# Transformations 2
## Dilations, a Graphical Approach

**Student Activity**

7  8  9  **10  11  12**

TI-Nspire™    Activity    Student    1 hour

## Introduction

The Transformations activities in this series involve using multiple functionalities of TI-Nspire to explore the concept of transformations of the plane – translations, dilations, reflections and combinations of these – and their effect on functions. The aim of this approach is to develop a sound understanding by linking graphical and algebraic representations of transformation problems and their solution and making sense of the algebra through visualisation of the relationship between the graph of the original function and the graph of the transformed function.

**This is the second transformation activity in the series, and it focuses on dilations.**

## Exploration 2. Dilations

In this part of the activity, you will explore the image of the line $3x - 2y = 6$ under the set of transformations below:

• translation of factor 1.5 **from** the y-axis (i.e. factor $\frac{3}{2}$ **parallel** to the x-axis)

• translation of factor 0.5 **from** the x-axis (i.e. factor $\frac{1}{2}$ **parallel** to the y-axis)

A variety of approaches will be used to determine the equation of the transformed function.

Please refer to the TI-Nspire document 'Transformation2'

Open the TI-Nspire document 'Transformation2' and select page 2.1. On page 2.1, $P$ is a point on the graph of the functional relationship with equation $3x - 2y = 6$. The equation of the graph has been entered as a relation, **rel1**$(x, y)$ (i.e. 'Relation 1').

The coordinates of the points shown have been stored as the variables, $P(xo, yo)$ and $P'(xi, yi)$.
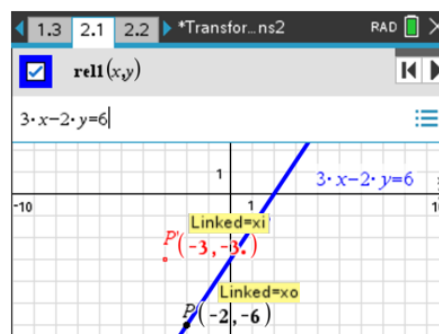
An easy way to enter (or edit) the equation of a graph as a relation:
menu > Graph Entry/Edit > Relation.

**Making $P'$ the image of $P$ under the transformation**    For $P'$ to be the image of $P$, it needs to undergo the dilations: 'factor 1.5 parallel to x-axis', therefore: $xi$ assigned the value $1.5xo$ and 'factor 0.5 parallel to the y-axis', therefore $yi$ assigned the value $0.5yo$.

What to do – select page 2.2. This page is a Notes application with two Maths Boxes. Click (⊡) the Maths Box so that a red dotted border appears, then input

- In the top box: $xi := 1.5 \cdot xo$ and press enter.

- In the bottom box: $yi := 0.5 \cdot yo$ and press enter.

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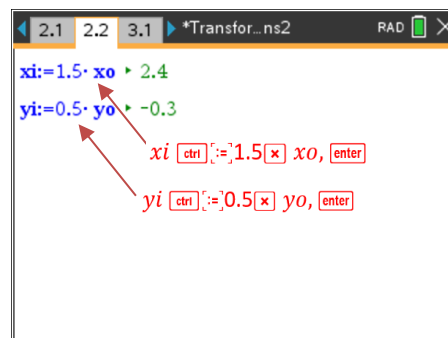
The coordinates of $P'$ are now $(1.5xo, 0.5yo)$. When point $P$ is moved along the graph on page 2.1, $P'$ will remain the image of $P$ under the transformation.

**Graph and equation of the transformed function**

$P$ can be animated using the control buttons ▮◀ ▶ernatively, $P$ can be grabbed (ctrl+⌖) and dragged along the graph. Press esc to release the 'grabbed' $P$.
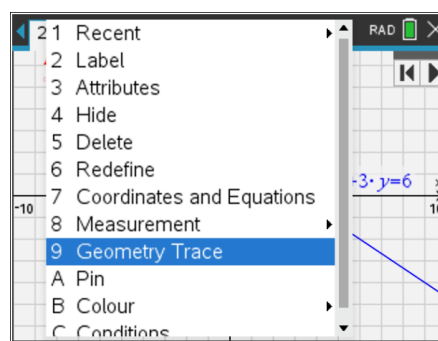
### Question 1
On page 2.1, use the control buttons to move point $P$, and observe the motion of point $P'$.
Describe the relationship between the positions of $P$ and $P'$ as the points move.
Sample response. $P'$ moves in a linear path, but with a gradient that is less steep than that of the graph of the original function.

