EXPLORATIONS

# CHAPTER 5

## Differential Calculus

It is assumed that the reader has worked through chapters one and two. In this chapter, various features of the TI-86 will be used to study some of the important ideas covered in a beginning differential calculus course. More precisely, we will consider problems dealing with limits, derivatives, and optimization.

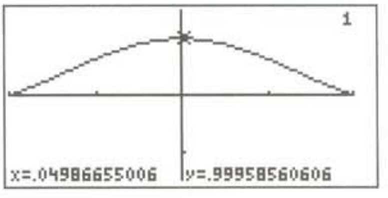
## §1 – Graphical and Numerical Investigation of $\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta}$

Perhaps the most famous limit in all of calculus is

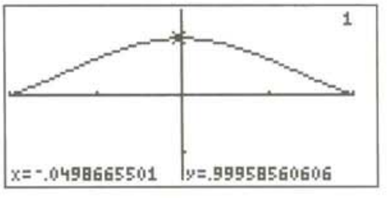
$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

where $\theta$ is in radians. With the aid of the TI-86, we can give compelling graphical and numerical evidence that the limit is 1. Before proceeding, check that the MODE settings are as in (1.1.5) of Chapter 1. In particular, make sure the calculator is in radian mode.

1. Then graph $y1 = (\sin x)/x$ in the window $[-\pi, \pi, 2] \times [-1.5, 1.5, 1]$ and trace on this graph near $x = 0$ as illustrated in (5.1.1) and (5.1.2) to see that it is reasonable to conjecture that the limit in question has value one.

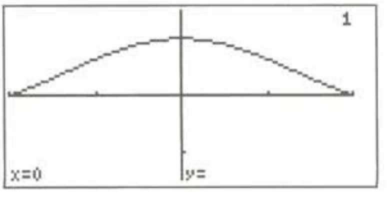



(5.1.1)




(5.1.2)

3. Also check, as was done in (5.1.3), that $y1$ is not defined at $x = 0$.

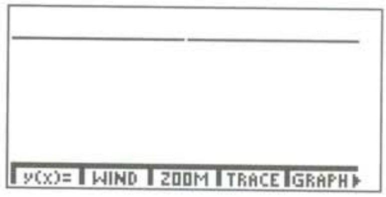



(5.1.3)

4. In (5.1.5) we show the graph of $y1$ in the window $[-0.05, 0.05, 0] \times [-1.5, 1.5, 1]$ with the axes turned off, as indicated in the GRAPH FORMAT settings shown in (5.1.4).

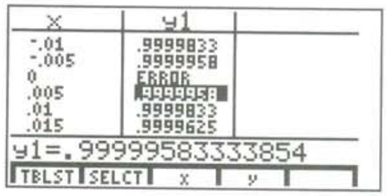

(5.1.4)

5. It is interesting to note the appropriate hole located at (0,1) on the graph displayed in (5.1.5) and that the graph gives confirmation of the important idea that "locally" curves are straight lines.




(5.1.5)

6. Finally, in (5.1.6), we have used TABLE with **ΔTbl** = 0.005, and **TblStart** = −0.01 for further numerical evidence that the limit in question is 1.




(5.1.6)

## §2 – Why Use Radian Measure?

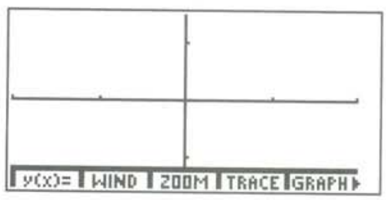
1. We assume now that the axes have been turned back on in the GRAPH FORMAT settings. It is an interesting problem to consider

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

where $\theta$ is in degrees rather than radians. Change the MODE settings as indicated in (5.2.1) so that the calculator is in degree mode.

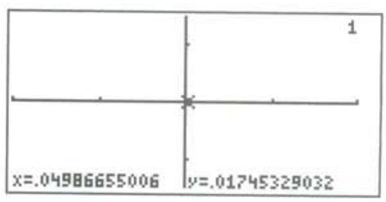

(5.2.1)

2. Figure (5.2.2) then shows the display when the TI-86 is asked to graph $y1$ in the window $[-\pi, \pi, \pi/2] \times [-1.5, 1.5, 1]$.



