

Activity 8

Collecting Pens (Continued)

In *Activity 7: Collecting Pens*, you used a calculator to generate random numbers for a Monte Carlo simulation to find out how many boxes of cereal a customer would have to buy to collect all six pens. In this activity, you will use a program to run many additional trials for the same problem.

The Problem

Recall the problem from *Activity 7: Collecting Pens*:

The Kellogg Company (the cereal makers) once placed a free felt-tipped marker in each box of *Kellogg's® Raisin Bran* they distributed. The back of the box proclaimed:

FREE INSIDE — BRUSH MARKER ...

You'll find one of the 6 washable colors in each specially marked package of *Kellogg's® Raisin Bran* cereal.

Start collecting all 6 NOW!

Assuming that equal numbers of each of the six pen colors were randomly distributed in the packaging of the cereal by the Kellogg company, about how many boxes of cereal do you think a customer would have to buy in order to collect all six of the pens?

Although the calculator was used to generate the random numbers, you and your classmates had to do most of the work to collect and organize the results for several trials of the simulation. In this activity, you will see how the calculator can be instructed to do most of this work for you.

This activity continues with an investigation of the problem introduced in Activity 7: Collecting Pens. The major focus here is upon the discussion and implementation of a calculator algorithm for carrying out many trials of the simulation used in the preceding activity. Students will study and download a prewritten program into their calculators using the calculator linking capabilities. (Note that the TI-80 does not have linking capabilities. If you are using TI-80s, you will have to enter the program into each calculator manually.) A page of keystroke instructions for entering the program into the calculator is provided. Students will run the program and analyze the results in a manner similar to that used in Activity 7. Make certain that students are familiar with Activity 7 before they begin Activity 8.

Students may find it helpful to identify most of these variables in the setting of the “tally” boxes used earlier:

Customer (C) ____

Pen P	Tally
R (1)	
T (2)	
Y (3)	
B (4)	
G (5)	
V (6)	

Total boxes B purchased: _____

Ask students if they see why the variable L was used instead of B to represent the count of the number of blue pens purchased. (B is used for a different purpose here.)

Your graphing calculator understands a programming language that can be used to teach the calculator to carry out complicated tasks. Rather than trying to write an original program, let’s start by studying a prewritten program. Table 8.1 on the next page gives a listing of a program called **SIXPENS** which was written to simulate up to 99 trials of the six-pens collection problem. You may find it helpful to associate the variables (storage locations) used by the program with the following essential characteristics of the real world problem:

- N** The **N**umber of customers for which the simulation will be conducted.
- C** Keeps count of the **C**ustomers who are currently buying cereal.
- B** Keeps count of how many **B**oxes the current customer has purchased.
- P** Remembers the **P**en color obtained from the last box of cereal.
- R** Keeps count of how many **R**ed pens the current customer has collected.
- T** Keeps count of how many **T**an pens the current customer has collected.
- Y** Keeps count of how many **Y**ellow pens the current customer has collected.
- L** Keeps count of how many **bL**ue pens the current customer has collected.
- G** Keeps count of how many **G**reen pens the current customer has collected.
- V** Keeps count of how many **V**iolet pens the current customer has collected.
- L1** A **list** containing the number of boxes of cereal purchased by each customer.

Study the program code and the explanation of the code. It is not essential that you understand everything about the program, just that you have some idea of what it accomplishes. Later, you will run this program on your calculator.

