Solving Systems involving Quadratic Equations

Conic Sections

Contributed by Jill Gough

Plot the graphs of the following systems of equations and determine the number of real solutions of the system of equations. Then find the solution of the system algebraically.

1. Example: $y^2 = 4 \cdot x$ and $x + y = 3$.

a. Solve each equation for y.

$\text{solve}\left(y^2 = 4 \cdot x, y\right)$

$y = 2\sqrt{x}$ and $x \geq 0$ or $y = -2\sqrt{x}$ and $x \geq 0$

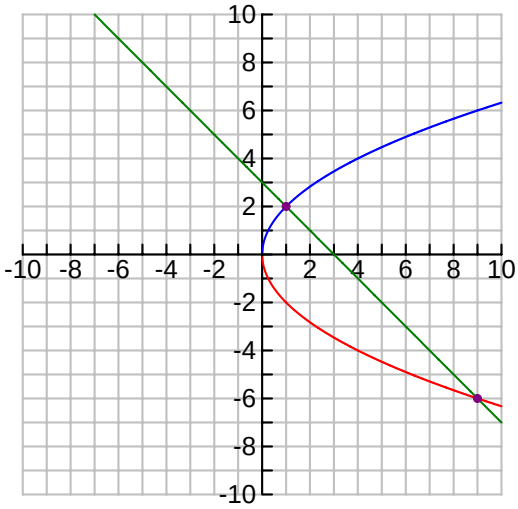
$\text{solve}(x + y = 3, y)$ $y = 3 - x$

b. Graph the system of equations by selecting Graph and entering the functions to plot the system.

c. Find the points in intersections by selecting Calculate Intersection.

d. Copy the solutions and paste them into the document.
The solutions to this sytem are (1., 2.) and (9., -6.).

e. Verify these solutions algebraically by the method of substitution.



$\text{solve}(x + y = 3, y)$ $y = 3 - x$ Solve for y in one equation.

$y^2 = 4 \cdot x \mid y = 3 - x$ $(x - 3)^2 = 4 \cdot x$ Substitute y into the other equation in the system.

$\text{solve}(\text{ans}, x)$ $x = 9$ or $x = 1$ Solve for x.

$y = 3 - x \mid x = 9$ $y = -6$

$y = 3 - x \mid x = 1$ $y = 2$ Substitute each x into the y-equation.

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1. $x^2 + 4 \cdot y^2 = 16$ and $x - y = 4$.

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2. $x^2 + y^2 = 25$ and $x - 2 \cdot y = -2$.

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3. $x^2 - 2 \cdot y^2 = 4$ and $x + 2 \cdot y = 2$.

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4. $x^2 - y = 1$ and $x - 2 \cdot y = -1$.

In the earlier exercises, the solution to a system of a line and a circle, an ellipse, or a hyperbola was found. Now investigate finding the solution of a system of two conics.

2. Example: $2\cdot x^2 - 3\cdot y^2 = 6$ and $6\cdot x^2 + y^2 = 58$

a. Solve each equation for y.

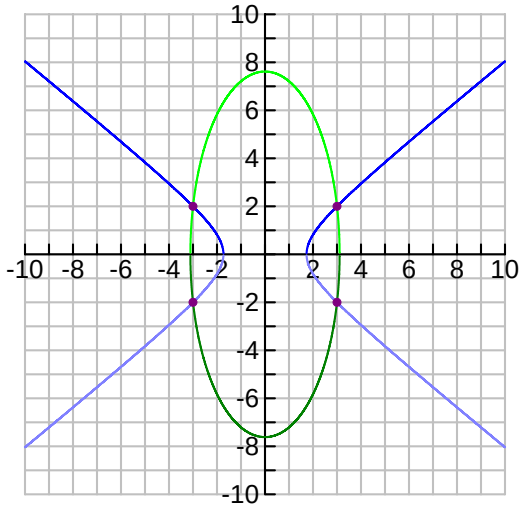
$\mathbf{solve}\left(6\cdot x^2 + y^2 = 58,\ y\right)$

$y = -\sqrt{-2\cdot\left(3\cdot x^2 - 29\right)}$ and $3\cdot x^2 - 29 \le 0$ or $y = \sqrt{-2\cdot\left(3\cdot x^2 - 29\right)}$ and $3\cdot x^2 - 29 \le 0$

$\mathbf{solve}\left(2\cdot x^2 - 3\cdot y^2 = 6,\ y\right)$

$y = \frac{\sqrt{6\cdot\left(x^2 - 3\right)}}{3}$ and $x^2 - 3 \ge 0$ or $y = \frac{-\sqrt{6\cdot\left(x^2 - 3\right)}}{3}$ and $x^2 - 3 \ge 0$

b. Graph the system of equations and find the points of intersection



The solutions to this system are (-3., 2.), (-3., -2.), (3., 2.), and (3., -2.).

c. Verify the system algebraically.

$\mathrm{solve}\left(2\cdot x^2 - 3\cdot y^2 = 6,\ y\right)$

$y = \frac{\sqrt{6\cdot\left(x^2 - 3\right)}}{3}$ and $x^2 - 3 \ge 0$ or $y = \frac{-\sqrt{6\cdot\left(x^2 - 3\right)}}{3}$ and $x^2 - 3 \ge 0$

$6\cdot x^2 + y^2 = 58 \mid y = \frac{\sqrt{6\cdot\left(x^2 - 3\right)}}{3}$ $\qquad \frac{20\cdot x^2}{3} - 2 = 58$

$\mathrm{solve}\left(\mathrm{ans},\ x\right)$ $\qquad x = 3$ or $x = -3$

$y = \frac{\sqrt{6\cdot\left(x^2 - 3\right)}}{3} \mid x = 3$ $\qquad y = \frac{\sqrt{6\cdot\left(x^2 - 3\right)}}{3} \mid x = -3$

$y = 2$ $\qquad y = 2$

$y = \frac{-\left(\sqrt{6\cdot\left(x^2 - 3\right)}\right)}{3} \mid x = 3$ $\qquad y = \frac{-\left(\sqrt{6\cdot\left(x^2 - 3\right)}\right)}{3} \mid x = -3$

$y = -2$ $\qquad y = -2$

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1. $x^2 + y^2 = 4$ and $x^2 + 2 \cdot y = 4$.

2. $x^2 + y^2 = 25$ and $x^2 + 4 \cdot y^2 = 64$.

3. $x^2 - y^2 = 9$ and $y^2 - 2 \cdot x = 6$.

4. $3 \cdot x^2 + 2 \cdot y^2 = 59$ and $2 \cdot x^2 + y^2 = 34$.