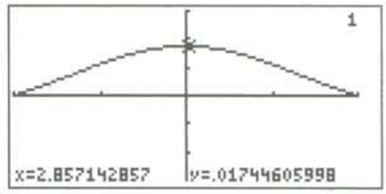

(5.2.2)

3. It appears that $y1$ has not been graphed, but selecting ⟨**TRACE**⟩ and moving the cursor one pixel unit to the right of zero gives (5.2.3).



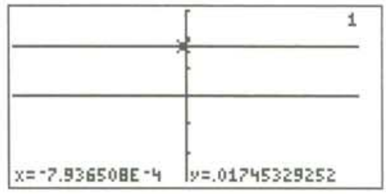

(5.2.3)

4. Consequently, the $y$ value here indicates that a change in the viewing window is needed. Figure (5.2.4) is the display obtained by graphing $y1$ in the viewing window $[-180, 180, 90] \times [-0.03, 0.03, 0.01]$ and using TRACE to move the cursor one pixel unit to the right of zero.



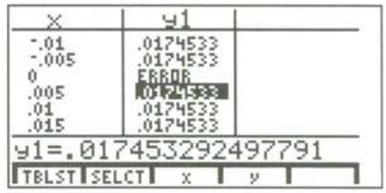

(5.2.4)

5. Similarly, in (5.2.5) we graphed $y1$ in the viewing window $[-0.05, 0.05, 0] \times [-0.03, 0.03, 0.01]$ and used TRACE to move the cursor one pixel unit to the left of zero.




(5.2.5)

6. In (5.2.6) we have used TABLE with **ΔTbl** = 0.005 and **TblStart** = −0.01 for further numerical evidence that the limit in question does exist and is approximately 0.0174533. It is an important result in calculus that this limit is in fact $\pi/180 \approx 0.01745329252$ because, as discussed in the paragraph below, it is a major reason why degree measure is not appropriate in the calculus.




(5.2.6)

The differentiation and integration formulas involving the trigonometric functions depend on the value of

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}.$$

For example, if we opted to use degree measure rather than radian measure in calculus, then the derivative of $\sin\theta$ would be $\left(\frac{\pi}{180}\right)\cos\theta$ rather than $\cos\theta$.

The resulting simplicity of the differentiation and integration formulas when using radian measure is a compelling reason for its use in calculus.

## §3 – Using seq to Investigate Limits

The **seq** command, accessed by pressing [2nd] [MATH] ⟨**MISC**⟩, can be put to good use in the investigation of limits. We illustrate by again considering the limit in §1; *i.e.*,

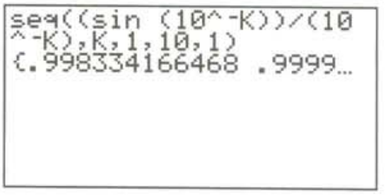
$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}, \text{ where } \theta \text{ is in radians.}$$

*(Note: The* **seq** *command is discussed in §1 of Chapter 6, and it is suggested that the reader take a short detour to that section before proceeding.)*

1a. With the calculator in radian mode, enter and execute the command

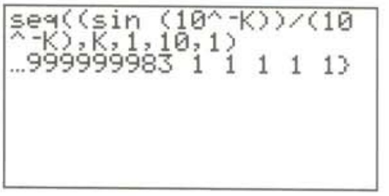
**seq**((sin(10^–K))/(10^–K), K, 1, 10, 1)

from the home screen to generate the sequence of ten values of $(\sin x)/x$ for values of $x$ which are negative integer powers of 10 ranging from $10^{-1}$ to $10^{-10}$.




(5.3.1)

Figure (5.3.1) is the resulting display after execution of the command, and (5.3.2) is obtained by using the right arrow key to scroll to the end of the sequence. The resulting sequence of numbers is consistent with the conclusion made in §1 that the limit in question has value 1.



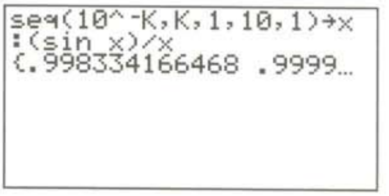

(5.3.2)

We can obtain the same numerical results shown in (5.3.1) and (5.3.2) by an *alternate procedure*, which takes advantage of the list arithmetic capabilities of the TI-86.

