First Order Linear Differential Equations

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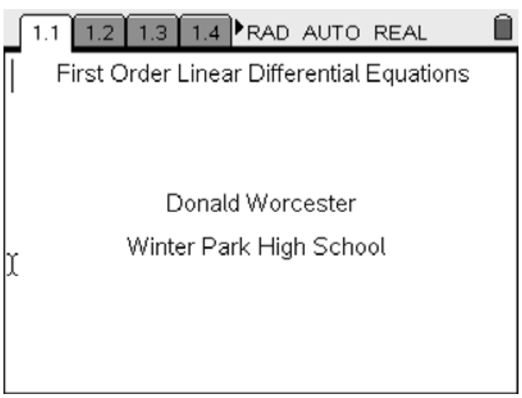
Materials Needed: TI-Nspire CAS unit
TI-Nspire Navigator
FirstOrderLinearDE.tns file
IntegratingFactorDerivation.tns file
GeneralSolutionofFirstOrderLinearDE.tns file

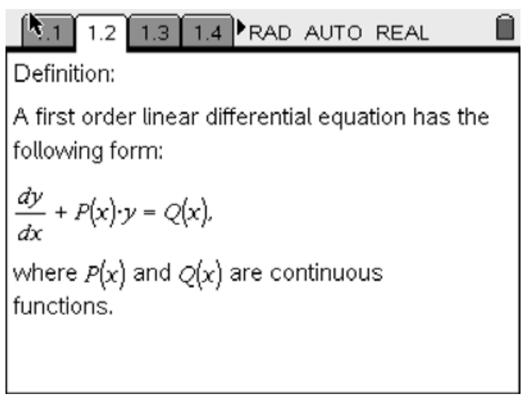
Courses: AP Calculus BC
Differential Equations

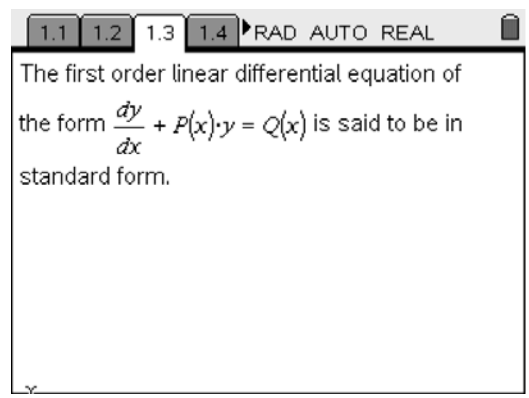
Objective: This activity will introduce students to first order linear differential equations.

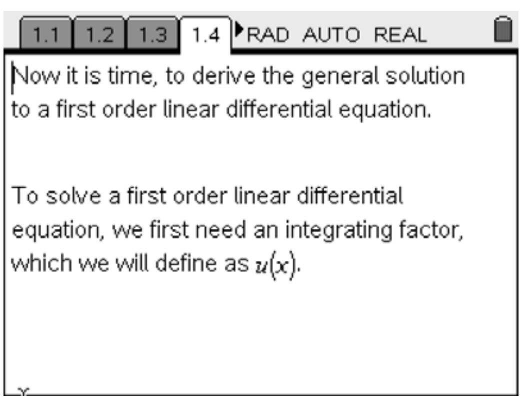
Procedures: Send the FirstOrderLinear DE.tns file to the students.

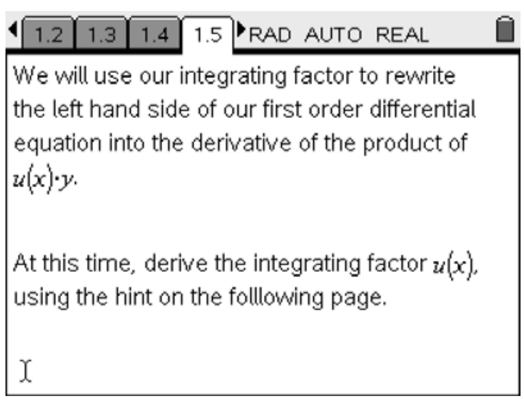
The students should review pages 1.1 – 1.6 on their own. You should give students a few minutes to read these pages, and then discuss the information with them.