Table 8.1. The SIXPENS Calculator Program

Program Code	Explanation of Code
PROGRAM: SIXPENS	Program is named SIXPENS
:ClrHome	Clears the home screen of all numbers and text.
:ClrList L1	Clears L1 which will be used to store results for each customer.
:Input "TRIALS?(≤99)",N	Lets the user define the number of trials (customers) when the program is run.
:Disp "RESULTS:"	Places the text (or word) RESULTS : on the home screen.
:For(C,1,N)	Defines the beginning of a set of instructions to be repeated for each customer.
:0→B	Sets the number of boxes purchased by the customer to 0.
:0→R:0→T:0→Y :0→L:0→G:0→V	Puts 0 into each storage location that keeps track of how many of each of the six pen colors have been collected by a customer.
:Lbl 1	A location label, similar to a bookmark, that marks a place that will be returned to later in the program.
:1+B→B	Customer buys a box of cereal; number of boxes is increased by 1.
:iPart 6rand+1→P	A random pen color is found in the box of cereal.
:If P=1:R+1→R :If P=2:T+1→T :If P=3:Y+1→Y :If P=4:L+1→L :If P=5:G+1→G :If P=6:V+1→V	Depending upon which color of pen was in the box, one of the counters for the pen colors is increased by 1.
:If RTYLGV=0:Goto 1	If any of the pen counts is 0, the product of all of the counts is 0 and the program returns to Lbl 1 (the bookmark); otherwise, the customer has at least one of every pen color and the program goes to the next line.
:B→L1(C)	Stores the number of boxes purchased by this customer into the list of all customer purchases. This is done only after the customer has at least one of every pen color.
:Disp B	Displays the number of boxes of cereal purchased by current customer.
:End	Defines the end of the instructions for a customer; all instructions following the For statement are repeated if more customers remain.

When discussing the program, it may prove helpful to work through a simulation with an overhead die, or the calculator ViewScreen and the command *iPart 6rand+1*, pointing out the decisions that we make and how the calculator is instructed to make and record those same decisions. A transparency master is included in the Appendix to aid in such a discussion.

The TI-83 uses parentheses in this command:
iPart (6rand+1)→P

You may need to remind the students that the only things they will see on the home screen while the program is working are the text or requested variable output listed after *Input* and *Disp* commands. Have them determine what the text or output will be before running the program.

A transparency master is included in the Appendix that provides linking and downloading instructions for the students using TI-82 or TI-83 calculators.

If you are using TI-83s, the restriction of no more than 99 trials is not necessary. In fact, if sufficient memory exists, the TI-83 can handle up to 999 trials. You may want to have all students do the same number of trials initially (15, for example), so that you can quickly check for problems and determine if the students are comfortable with the program. You could then have them estimate the length of time the program needs to run to complete their next simulation based on how long it took to run 15 trials.

You may want to ask if they know in which list the data is being stored. The display of the results of each trial serves only to provide the calculator user with something to watch while the program is running. It is interesting to attend to the length of time between trials. Students enjoy estimating how large the result will be when the time period begins to get large.

Using the Calculator

Once you have studied and discussed the **SIXPENS** program, follow the instructions provided by your teacher to download a copy of **SIXPENS** into your calculator. Then, if necessary, link your calculator with that of another classmate to download a copy of the program into his or her calculator. Once everyone has a copy of the program, you can proceed with the remainder of the activity.

1. To run the program, press **PRGM** and use **▼** to highlight the **SIXPENS** program name under the **EXEC** menu.
2. Once **SIXPENS** is highlighted, press **ENTER** twice to run the program.

You will see the text **TRIALS?(≤99)** on the screen, as you probably predicted. This is a prompt asking you to tell the calculator how many customers you would like to use in your simulation.

3. Enter some number less than or equal to 99 and press **ENTER**.

You should see the “working” indicator in the upper right corner of the display. A sequence of numbers should begin to appear on the display. Each of these numbers represents the number of boxes of cereal purchased by a customer to obtain all six of the pen colors. Don’t worry about trying to write down these numbers; each is being stored in the calculator so that you can examine and work with them later. Watch the display as the numbers are generated. You can usually tell if a customer is buying a large number of boxes just by the length of time it takes before the number is printed.

Once purchases for all customers have been simulated and displayed (this could take several minutes if you are simulating a large number of customers), the word **Done** will show on the screen.