1b. From the home screen, type in the entry line

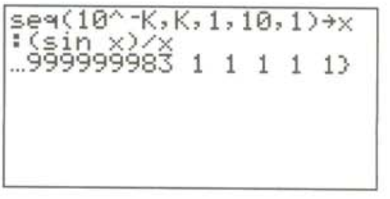
**seq**(10^–K, K, 1, 10, 1) → $x$: $(\sin x)/x$

as indicated in (5.3.3). Upon pressing [ENTER], the TI-86 executes the above entry line as follows. First it computes the list of ten values in which the ith element is $x_i = 10^{-i}$, and temporarily stores this list under the name $x$. Next for each of the ten $x_i$ values, it computes $(\sin x_i)/x_i$ and displays the new list in which the ith element is $(\sin x_i)/x_i$.




(5.3.3)

As before, you will need to scroll to see the entire list. From (5.3.3) and (5.3.4) we see that the results are exactly the same as obtained in (5.3.1) and (5.3.2).




(5.3.4)

There is yet another alternate means of obtaining this same output:

1c. From the home screen, execute the entry line

**seq**(10^–K, K, 1, 10, 1) → H: (sin $H$) / $H$.

This method is usually discouraged because you will be storing the resultant list in a variable $H$, which is much more permanent than the variable $x$. Of course, if you plan to use the list **seq**(10^–K, K, 1, 10, 1) many times, then you would probably prefer this method.

***Tip:*** *You should never store a list you plan to use often under the variable name $x$, since the TI-86 uses the variable name $x$ in many of its internal algorithms and as soon as it needs to use $x$ in such an algorithm then it will delete the list you have stored as $x$.*

If you use $x$ as the list variable name, you can also use the functions you have in the ⟨**y(x)=**⟩ graphing editor. For example, since $y1 = (\sin x)/x$, then the entry line **seq**(10^–K, K, 1, 10, 1) → $x$: $y1$ will yield the same output as shown in (5.3.1)–(5.3.4)

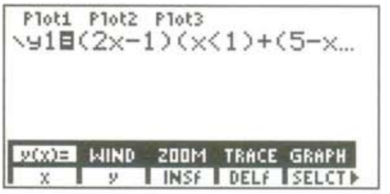
## §4 – A Piecewise Function Limit Problem

1. Piecewise functions also give rise to interesting and important limit problems. In (5.4.1) we have entered the piecewise function

$$f(x)=\begin{cases}2x-1, & \text{if } x<1\\ 5-x^2, & \text{if } x\geq 1\end{cases}$$

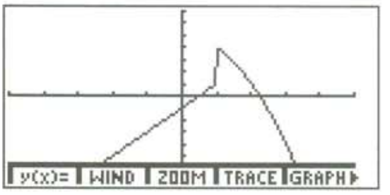
as $y1$ in the ⟨**y(x)=**⟩ graphing editor by setting

$$y1=(2x-1)(x<1)+(5-x^2)(x\geq 1).$$



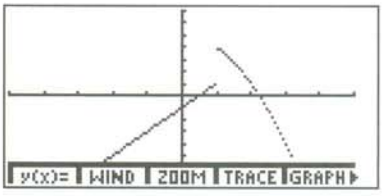

(5.4.1)

2. Figure (5.4.2) shows the graph of $y1$ in the viewing window $[-5, 5, 1] \times [-7, 7, 1]$ using the default Line style.



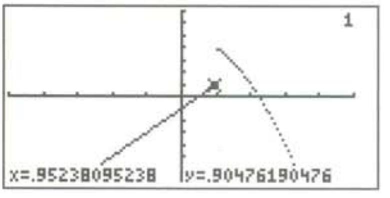

(5.4.2)

3. A better representation of the graph in this viewing window is shown in (5.4.3), where we have opted for the Dot style.

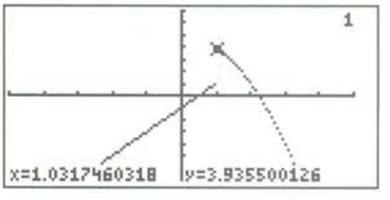



(5.4.3)

4. Next using TRACE, we obtain the results displayed in (5.4.4) and (5.4.5), which would seem to indicate that $\lim_{x\to 1^-} f(x)=1$ and $\lim_{x\to 1^+} f(x)=4$ so that $\lim_{x\to 1} f(x)$ does not exist.