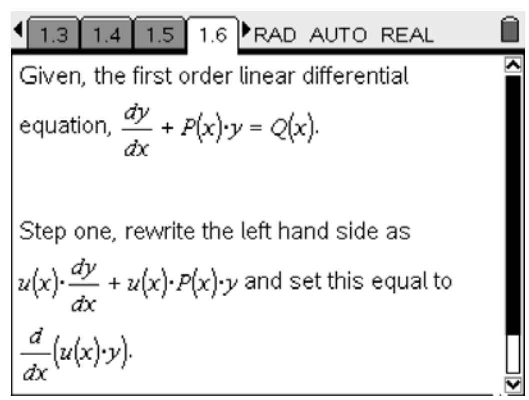




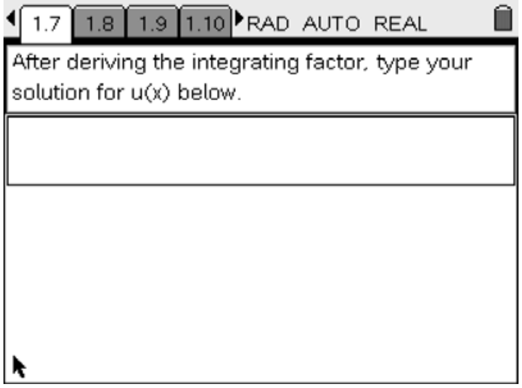




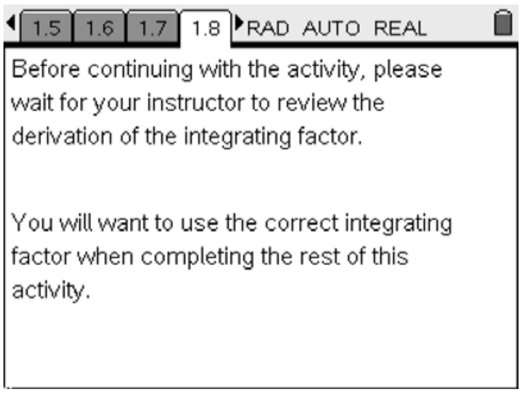


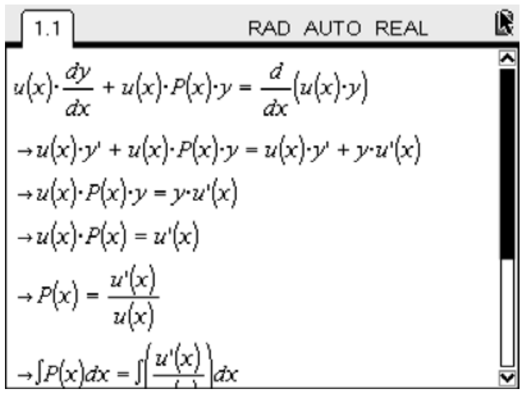


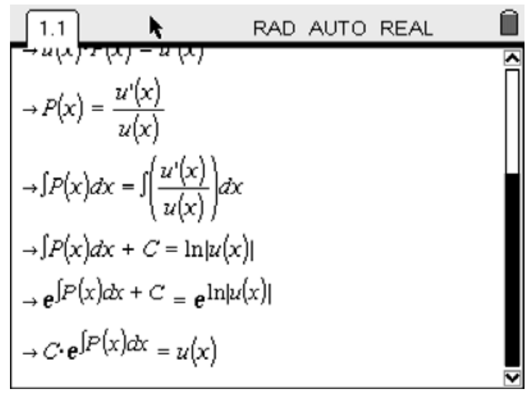
Students should enter their solution for $u(x)$ on page 1.7.



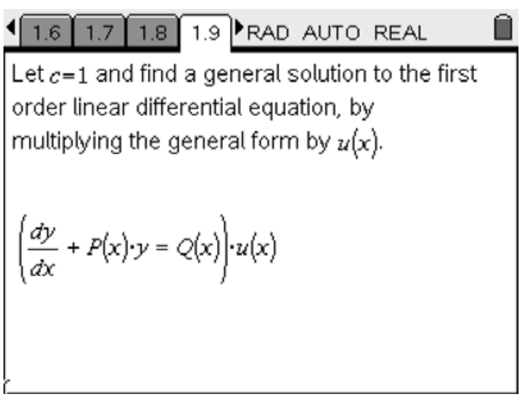
Once all of the students have derived (or attempted to derive) a value of $u(x)$, you should send all students the IntegratingFactorDerviation.tns file and review the derivation with them.