### Path of $P'$
Return $P$ to the original position by clicking the left control buttom. The 'Ge ▮◀ etry Trace' tool will be used to obtain an outline of the path of $P'$.

What to do. To activate this tool, move the cursor to the point $P'$ and open the context menu by pressing ctrl+menu. Select 'Geometry Trace', then animate $P$. A trace of $P'$ will be visible. When $P'$ is outside the window settings, return $P'$ to the starting position, then exit the tool by pressing esc.

### Question 2
a.  Use the trace of $P'$ to determine the equation of the transformed function.
b.  Test whether your equation in part b. is correct by graphing your equation on page 2.1, using the same set of axes as the graph of the original function. You should enter your equation as 'Relation 2' (menu > Graph Entry/Edit > Relation will ensure 'Relation' is selected. (The graph of your equation should contain all points traced by $P'$).
c.  Find (i) the gradient and (ii) the coordinates of the axes intercepts of the graph of the transformed function.
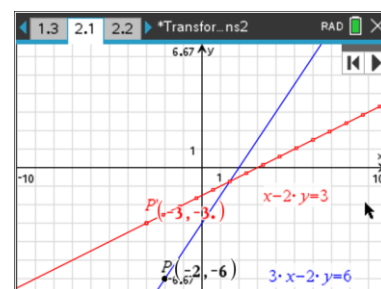
a.  $x - 2y = 3$  or  $2x - 4y = 6$  or $y = \frac{x}{2} - \frac{3}{2}$ (or equivalent)

b.  See screenshot on the right

c.  i. gradient $= \frac{1}{2}$ (by using $\frac{rise}{run}$ on graph, or rearranging eqn.)

   ii. When $y = 0, x = 3$ .      When $x = 0, y = -\frac{3}{2}$.      x-

   intercept (3,0)        y-intercept $\left(0, -\frac{3}{2}\right)$

### Question 3
What is the relationship between the gradient and axes intercepts of the graph of the transformed function and the gradient and axes intercepts of the graph of the original function?
Sample response. The gradient of the graph of the original function is $\frac{3}{2}$, whereas the graph of the transformed function has gradient of $\frac{1}{2}$. The dilations also change the axes intercepts as per the calculations in Question 2.c.ii. above.

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**Relationship between equation of the transformed and original functions**

Now consider how the equation of the transformed function could have been obtained from the equation of the original function, $3x - 2y = 6$.
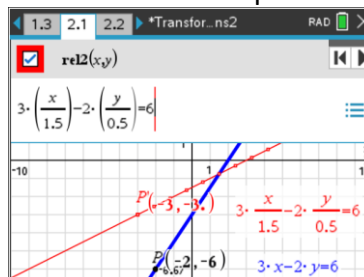
## Question 4

Which one of the following is the correct equation for the transformed function? Test your answer by entering the equation as 'Relation 2'. The graph should contain all points on the trace of $P'$.

A.  $3(1.5x) - 2(0.5y) = 6$

B.  $3(x + 1.5) - 2(y + 0.5) = 6$

**C.  $3\left(\dfrac{x}{1.5}\right) - 2(\dfrac{y}{0.5}) = 6$   (Correct equation)**

D.  $3\left(\dfrac{x}{1.5}\right) - 2(0.5y) = 6$

### Explaining the equation of the transformed function

We saw that the point $P(x, y)$ maps to point $P'(1.5x, 0.5y)$ under this transformation. That is, if $P'$ has coordinates $(x', y')$, then

$$x' = 1.5x \quad (eqn1)$$
$$y' = 0.5y \quad (eqn2)$$

## Question 5

Use the equations in blue above to explain why the graph of the transformed function - which contains all points on the trace of $P'$ - can be expressed as $3\left(\dfrac{x}{1.5}\right) - 2\left(\dfrac{y}{0.5}\right) = 6$.