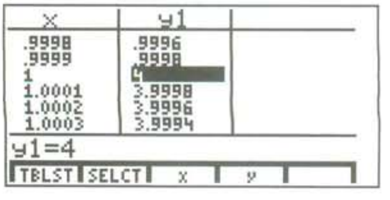



(5.4.4)




(5.4.5)

5. Using TABLE with **ΔTbl** = 0.0001 and **TblStart** = 0.9998, as illustrated in (5.4.6), gives more concrete evidence for these conclusions.




(5.4.6)

6. The most compelling evidence that $\lim_{x\to 1^+} f(x) = 4$ is given by executing the command shown in (5.4.7) and scrolling through the resulting list of ten values to obtain (5.4.8).

```
seq(1+10^-K,K,1,10,1)
→x:y1
{3.79 3.9799 3.99799…
```

(5.4.7)

```
seq(1+10^-K,K,1,10,1)
→x:y1
…999998 3.9999999998}
```

(5.4.8)

7. Similarly, (5.4.9)–(5.4.10) strongly indicate that $\lim_{x\to 1^-} f(x) = 1$.

```
seq(1-10^-K,K,1,10,1)
→x:y1
{.8 .98 .998 .9998 .…
```

(5.4.9)

```
seq(1-10^-K,K,1,10,1)
→x:y1
…9999998 .9999999998}
```

(5.4.10)

## §5 – Difference Quotients

A prominent feature in the study of calculus is the estimate of the value of a derivative using a difference quotient. This section illustrates a variety of ways to use the difference quotient $\dfrac{f(2+h)-f(2)}{h}$ to approximate the value of $f'(2)$ for $f(x) = x^{\sin x}$.

1. From the ⟨**y(x)=**⟩ graphing editor, enter the functions

   $y1 = x\hat{}\sin x$ and $y2 = (y1(2+x) - y1(2))\,/\,x$

   as in (5.5.1), so that $y1$ is the function $f$ and $y2$ is the corresponding difference quotient with $x$ serving the role of $h$.

```
 Plot1 Plot2 Plot3
\y1=x^sin x
\y2=(y1(2+x)-y1(2))/x

y(x)= WIND  ZOOM  TRACE GRAPH
  x     y   INSf  DELf  SELCT▸
```

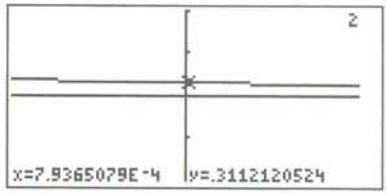
(5.5.1)

2. In (5.5.2) we have returned to the home screen and used $y2$ to compute, in two different ways, $\dfrac{f(2+h)-f(2)}{h}$ for an $h$ value of .005, obtaining 0.30628 as an estimate for $f'(2)$.

```
.005→x:y2
          .30628097038
y2(.005)
          .30628097038
```

(5.5.2)

3. In (5.5.3) we see the result of graphing $y2$ in the viewing window $[-0.05, 0.05, 0] \times [-2, 2, 1]$ and tracing near $x = 0$ to obtain 0.31121 as an estimate for $f'(2)$.

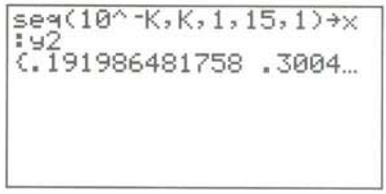



(5.5.3)

4. The method developed in §3 can be used to generate a list of difference quotients with decreasing values of $h$. Specifically, from the home screen, enter and execute the command

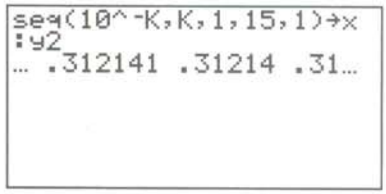
   **seq**(10^–K, K, 1, 15, 1) → $x$: $y2$

   to obtain (5.5.4).



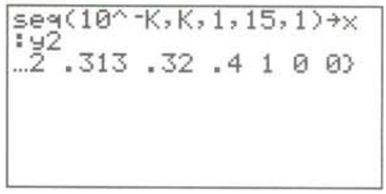

(5.5.4)

5. Using the right arrow to scroll through this list of numbers seems to confirm our estimate for $f'(2)$. See (5.5.5).




(5.5.5)

6. However, scrolling to the end of the resulting list of numbers (as is done in (5.5.6)) would seem to indicate that the value of $f'(2)$ is 0; but this, of course, is not the case.



