longer side to the shorter side of a golden rectangle is the golden ratio. Use the figure at the right to find the golden ratio.

Rectangle A



Vocabulary Tip

Two rectangles are similar if their corresponding sides are in proportion.

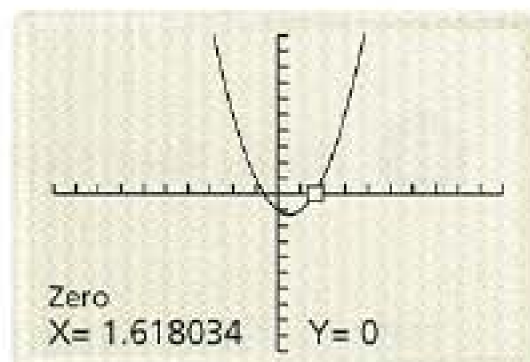
Relate $\dfrac{\text{longer side of } A}{\text{shorter side of } A} = \dfrac{\text{longer side of } B}{\text{shorter side of } B}$

Define Let x = longer side of rectangle A. Then $x - 1$ = shorter side of rectangle B.

Write $\dfrac{x}{1} = \dfrac{1}{x - 1}$

$x^2 - x = 1$ **Find cross-products.**

$x^2 - x - 1 = 0$ **Write in standard form.**



Graph the function $y_1 = x^2 - x - 1$. Use "zero" in the CALC feature to find the positive solution.

The ratio is about 1.62 : 1.

Quick Check 5 Solve each equation. When necessary, round to the nearest hundredth.

a. $x^2 - 2x = 4$ **b.** $x^2 + \frac{1}{2}x - \frac{1}{4} = 0$

Real-World Connection

Francisco José de Goya y Lucientes (1746–1828) was a Spanish painter. In *Portrait of a Man,* Goya placed the most significant elements of the painting within golden rectangles.