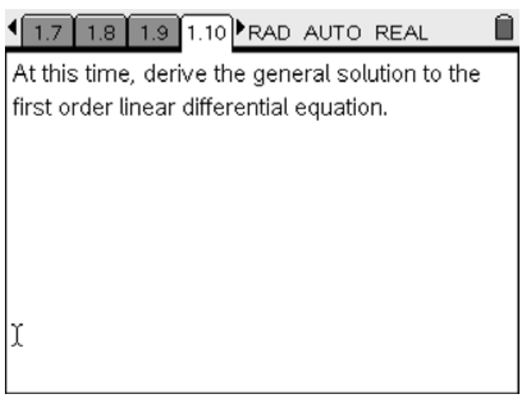


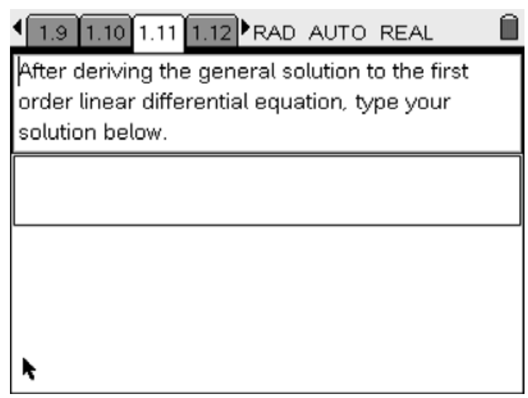




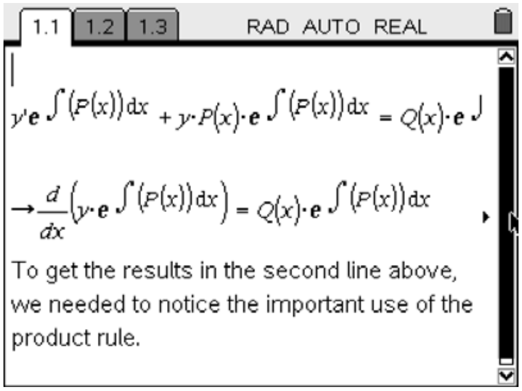
Students will now find a general solution to a first order linear differential equation. Students should review pages 1.9 – 1.10, and then enter their general solution on page 1.11.

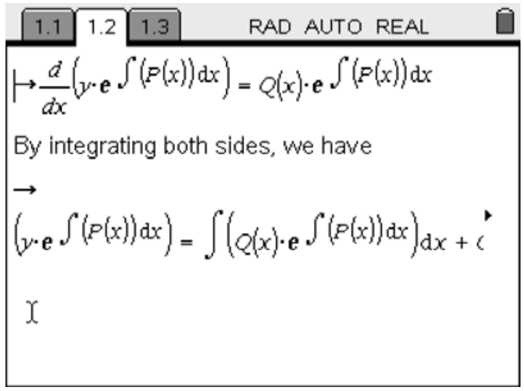


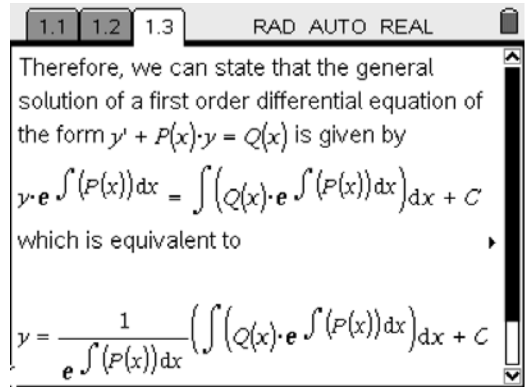




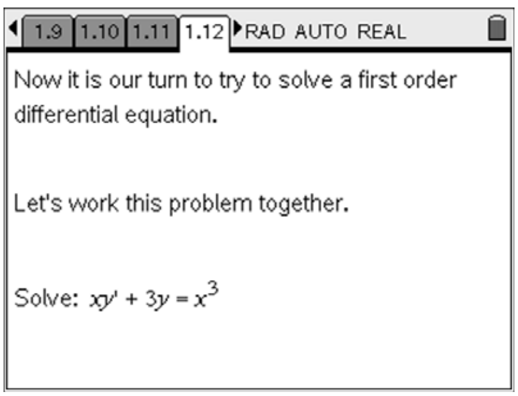
Once all of the students have derived (or attempted to derive) a general solution to the first order linear differential equation, you should send all students the GeneralSolutionofFirstOrderLinearDE.tns file and review the derivation with them.