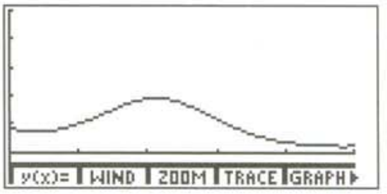

(5.5.6)

The problem is that we have exceeded the numerical capabilities of the TI-86. The values of $10^{-K}$ for $K$ greater than 13 are sufficiently small enough that the calculator thinks the numerator in our difference quotient is zero, and hence, the difference quotient has value zero. You should also note that the first element of the list has 12 digits, the second element of the list has 11 digits, the third element of the list has 10 digits, etc. Thus, it is clear that the calculator is steadily losing significant digits as it computes the difference quotient for ever smaller increment values. Thus, it is imperative on our part that we take into account the numerical capabilities of the calculator when using it to form difference quotients to estimate derivatives. As we will see in §6, the best estimate of $f'(2)$ in the current 15 element list is the 7th element 0.312141.

## §6 – Using ⟨TANLN⟩ in the GRAPH MATH Menu

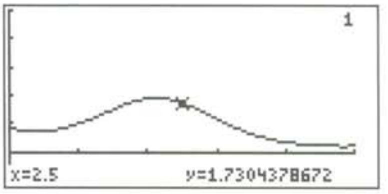
In this section we continue our investigation of $f'(2)$ for the function $f(x) = x^{\sin x}$ by using a feature of the GRAPH MATH menu that allows us to both draw the tangent line at the point $(2, f(2))$ and to obtain $f'(2)$.

1. Graph $y1 = x\wedge\sin x$ in the viewing window $[0, 5, 1] \times [-1, 5, 1]$ to obtain (5.6.1).



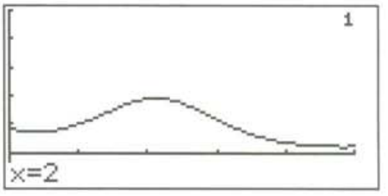

(5.6.1)

2. Press [MORE] ⟨**MATH**⟩ [MORE] [MORE] and select ⟨**TANLN**⟩ to obtain (5.6.2).



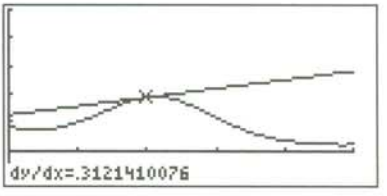

(5.6.2)

3. Type in 2 to obtain (5.6.3), and then press [ENTER] to obtain (5.6.4).




(5.6.3)

We see that the calculator's estimate for $f'(2)$ is 0.3121410076, an estimate which is, in fact, correct to 11 digit accuracy.




(5.6.4)

*Incidentally, the tangent line is indeed just a drawing.* To see this, press [EXIT], select ⟨**DRAW**⟩, press [MORE] [MORE], and select ⟨**CLDRW**⟩. The TI-86 then erases the tangent line and regraphs $y1$.

## §7 – Introduction of the Derivative Commands

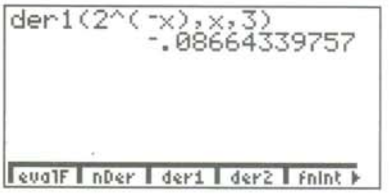
The TI-86 also has built-in commands for computing derivatives. The commands **der1** and **der2** use the rules of differentiation to compute the first and second derivatives of a function to 12-digit accuracy, and the command **nDer** is used to estimate a derivative using a certain difference quotient. The three commands are accessed via the CALC menu by pressing [2nd] [CALC]. The **der1** command has the syntax

**der1**(*expression*, *variable name*, *value*),

as do the commands **der2** and **nDer**. The *value* argument is optional. If omitted, the current value of the variable is used when the command is executed.

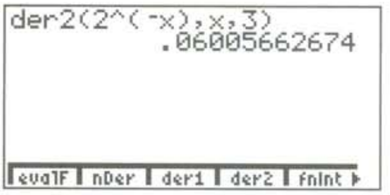
 

1. The built-in commands **der1** and **der2** work on most of the standard functions encountered in precalculus or beginning calculus; but will not work, for example, on a function defined as an integral as will be encountered in the next chapter. In (5.7.1) we have the calculator find the value of the derivative of $2^{-x}$ at $x = 3$ to 12-digit accuracy.




