EXPLORATIONS ™

Activity 3

Objectives

- Explore the effects of changes in parameters on the graph of a parabola that is given in vertex form
- Predict the location of the vertex based on the quadratic function

Exploring the Vertex Form of the Quadratic Function

Introduction

Any quadratic function (parabola) can be expressed in $y = a(x - h)^2 + k$ form. This form of the quadratic function (parabola) is known as the vertex form. In this activity, you will see how this form shows both the location of the vertex and the width and direction of the curve.

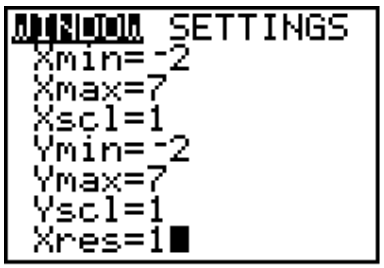
Quadratic functions come in two basic shapes: those that open up and those that open down. One of the important points in a quadratic function is its vertex. The vertex is the lowest point (minimum value) when the function opens up and the highest point (maximum value) when it opens down. When you can identify the vertex and the direction a parabola opens, you will be able to find the minimum or maximum value of quadratic functions expressed in vertex form.

These concepts will allow you to explore the concept of translation and be able to apply it to new situations.

The Graph of a Quadratic Function in Vertex Form

Using your graphing handheld, you will graph $y = (x - 2)^2 + 1$.

1. Press [Y=] and enter the equation in the **Y=** editor.
2. Press [WINDOW] and set the parameters as shown in the screen.



© 2004 TEXAS INSTRUMENTS INCORPORATED

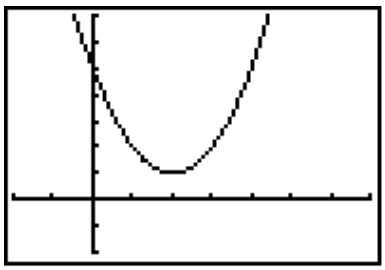
3. Press GRAPH.

What are the coordinates of the vertex?

_____

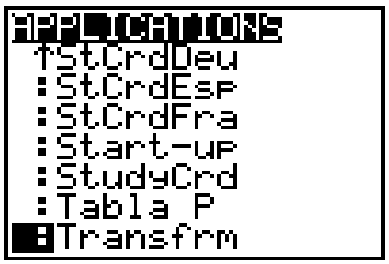
In what direction does it open?

_____

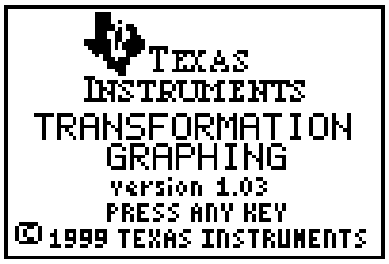


Studying the Effect of *A, B,* and *C*

1. Press APPS and select **Transfrm**.

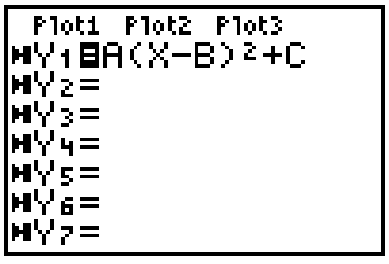


Note: *If you do not see the screen shown, select* **Continue**.



2. Display the **Y=** editor and enter the general vertex form of the quadratic function, $y = a(x - b)^2 + c$ by pressing ALPHA **A** (X,T,Θ,*n* − ALPHA **B**) x^2 + ALPHA **C**.

Note: *Clear any equations that are listed in the* **Y=** *editor and turn off any plots. Make sure you are in the* **Function** *mode* **(Func)**.



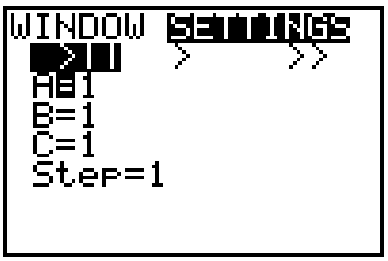
Note: *You entered* **Y=A(X – B)^2 + C** *in place of* **Y=A(X – H)^2 + K**, *which is the form commonly found in textbooks, because the Transformation Graphing App uses only the coefficients* A, B, C, *and* D.