Students should now be able to solve a first order linear differential equation. You will probably want to complete this example as a class to be sure that all students understand the process.



To solve this differential equation, you will first need to rewrite it into the standard form.

$$xy' + 3y = x^3$$

$$\Rightarrow y' + \frac{3}{x}y = x^2$$

Students should now identify $P(x) = \frac{3}{x}$ and $Q(x) = x^2$.

Calculating the value of the integrating factor,

$$u(x) = e^{\int P(x)dx}$$

$$u(x) = e^{\int \frac{3}{x}dx}$$

$$u(x) = e^{3\ln x}$$

$$u(x) = x^3$$

Therefore the general solution to the differential equation is

$$y = \frac{1}{u(x)}\left(\int Q(x)u(x)dx + C\right)$$

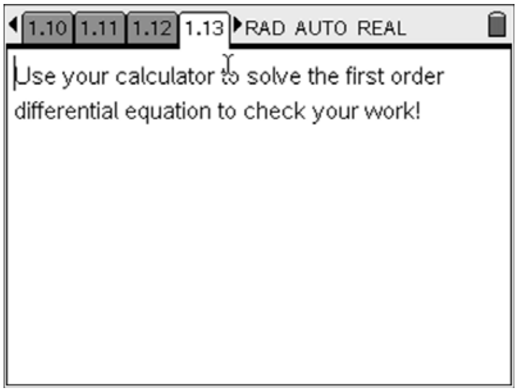
$$y = \frac{1}{x^3}\left(\int\left(x^2 \cdot x^3\right)dx + C\right)$$

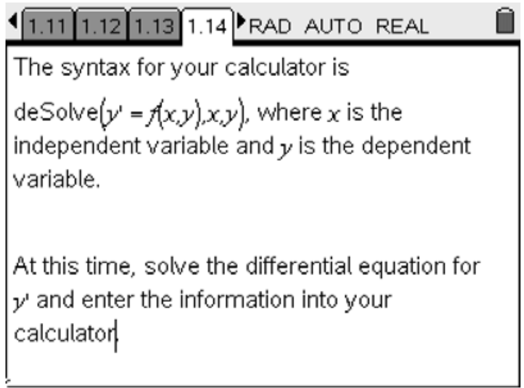
$$y = \frac{1}{x^3}\left(\int\left(x^5\right)dx + C\right)$$

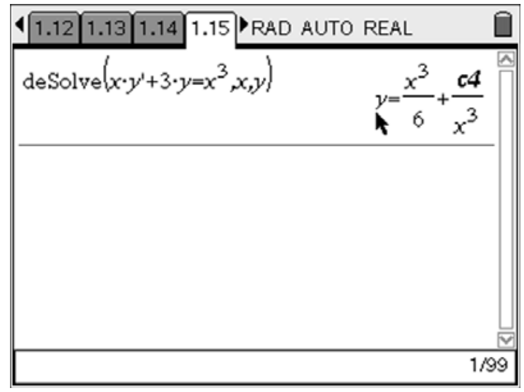
$$y = \frac{1}{x^3} \cdot \frac{x^6}{6} + \frac{C}{x^3}$$

$$y = \frac{1}{6}x^3 + \frac{C}{x^3}$$

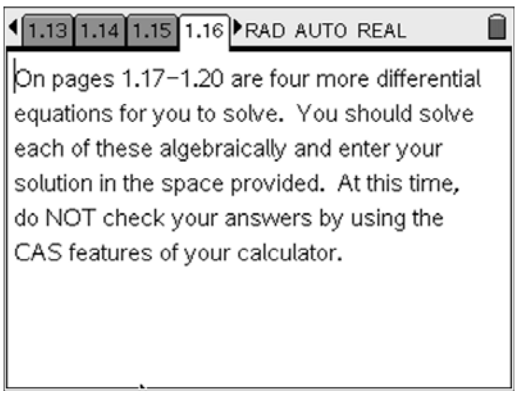
At this point, you should show students how to solve a first order linear differential equation with their Nspire-CAS calculators. The procedure can be found on pages 1.13 – 1.15.