(5.7.1)

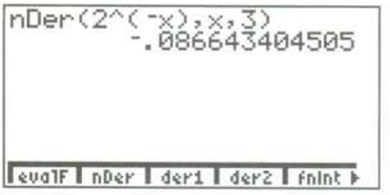
2. Similarly, (5.7.2) gives the value of the second derivative of $2^{-x}$ at $x = 3$ to 12-digit accuracy.




(5.7.2)

The built-in command **nDer** uses a symmetric difference quotient of the form $\dfrac{f(a+\delta)-f(a-\delta)}{2\delta}$ to estimate the derivative of $f(x)$ at $x = a$, where the current value of $\delta$ can be accessed by pressing [2nd] [MEM] ⟨**TOL**⟩. The default value of $\delta$ is $\delta = 0.001$. This default value was used to obtain the result in (5.7.3).

3. Figure (5.7.3) gives the numerical estimate for the derivative of $2^{-x}$ at $x = 3$.




(5.7.3)

It is important to understand how the **der1** and **der2** commands work. When the TI-86 executes the command

$$\textbf{der1}(2\text{^}(-x), x, 3)$$

it knows that the derivative of $2^{-x}$ is precisely $-2^{-x}\ln 2$, and so it computes this expression at $x = 3$ to get the numerical value of the derivative. Similarly, when the TI-86 executes the command

$$\textbf{der2}(2\text{^}(-x), x, 3)$$

it knows that the second derivative of $2^{-x}$ is precisely $2^{-x}(\ln 2)^2$, and so it computes this expression at $x = 3$ to get the numerical value of the second derivative.

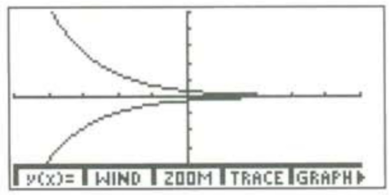
## §8 – Using the Derivative Command in Graphing

The built-in commands **der1**, **der2**, and **nDer** can also be used in the definition of functions in the ⟨**y(x)=**⟩ graphing editor.

1. In (5.8.1) we enter $y1 = 2^{-x}$ and $y2 = \mathbf{der1}(y1, x)$. (Note in $y2$, we did not enter the optional *value* argument since we will be using the current value of $x$ generated by the graphing algorithm. It would have been permissible to use $y2 = \mathbf{der1}(y1, x, x)$. In this setting the two $x$'s are being used for different purposes. The first $x$ denotes the variable, and the second $x$ denotes the current value of the variable.)

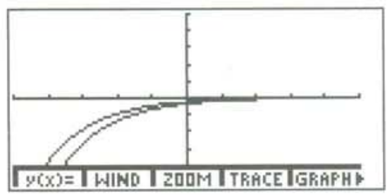

(5.8.1)

2. Figure (5.8.2) shows the graphs of these two functions in the window $[-5, 5, 1] \times [-15, 15, 3]$. From the graph in (5.8.2), it might be reasonable to conjecture that $y2 = -y1$.



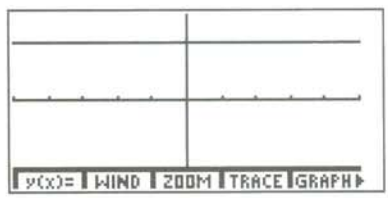

(5.8.2)

3. To check out this conjecture, let $y3 = -y1$. Deselect $y1$ for graphing, and graph $y2$ and $y3$ to see that the derivative of $2^{-x}$ is not $-2^{-x}$. See (5.8.3).




(5.8.3)

4. Finally, let $y4 = -y2/y1$, and graph only $y4$ in the viewing window $[-5, 5, 1] \times [-1, 1, 0]$ to get (5.8.4). This graph would seem to indicate that the derivative of $2^{-x}$ is a constant times $2^{-x}$.




(5.8.4)

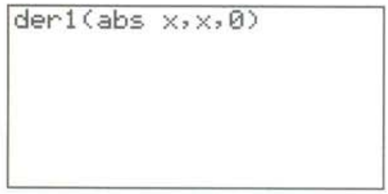
5. From (5.8.4), select ⟨**TRACE**⟩, and check that the $y$ coordinates are all 0.69314718056. Exit to the home screen, and then press [2nd] [$e^x$] [2nd] [ALPHA] **y** [ENTER] to get (5.8.5). Consequently, we have strong evidence that the derivative of $2^{-x}$ is $-2^{-x} \ln 2$.


