Parabolic secants

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Abstract: This activity involves some of the prerequisites of calculus relating to functions and equations. It also is an application of derivatives. It introduces students to an interesting property of parabolas and a method of proving that property using the TI-89 scripts. They then use the symbolic capacity of their calculator to generalize upon specific results.

NCTM Principles and Standards:
Algebra standards
a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

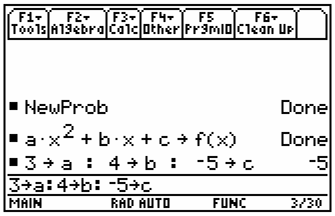
Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

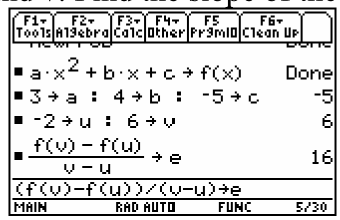
Reasoning and Proof Standard
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Key topic: Prerequisites of calculus relating to functions and equations. Application of Derivatives - Mean Value Theorem. Scripts, formal proofs.

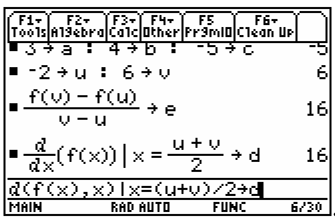
Degree of Difficulty: Elementary
Needed Materials: TI-89 calculator
Situation: Parabolas have many interesting properties. In this activity we'll use calculus to investigate one of them with the aid of the TI-89 calculator. Consider a line, called a secant line, which crosses a parabola in two points. What is the relationship between the slope of the secant line and the slope of the tangent line to the parabola at the point whose x-coordinate is the average of the x-coordinates of the two points?

Choose arbitrary values for the coefficients of a parabola in the form of $ax^2 + bx + c$ and store this as f(x).

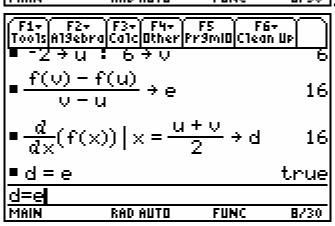


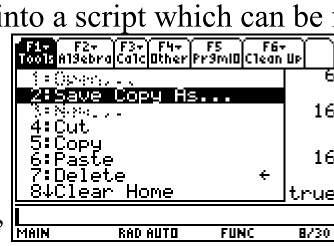
Now choose two arbitrary values for the x-coordinates of the two points on the parabola and store them as u and v: Find the slope of the secant line through these two points and store that result as e: 

Now find the derivative of f(x) at the point the point whose x-coordinate is the average of the x-coordinates of the two points: $x = \dfrac{u+v}{2}$ and store this as d:

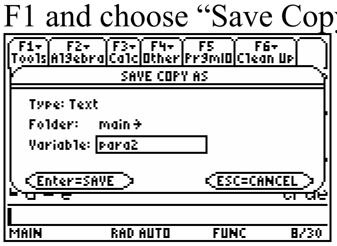


. Finally compare the values of d and e:

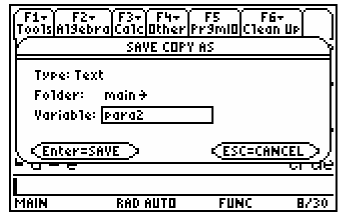


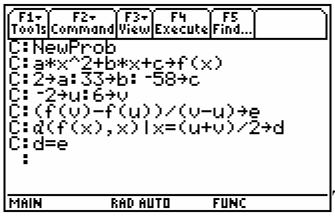
We have shown for this parabola that the slope of a secant line is the slope of the tangent line at the average of the x-coordinates of the points where the secant line crosses the parabola. Is this property always true? You could scroll back and change the coefficients of the parabola and execute the steps again, but we'll take a different tack. We'll turn what we've written into a script which can be followed for any parabola: Press F1 and choose "Save Copy As" 

I named this script "para2"

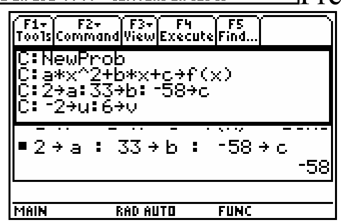


From the application menu APPS choose 8:Text Editor:

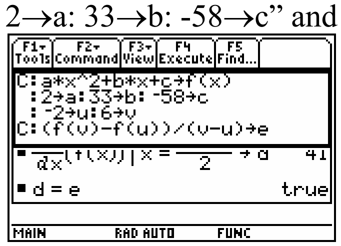
and open the script named para2: 

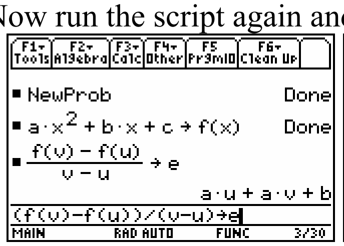


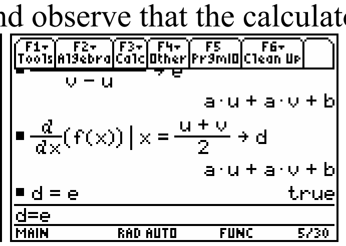
This script contains the commands that were originally typed in. We can play this script for other choices of a, b, and c: First press [F3] to change the view to A: Script View 

Pressing [F4] executes each line. Change the values of a, b, and c: 

and continue pressing [F4] to see that the property is true with these choices.

Is it always true? Go back to the line "C: 2→a: 33→b: -58→c" and choose 4:Clear Command from the [F2] Command menu: 

which will erase the C: in the line "C: 2→a: 33→b: -58→c". Do the same for the line "C:-2→u:6→v" Now run the script again and observe that the calculator creates a proof of this property: 



Scripts can be very useful in proving properties. Here we showed that for a parabola the secant line is parallel to the tangent line at the average of the x-coordinates of the points where the secant line crosses the parabola. This is an example of the Mean Value Theorem.