Rearrange *eqn1* to $x = \dfrac{x'}{1.5}$ and *eqn2* to $y = \dfrac{y'}{0.5}$. Substitute in $3x - 2y = 6$.

$3\left(\dfrac{x'}{1.5}\right) - 2\left(\dfrac{y'}{0.5}\right) = 6$. Hence transformed function is $3\left(\dfrac{x}{1.5}\right) - 2\left(\dfrac{y}{0.5}\right) = 6$.

### Question 6

On page 2.1, navigate to the Graph Entry ([menu] > Graph Entry/Edit > Relation). Turn off 'Relation 2' and for 'Relation 3' input $\text{rel1}\left(\dfrac{x}{1.5}, \dfrac{y}{0.5}\right)$, as shown on the right.

a.  What is the relationship between the graph of 'Relation 3' and the trace of $P'$ ?  'Relation 3' should contain all points on the trace of $P'$.
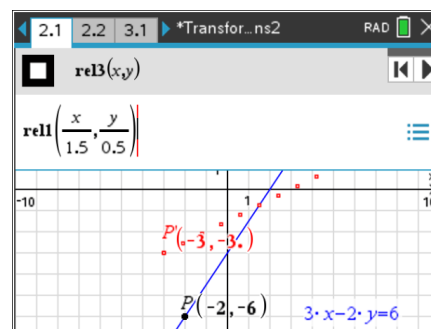
b. Confirm that the equation of 'Relation 3' is equivalent to the equation in Question 6 above.

$\text{rel1}(x, y)$ is     $3x - 2y = 6$  therefore

$\text{rel1}\left(\dfrac{x}{1.5}, \dfrac{y}{0.5}\right)$ is  $3\left(\dfrac{x}{1.5}\right) - 2\left(\dfrac{y}{0.5}\right) = 6$

**Generalising the equation of transformed functions involving translations**

You will now consider the form of the equation of the transformed function for any functional relationship, whether linear or non-linear.

You will explore the equations of the transformed functions for all the cases in **Table 1** below.

**What to do. Step 1. Erase old Geometry Trace.** Navigate to page 3.1. Erase the old geometry trace before starting the next problem from Table 1.

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Erasing the 'Geometry Trace': move the cursor to a blank part of the graphing window and open the context menu (⟨ctrl⟩+⟨menu⟩).  Select 'Erase Geometry Trace'.

**Step 2. Edit Equation.** On page 3.1, replace the equation of 'Relation 1' with the equation of the original function for the next problem in Table 1.

For example, second problem from Table 1: Edit 'Relation 1' to $y = \sqrt{x + 5} - 2$ , as shown on the right.
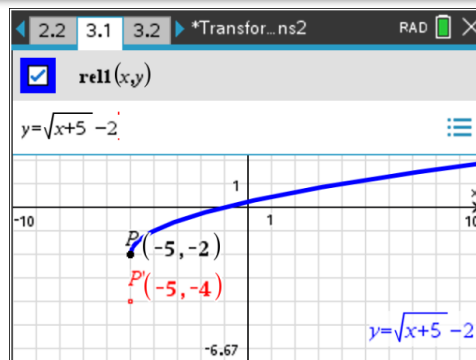


**Step 3. Edit dilations.** On page 3.2, edit the Maths Boxes to match the set of translations listed for that problem in Table 1.

**Step 4. Obtain a trace of $P'$ .**

Return $P$ to the original position by clicking the left control button  .

Activate the Geometry Trace tool: move the cursor to the point $P'$ and open the context menu by pressing ⟨ctrl⟩+⟨menu⟩. Select 'Geometry Trace', then animate $P$ by clicking the right control button  . A trace of $P'$ will be visible. When $P'$ is outside the window settings, return $P'$ to the starting position, then exit the tool by pressing ⟨esc⟩.