(5.8.5)

## §9 – A Limitation of nDer

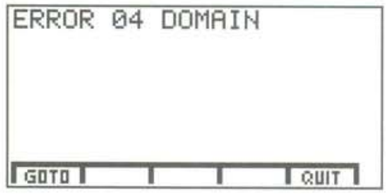
The absolute value function is the most prominently mentioned example of a function which fails to be differentiable at a point. So **der1**(*abs* $x$, $x$, 0) and **nDer**(*abs* $x$, $x$, 0) are two important calculations to try with the TI-86.

1. Figures (5.9.1) and (5.9.2) are reassuring, but (5.9.3) should give reason for concern.



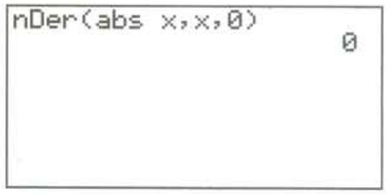

(5.9.1)

2. The built-in command **der1** knows that the absolute value function is not differentiable at $x = 0$, as (5.9.2) indicates.




(5.9.2)

3. But (5.9.3) shows the built-in command **nDer** does not know that the absolute value function is not differentiable at $x = 0$. This is due to the fact that **nDer** computes a symmetric difference quotient centered at the place where it has been asked to compute the value of the derivative. (Refer to §7.) Since the absolute value function is symmetric about the $y$-axis, **nDer**(*abs* $x$, $x$, 0) will produce zero.




(5.9.3)

## §10 – A Limitation of der1

In this section, we compare the graphs of

$$y1 = \sin x + \ln x,\ y2 = \mathbf{der1}(y1, x),$$

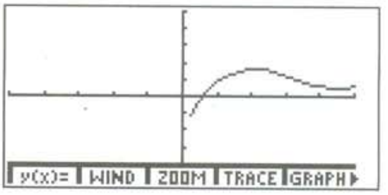
$$y3 = \mathbf{nDer}(y1, x), \text{ and } y4 = \cos x + \frac{1}{x}$$

in the viewing window $[-5, 5, 1] \times [-5, 5, 1]$.

1. Figure (5.10.1) shows the four functions entered in the $\langle$**y(x)=**$\rangle$ graphing editor. To speed up the graphing process, set **xRes** = 3 for all graphs in this section. The loss in resolution is minimal.

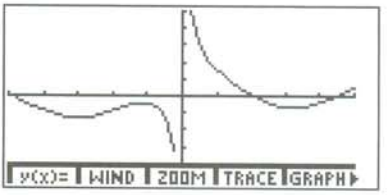

(5.10.1)

2. The graph of $y1$ is shown in (5.10.2). This graph correctly shows that $y1$ is only defined for $x$ greater than zero (because $\ln x$ is only defined for positive $x$).



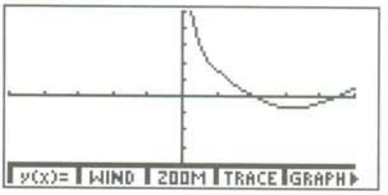

(5.10.2)

3. However, the graph of $y2$ shown in (5.10.3) incorrectly exists for both positive and negative values of $x$. In fact, the graph of $y2$ is exactly the same as the graph of $y4$. So the **der1** command knows the form of the exact derivative but does not impose on the derivative the restricted domain of the original function.




(5.10.3)

4. The graph of $y3$ is shown in (5.10.4). Note that this graph of $y3$ correctly exists for only positive values of $x$. Since **nDer**($y1$, $x$) relies on values from $y1$, the restricted domain of $y1$ is correctly imposed on $y3$. Consequently, there may be times when the use of **nDer** would be more appropriate than the use of **der1**, even in a case when the calculator knows the form of the exact derivative.




(5.10.4)

## §11 – An Optimization Problem

In Chapter 2 we saw the use of the TI-86's built-in **fMax** and **fMin** commands to find relative extrema. In this section, we will link some of those ideas to solving optimization problems in calculus. In particular, to solve a certain optimization problem, it is required to find the value of $x$ in the closed interval [0, 2000], as well as the corresponding $T(x)$ value, for which the function

$$T(x) = \frac{x}{6} + \frac{\sqrt{(2000-x)^2 + 600^2}}{4}$$

is minimized. One could use exact calculus techniques and time consuming tedious algebra to find that

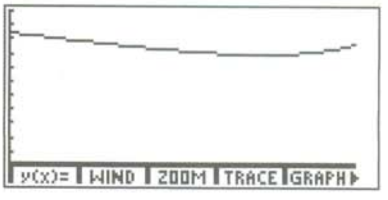
$$x = 2000 - \frac{1200}{\sqrt{5}} \text{ and } T(x) = 50\sqrt{5} + \frac{1000}{3}$$

but a graphical and numerical solution is much simpler.