If the Play-Pause mode (**>||**) is not selected at the left of **Y1**, press ◀ until the cursor is over the symbol and then press ENTER until the correct symbol is selected.

© 2004 TEXAS INSTRUMENTS INCORPORATED

3. Press WINDOW ▲ to display the settings screen.

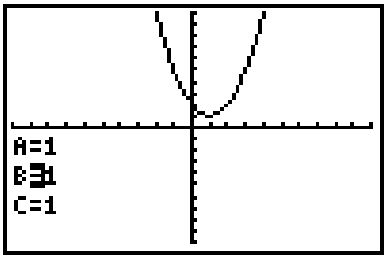
4. As a starting place, enter the values shown. To make these selections, press ▼ **1** ▼ **1** ▼ **1** ▼ **1**. This defines the starting values for the coefficients and the increment by which you want to change the coefficients.



Studying the Effect of *B*

4. Press ZOOM **6** to select **6:ZStandard** and display the graph. The graph shows the pre-selected values for *A*, *B*, and *C*. Both the *x*- and *y*-axis range from ⁻10 to 10 with a scale of 1.

4. Highlight **B=**. You will start your study with the effect of *B*.



5. Press ▶ to increase the value of *B* by the pre-selected step value (1 in this example). The graph is automatically redrawn showing the effect of this change on *B*. Continue to press ▶ until you have an idea of how changing *B* affects the graph.

6. Press ◀ to decrease the value of *B* by the pre-selected step value. Did the graph move in the direction you expected?

Questions for Discussion

1. What effect does changing the value of *B* have on the curve?

2. If **B=3**, where is the vertex? _____

   If **B=5**? _____

   If **B=⁻2**? _____

   If **B=2**? _____

3. Based on your answers above, make a statement about the relationship between the value of *B* and the position of the vertex of the parabola.

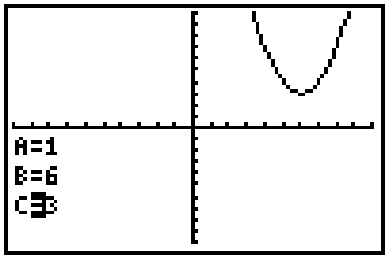
   _____

In the vertex form, $y = a(x - b)^2 + c$, the value of *B* gives the *x*-coordinate of the vertex. Be careful, though. Notice the form has $x - b$. If you had the equation $y = (x - 3)^2$ then $b = 3$ and the vertex is at $x = 3$. For the equation $y = (x + 1)^2$ the *B* would be ⁻1 with the vertex at $x = ⁻1$.

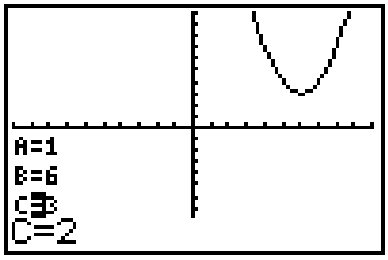
© 2004 TEXAS INSTRUMENTS INCORPORATED

Studying the Effect of *C*

1. Highlight the **C=**. Press ▶ several times, and notice the change in the graph. Press ◀ several times, and notice this change.



2. Predict where the vertex of the parabola will be if **C=2**. Enter **2**, and check your prediction.

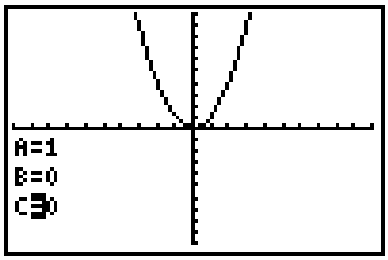


3. What effect does a change in the value of *C* have on the vertex of the parabola?

_____

Check Your Understanding So Far

The graph of $y = x^2$ is shown at the right. The following equations are vertical and horizontal translations of $y = x^2$. Use what you have learned about translation of the vertex of a quadratic function to determine the vertex of the graph of each equation below. Check your answer using the Transformation Graphing App.