Selecting the correct point when $P$ and $P'$ are close together. Move the cursor close to the point you wish to select. Press the ⟨tab⟩ key until the desired point is displayed. (Repeated use of the <tab> key will cycle through all nearby points and other object).

**Step 5. Equation of transformed function: its graph should contain all points on the trace of $P'$.**
- Write down an equation which you believe corresponds with the transformed function.
- Test whether your equation contain all points on the trace of $P'$. On page 3.1, enter your equation for the transformed function as 'Relation 2'.
- On page 3.1, also enter the transformed function as 'Relation 3' in the form: $\text{rel1}\left(\frac{x}{a}, \frac{y}{b}\right)$, as was shown in Question 7 above. The graph of 'Relation 3' should also contain all points on the trace of $P'$.

**Step 6. Use algebra to confirm the equation of the transformed function.**
For dilations of factor $a$ from the y-axis and factor $b$ from the x-axis, the point $P(x, y)$ maps to point $P'(a \cdot x, b \cdot y)$; that is, if $P'$ has coordinates $(x', y')$, then $x' = a \cdot x$ and $y' = b \cdot y$ .
Use a method like the one used in Question 6 above to confirm algebraically the equation of the transformed function. Repeat Steps 1 – 6 for the next problem in Table 1.

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**Question 8**

Follow Steps 1 – 6 above for each original equation below and complete the table.

| Original equation | Dilation of factor:    **TABLE 1** | Equation of transformed function |
|---|---|---|
| a. $y = \sqrt{x} + 5$ | 2 from the y-axis (parallel to x-axis) | $y = \sqrt{\left(\frac{x}{2}\right)} + 5$ (or equivalent) |
| b. $y = \sqrt{x+5} - 2$ | 2 from the x-axis (parallel to y-axis) | $\left(\frac{y}{2}\right) = \sqrt{x+5} - 2$ or $y = 2\sqrt{x+5} - 4$ |
| c. $2x - 5y + 10 = 0$ | $\frac{1}{2}$ from the y-axis and<br><br>3 from the x-axis | $2\left(\frac{x}{\frac{1}{2}}\right) - 5\left(\frac{y}{3}\right) + 10 = 0$ or<br><br>$4x - \frac{5y}{3} + 10 = 0$ or $12x - 5y + 30 = 0$ |
| d. $y = \frac{x^2-4}{8}$ | $\frac{1}{2}$ from the y-axis | $y = \dfrac{\left(\frac{x}{\frac{1}{2}}\right)^2 - 4}{8}$ or $y = \frac{(2x)^2-4}{8}$ or $y = \frac{x^2-1}{2}$ |
| e. $y = \frac{x^2-4}{8}$ | $\frac{1}{2}$ from the x-axis | $\left(\frac{y}{\frac{1}{2}}\right) = \frac{x^2-4}{8}$ or $y = \frac{x^2-4}{16}$ |

**Question 11**

Show all working used in Step 6 above to obtain the equation of the transformed function using algebra.
Graph your equation to verify that it is an equivalent form of the equation from Step 5.

a. $x' = 2x$ , $y' = y$ therefore $x = \frac{x'}{2}$ , $y = y'$ . Substitute into equation of original function, as shown in third column of Table 1.

b. $x' = x$ , $y' = 2y$ therefore $x = x'$ , $y = \frac{y'}{2}$ . Substitute into equation of original function, as shown in third column of Table 1.

c. $x' = \frac{1}{2}x$ , $y' = 3y$ therefore $x = \frac{x'}{\left(\frac{1}{2}\right)}$ , $y = \frac{y'}{3}$ . Substitute into equation of original function, as shown in third column of Table 1.

d. $x' = \frac{1}{2}x$ , $y' = y$ therefore $x = \frac{x'}{\left(\frac{1}{2}\right)}$ , $y = y'$ . Substitute into equation of original function, as shown in third column of Table 1.

e. $x' = x$ , $y' = \frac{1}{2}y$ therefore $x = x'$ , $y = \frac{y'}{\left(\frac{1}{2}\right)}$ . Substitute into equation of original function, as shown in third column of Table 1.

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