1. Enter the function

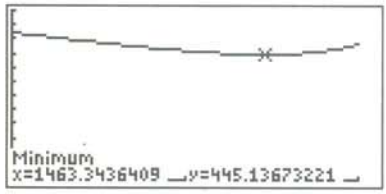
   $$y1 = x/6 + \sqrt{((2000-x)^2 + 600^2)}/4$$

   and graph in the window $[0, 2000, 500] \times [0, 600, 50]$ with **xRes** = 1 to obtain (5.11.1).




(5.11.1)

2. Press [MORE] ⟨**MATH**⟩ ⟨**FMIN**⟩. Enter 0 for **Left Bound**, 2000 for **Right Bound**, omit a value for **Guess**, and press [ENTER] to obtain (5.11.2). From (5.11.2) we conclude that the function in question is minimized when $x$ is about 1463.34 and the minimum value is about 445.14. According to calculus theory, the derivative of $y1$ at this minimizing $x$ value should be 0, and we expect the second derivative of $y1$ for this $x$ to be positive since the graph is concave up there.




(5.11.2)

3. We verify that this is the case. From (5.11.2), press [2nd] [QUIT] to return to the home screen. Check, as shown in (5.11.3), that the current value of $x$ is still the minimizing value of $x$ for the problem at hand.

```
x
                 1463.34364091
```

(5.11.3)

4. Then execute the commands **der1**($y1$, $x$), and **der2**($y1$, $x$) as shown in (5.11.4) to evaluate the indicated derivatives at the current value of $x$.

```
x
                 1463.34364091
der1(y1,x)
                    -7.67574E-9
der2(y1,x)
              1.72536090304E-4
```

(5.11.4)

## Exercises

1. Consider $y1 = 20x^3 + 2x^2 - 10$.

   (a) Graph $y1$ in the viewing window $[-2, 2, 1] \times [-30, 20, 5]$.

   (b) Study the roots of the derivative of $y1$ to find and classify the local extrema of $y1$.

2. One of the important limits in mathematics is $\lim_{x \to 0} (1 + \alpha x)^{1/x} = e^{\alpha}$.

   (a) Graphically verify this limit result for the case $\alpha = \dfrac{1}{2}$ by graphing $y1 = e^\wedge(1/2)$ and $y2 = (1 + x/2)^\wedge(1/x)$ together in the viewing window $[-1, 1, 0] \times [0, 3, 0]$ and using the ⟨**TRACE**⟩ and ⟨**ZIN**⟩ options appropriately.

   (b) After completing (a), use the TABLE feature of the TI-86 to give numerical evidence of the value of this limit.

3. Consider $f(x) = \sin^2 x$. It is a routine problem for beginning calculus students to see that $f'(x) = 2\sin x \cos x = \sin(2x)$. Let $y1 = \sin x$ and $y2 = (y1)^2$, then let $y3$ = **der1**($y2$, $x$), and $y4 = \sin(2x)$. Use the Animate graph style for $y4$, and graph $y3$ and $y4$ on $[-2\pi, 2\pi, \pi/2] \times [-2, 2, 1]$. Comment on the resulting graph activity.

4. Use the TI-86 to graph the derivative in order to find the points on the graph of $y = x^3 + \frac{3}{2}x^2 - 8x + 2$ where the slope is equal to 10.

5. Analyze $\lim\limits_{\theta \to 0} \frac{\sin(3\theta)}{\theta}$ in a manner similar to what was done in §3.

6. Find the minimum value of $T(x) = -\frac{x}{8} + \frac{\sqrt{(2000 - x)^2 + 600^2}}{2}$ on the interval [0, 4000].

7. A familiar property of exponential functions is that the ratios of function values for equally spaced domain values are constant. Use the TABLE feature of the TI-86 to give numerical evidence that **der1**$\left(3^x,\ x\right)$ is an exponential function. Then go to the home screen, and give numerical evidence that the derivative of $3^x$ is $3^x \ln 3$.