Eye Spy

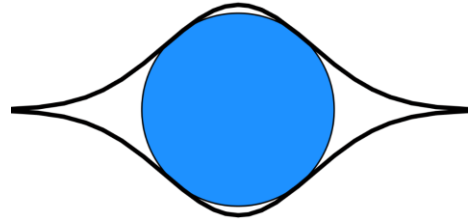TI-Nspire™   Investigation   Student   50 min

Problem to Solve

This problem is called "Eye Spy" because the combination of graphs looks a little bit like an eye. The top outline of the eye is defined by the function:

$$f(x) = e^{-x^2}$$

The iris of the eye is defined by the relation:

$$x^2 + y^2 = r^2 \quad \text{where } r \text{ is the radius of the iris.}$$

The aim of this investigation is to determine the maximum area of the iris where it is contained within the outline of the eye.

Question: 1.

Determine a function for the lower outline of the eye.

Question: 2.

Let P represent the point where the outer edge of the iris connects with the outline of the eye.

a) Write an equation relating $f(x)$ and the equation to the iris at point P.

b) Write an equation for the area of the iris in terms of x.

c) Use calculus to determine the value of x for which the area of the iris is a maximum given the iris fits completely inside the outline of the eye.

Question: 3.

a) Write an equation relating $f'(x)$ and the gradient of the iris at point P.

b) Determine the value of x and corresponding maximum area of the iris.

Extension

Apply appropriate transformations to the original eye shape such that the maximum iris area occurs at the point of inflection on the graph.

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TEXAS INSTRUMENTS