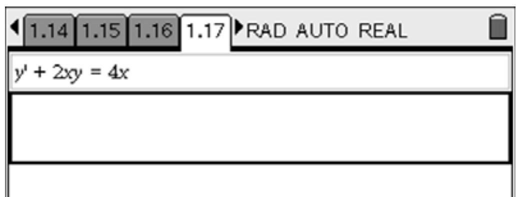


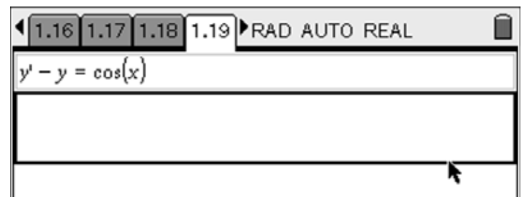


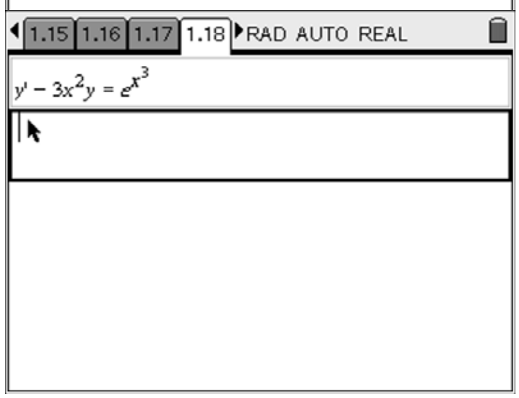


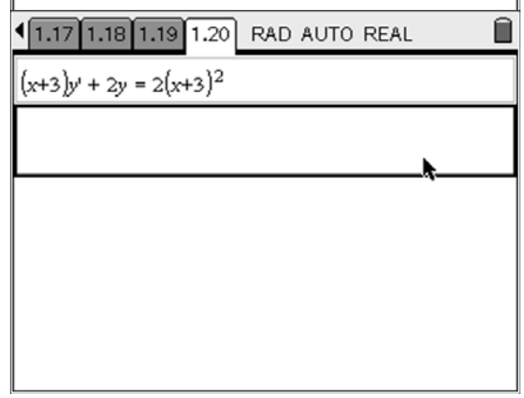
Students should now attempt to solve the four differential equation on pages 1.17 – 1.20.



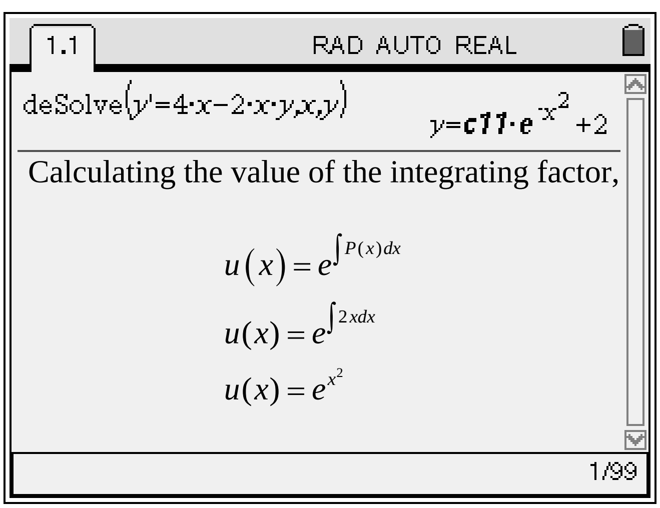








The solutions to the differential equations appear below.