The program has used the list **L1** to store the number of boxes of cereal each of your customers had to purchase in order to collect all six pen colors. These are the same numbers that you saw displayed on the screen as the program was running.

4. To examine this list, press $\boxed{\text{STAT}}$ $\boxed{\text{ENTER}}$. You can use the blue arrow keys to scroll up and down this list.

✍ Go to the **Questions** section and answer #1.

Analyzing the Results

This data can be examined further to search for information that would be helpful in formulating an answer to the original question: “How many boxes of cereal would the average customer expect to buy in order to collect all six pen colors?”

You will use the calculator to look at three techniques for examining this data more closely:

- Sorting the data
- Obtaining more extensive summary statistics
- Drawing a histogram and a drawing a box-and-whisker plot of the data

The separate analyses performed here are very similar to those the students were led through in Activity 7. Students are not provided with as much detail for performing the analyses here. Instead, they are ask to provide such details for themselves.

Sorting the Data

The data would be easier to interpret if it were ordered numerically from the lowest to the highest value.

1. Press $\boxed{\text{STAT}}$ $\boxed{2:\text{SortA}}$. The screen will display the partial command SortA (. SortA is short for “**sort** the list entries into **Ascending** order.”
 2. Press $\boxed{2\text{nd}}$ $\boxed{[L1]}$ $\boxed{)}$ $\boxed{\text{ENTER}}$ to tell the calculator that you wish to sort the list L1. The calculator will inform you it is **Done**.
 3. Press $\boxed{\text{STAT}}$ $\boxed{\text{ENTER}}$ to retrieve the data screen. Observe that the list L1 is now sorted from the lowest to the highest value.
- ✍ Move through the list and note the minimum and maximum values. Now answer questions #2 and #3 in the **Questions** section.

Obtaining Summary Statistics

It is sometimes very difficult to examine an entire set of data in order to make summarizing statements about that data. In such cases, examining summary statistics on the data often proves more helpful.

You may want to ask if students understand why they need only to mention L1 (and not L2 as was done in Activity 7) in the 1-Var Stats command. (The data is in a list rather than a frequency table.)

1. Press **[STAT]** **[▶]** **[ENTER]** **[2nd]** **[L1]** **[ENTER]** to instruct the calculator to produce a list of statistics summarizing information on the number of boxes purchased by all customers represented in the simulation.

Examine this set of statistics. There are two screens of numbers and the second can be examined by pressing **[◀]** several times.

✍ Answer #4 in the **Questions** section.

Drawing a Histogram and a Box-and-Whisker Plot of the Data

You can create a histogram and a box-and-whisker plot of this data just like you did in *Activity 7: Collecting Pens*. Recall that there are three steps to create this plot:

1. Clear or disable any functions stored in the **[Y=]** menu.
2. Define an appropriate viewing window.
3. Define and activate the appropriate **[STAT PLOT]** features.

Discuss with others in your group how to accomplish each of these three tasks. The most crucial, and often the most challenging of these tasks, is determining the values needed to define an appropriate viewing window.

✍ Write the values you decide upon for the window parameters and the reasoning for your decisions in Table 8.2, **Window Parameters**, in #5 in the **Questions** section. Then answer #6 through #8.

Questions

1. What was the largest number of boxes of cereal any one of your customers purchased to obtain all six pen colors? What was the smallest?

◆ *Return to page 81, Analyzing the Results.*

2. As you scroll through the sorted data, are you finding that all values between the minimum and the maximum are represented in the list? Do some values tend to occur more often than others?

3. If you had to decide “How many boxes of cereal would the average customer expect to buy in order to collect all six pen colors?” based solely upon your examination of this list of data, what response would you give and why?

◆ *Return to page 82, Obtaining Summary Statistics.*

4. Based upon these summary statistics, how many boxes of cereal does the “average” customer expect to buy in order to collect all six pen colors? Explain your answer. How does this answer compare to previous responses you have made for this question?

