Local Linearity Discovery

ID: 10890

Time required
10–15 minutes

Activity Overview

This is a short activity designed for a first time user. In this activity, students will explore zooming in on various functions including piecewise functions. They will investigate the concept of local linearity. This introductory calculus activity has strong connections to many calculus concepts including slope, limit of a difference quotient, the definition of the derivative, and the criteria for differentiability.

Topic: Local Linearity

- *Slope of the tangent*
- *Connections to definition of the derivative and differentiability*

Teacher Preparation and Notes

- *Students will begin the activity with a blank TI-Nspire document.*
- *Students will need to press* ctrl + ▶ *to advance to the next page. They will need to press* ctrl + tab *to move to a different application within the same page.*
- ***To download the solution TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "10890" in the quick search box.***

Associated Materials

- *LocalLinearity_Student.doc*
- *LocalLinearity.tns*
- *LocalLinearity_Soln.tns*

Suggested Related Activities

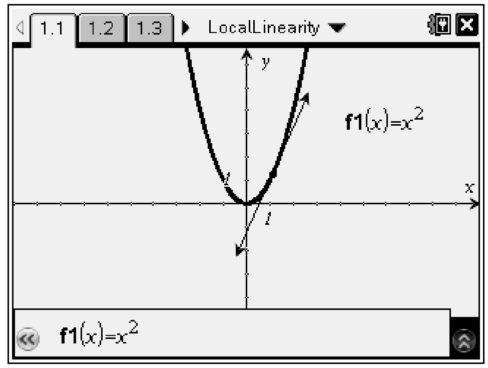
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Local Linearity (TI-Nspire technology) — 10090*
- *Local Linearity, Differentiability and Limits of Difference Quotients (TI-84 Plus) — 5585*
- *Continuity and Differentiability of Functions (TI-Nspire technology) — 8498*
- *Differentiability and Continuity (TI-84 Plus and TI-Navigator) — 5417*
- *Piecewise Functions: Continuous-Differentiable (TI-89 Titanium) — 3293*

Part 1 – Draw a Tangent Line by Hand

Students will begin the activity by drawing a line tangent to the graph of $y = x^2$ at some point in the first quadrant. Students can approximate the slope and equation of the tangent line using the grid points on the graph

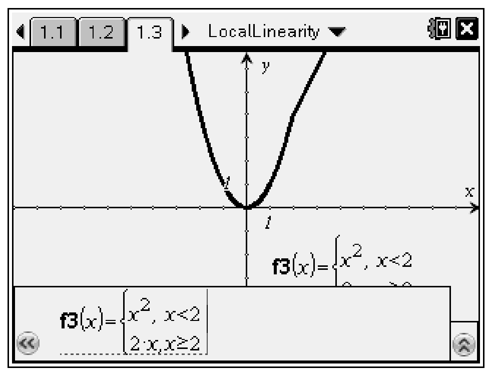
Part 2 – Draw and Explore Tangent using Technology



In a new TI-Nspire document, students will graph the function **f1**(x) = x^2 and draw a tangent line as they did in Problem 1. Note: to escape from an undesired location while using the TI-Nspire handheld, try pressing esc.

If students zoom in more than 10 times (that is a 1024× zoom), or if the window size has a width near 0.01, they may 'miss' the point of tangency when zooming in again. Students can press ctrl + esc (or ctrl + Z) to undo. Or, if they know the exact point they are trying to zoom in on, e.g. (2, 4) for Problem 3, use **MENU > View > Show Axes End Values** so they will know which direction to drag the screen.

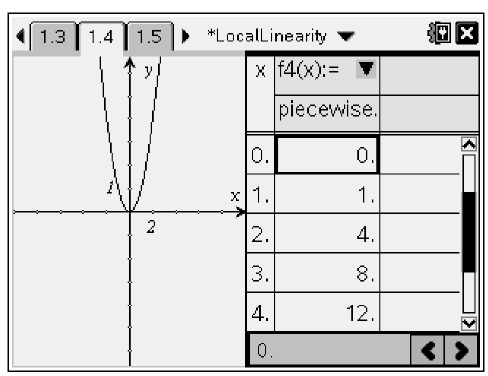
Parts 3 and 4



In Problems 3 and 4, students will graph piecewise functions and observe whether or not the graph is local linear in the neighborhood of (2, 4). Students will observe that some piecewise functions have the local linearity property, while other piecewise functions lack this property.

Note: When entering the piecewise functions, you may want to show students how to use the templates in the catalog (catalog key) instead of using the template key. On the TI-Nspire handheld, press the catalog key and then the number of the templates tab. The benefit of using the templates in the catalog is that a template description appears at the bottom.

Extension



Connect the concept of local linearity to the definition of the derivative, continuity and differentiability. On a Graphs page use ctrl + T *to explore the slope numerically. To change the start and step of the table press* ***MENU > Table > Edit Table Setting****.*

Solutions – Student worksheet

Part 1

- Answers may vary.

Part 2

- Sample answer: when examined close up, the tangent line and the graph of $y = x^2$ look to be the same line.
- Sample answer: this behavior will not occur for all other functions because some functions are not continuous.

Part 3

- Sample answer: when zoomed in, the line does not appear to be linear. There is a cusp where the slopes on both sides of (2, 4) are not equal.

Part 4

- Sample answer: This function does appear to be locally linear in the neighborhood of (2, 4). For this function, there is not a cusp at (2, 4) like there was in Problem 3.

Part 5

- Sample answer: as the interval of a graph is repeatedly zoomed in, Δx becomes closer to zero.