Calculating the value of the integrating factor,

$$u(x) = e^{\int P(x)dx}$$

$$u(x) = e^{\int 2x dx}$$

$$u(x) = e^{x^2}$$

$P(x) = 2x$ and $Q(x) = 4x$

$$y = \frac{1}{u(x)}\left(\int Q(x)u(x)dx + C\right)$$

$$y = \frac{1}{e^{x^2}}\left(\int \left(4xe^{x^2}\right)dx + C\right)$$

$$y = \frac{4}{e^{x^2}}\left(\int \left(xe^{x^2}\right)dx + C\right) \qquad u = x^2, \quad du = 2xdx$$

$$y = \frac{2}{e^{x^2}}\left(\int \left(e^{u}\right)du + C\right)$$

$$y = \frac{2}{e^{x^2}}\left(e^{x^2} + C\right)$$

$$y = 2 + \frac{C}{e^{x^2}}$$

1.1 | 1.2 | RAD AUTO REAL

deSolve$(y'=\cos(x)+y,x,y)$

$$y=\frac{-\cos(x)}{2}+\frac{\sin(x)}{2}+c12\cdot e^{x}$$

Calculating the value of the integrating factor,

$$u(x)=e^{\int P(x)dx}$$

$$u(x)=e^{-\int dx}$$

$$u(x)=e^{-x}$$

1/99

$P(x)=-1$ and $Q(x)=\cos x$

$$y=\frac{1}{u(x)}\left(\int Q(x)u(x)dx+C\right)$$

$$y=\frac{1}{e^{-x}}\left(\int\left(e^{-x}\cos x\right)dx+C\right)$$

Use integration by parts.

$$y=\frac{1}{e^{-x}}\left(\frac{1}{2}e^{-x}\left(\sin x-\cos x\right)+C\right)$$

$$y=\frac{1}{2}\left(\sin x-\cos x\right)+Ce^{x}$$

1.1 | 1.2 | 1.3 | 1.4 | RAD AUTO REAL

deSolve$\left(y'=e^{x^3}+3\cdot x^2\cdot y,x,y\right)$ $\quad y=(x+c15)\cdot e^{x^3}$

Calculating the value of the integrating factor,

$$u(x)=e^{\int P(x)dx}$$

$$u(x)=e^{\int -3x^2dx}$$

$$u(x)=e^{-x^3}$$

1/99

$P(x)=-3x^2$ and $Q(x)=e^{x^3}$

$$y=\frac{1}{u(x)}\left(\int Q(x)u(x)dx+C\right)$$

$$y=\frac{1}{e^{-x^3}}\left(\int\left(e^{x^3}\cdot e^{-x^3}\right)dx+C\right)$$

$$y=\frac{1}{e^{-x^3}}\left(\int dx+C\right)$$

$$y=\frac{1}{e^{-x^3}}\left(x+C\right)$$

1.1 | 1.2 | 1.3 | 1.4 | RAD AUTO REAL

deSolve$\left(y'=\frac{2\cdot(x+3)^2-2\cdot y}{x+3},x,y\right)$

$$y=\frac{x^4+12\cdot x^3+54\cdot x^2+108\cdot x+2\cdot c14+81}{2\cdot(x+3)^2}$$

Calculating the value of the integrating factor,

$$u(x)=e^{\int P(x)dx}$$

1/99

$$u(x)=e^{2\int\frac{dx}{x+3}}$$

$$u(x)=e^{2\ln|x+3|}$$

$$u(x)=(x+3)^2$$

$$(x+3)y'+2y=2(x+3)^2$$

$$y'+\frac{2}{x+3}y=2(x+3)$$

$P(x)=\frac{2}{x+3}$ and $Q(x)=2(x+3)$

$$y=\frac{1}{u(x)}\left(\int Q(x)u(x)dx+C\right)$$

$$y=\frac{1}{(x+3)^2}\left(\int 2(x+3)\cdot(x+3)^2dx+C\right)$$

$$y=\frac{2}{(x+3)^2}\left(\int (x+3)^3dx+C\right)$$

$$y=\frac{1}{2(x+3)^2}\left((x+3)^4+C\right)$$

$$y=\frac{(x+3)^2}{2}+\frac{C}{(x+3)^2}$$

After the students have solved each of their differential equations algebraically, collect their solutions using the TI-Nspire Navigator system.

As you are showing the results to the four questions to the class, using the slide show within Class Analysis, discuss the different format that the answers have been expressed. It is important to point out to the students that answers may be equivalent and correct but look different.

While reviewing the results, you should have the students solve each of these differential equations using their TI-Nspire CAS handheld. Using the TI-Nspire Navigator system, select four different students to present their solutions to one of the four differential equations. Discuss the solution the calculator gave versus the one that was derived algebraically from the students.