We expect the students to select the mean of the data. There will be some variability in the means from each of their trials but if the number of trials has been large each time, the difference in the means should be minimal. You may wish to discuss the differences in interpretation between using the mean and using the median to answer this question.

◆ *Return to page 82, Drawing a Histogram.*

Xmin could be set to 5 (one less than the minimum possible number of boxes purchased) and *Xmax* to two more than the maximum number found during the simulation. This selection permits the entire range of values to be displayed and still permits a small border at each end of the graph. Students do not need to have these exact values but their choices should not stray too much from these without good rationale. *Xscl* should be set to 1 so that the histogram shows the frequency counts for all whole numbers between the minimum and maximum. *Ymin* should be set to zero (or slightly less) and *Ymax* to the largest frequency (or slightly larger) to permit sufficient space for the height of the histogram on the window. The value set to *Yscl* is arbitrary but should be reasonable and based upon the distance between the minimum and maximum values for the *y*-variable.

Here students will likely select the median (middle) value for the "average."

5. Write the values you selected for the viewing window.

Table 8.2. Window Parameters

Parameter	Value Selected	Reasons for Your Selection
Xmin		
Xmax		
Xscl		
Ymin		
Ymax		
Yscl		

6. Once you have selected and set your viewing window parameters, instruct the calculator to draw the histogram. What does the histogram tell you about the number of boxes of cereal most customers will need to buy?

7. Instruct the calculator to draw the box-and-whisker plot for this data.

Using only the information from the box-and-whisker plot and a trace of its five critical points, how would you answer the question concerning the number of boxes of cereal that need to be purchased? How does this response differ from your earlier response to the same question?

Note: The TI-80 does not have the Rcl feature found on the TI-82 and TI-83 calculators. TI-80 users will need to enter the program used in the Problems for Additional Exploration by hand.

The **EIGHTPEN** program (changes from **SIXPENS** are in **bold**) :

```

ClrHome
ClrList L1
Input
"TRIALS?(≤99)",N
Disp "RESULTS:"
For(C,1,N)
0→B
0→R:0→T:0→Y
0→L:0→G:0→V
0→M:0→O
Lb1 1
1+B→B
iPart 8rand+1→P
If P=1:R+1→R
If P=2:T+1→T
If P=3:Y+1→Y
If P=4:L+1→L
If P=5:G+1→G
If P=6:V+1→V
If P=7:M+1→M
If P=8:O+1→O
If RTYLGVMO=0:Goto 1
B→L1(C)
Disp B
End

```

Problems for Additional Exploration

Suppose the Kellogg company decides to put eight different colors of pens in their cereal instead of six. (They add **Orange** and **Magenta** colors to the collection.)

1. Decide upon the modifications needed for the **SIXPENS** program to handle the eight-pens collection problem.

Once you have decided upon the needed changes to the **SIXPENS** program, you can use the **RCL** feature on the calculator to copy the **SIXPENS** program into another program storage location.

- a. Create a new program area with the name **EIGHTPEN**: . Press **PRGM**, use **▶** to highlight **NEW**, and press **ENTER**.
 - b. Type in the name **EIGHTPEN** and press **ENTER**.
 - c. Copy the contents of the **SIXPENS** program into the **EIGHTPEN** program area. Press **2nd** **[RCL]** **PRGM**, highlight **EXEC**, highlight the line containing the program name **SIXPENS**, and press **ENTER** **ENTER**. You can now edit the old program to fit the needs of the new problem.
 - d. Once the **EIGHTPEN** program is completed, press **2nd** **[QUIT]** to get to the home screen.
 - e. Press **PRGM**, highlight **EXEC**, highlight the line containing the program name **EIGHTPEN**, and press **ENTER** **ENTER** to run the program.
2. Use the techniques of this activity to investigate and answer the eight-pens collection problem.