$y = (x - 2)^2$ Vertex: _____

$y = (x - 2)^2 + 3$ Vertex: _____

$y = (x + 1)^2 + 3$ Vertex: _____

$y = (x + 1)^2 - 2$ Vertex: _____

$y = (x + 5)^2$ Vertex: _____

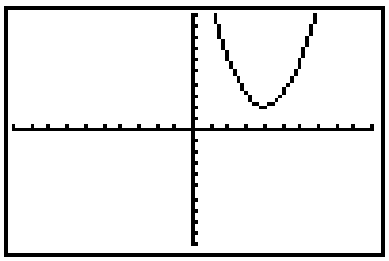
$y = x^2 - 2$ Vertex: _____

© 2004 TEXAS INSTRUMENTS INCORPORATED

What is the most likely equation of the parabola (quadratic function) graphed at the right?

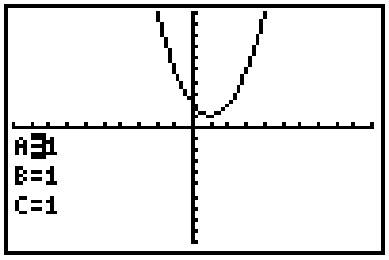
Note: *The scale is 1.*

_____



Studying the Effect of *A*

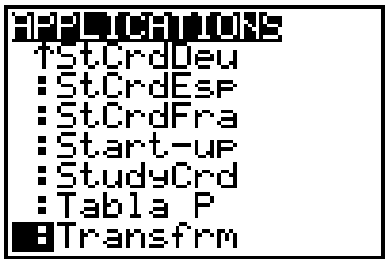
1. Return to the Transformation Graphing App screen, and press ▲ until the **A=** is highlighted.



2. Use the same discovery method you used with *B* and *C* to investigate the effect of *A* on the graph of the parabola. Be sure to let *A* be both negative and positive.

3. Deactivate the Transformation Graphing App before continuing.

a. Press APPS and select **Transfrm**.



b. Select **Uninstall**.

© 2004 TEXAS INSTRUMENTS INCORPORATED

Question for Discussion

1. What effect does changing the value of *A* have on the graph? Be sure to discuss both magnitude and sign change (positive and negative values).

_____

_____

Check Your Understanding

Match each equation from items **1–5** with its graph in column **2**. Be sure to look at all the equations and compare them before you answer any questions. Match equations and graphs first without using your graphing handheld, and then verify your answers using your graphing handheld.

Note: *These examples only investigate changes in the value of* A.

1. $y = 3(x - 2)^2 + 3$

_____

2. $y = {}^{-}(x - 2)^2 + 3$

_____

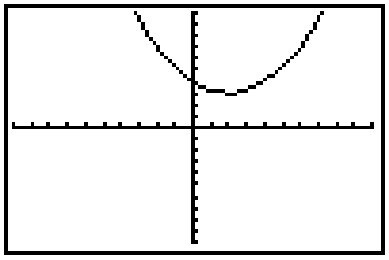
3. $y = 0.25(x - 2)^2 + 3$

_____

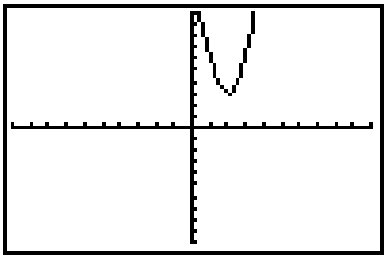
4. $y = {}^{-}2(x - 2)^2 + 3$

_____

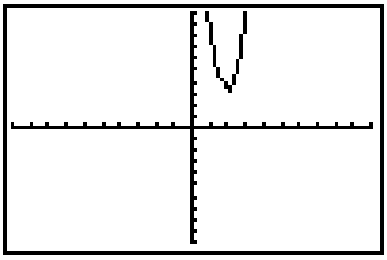
a.



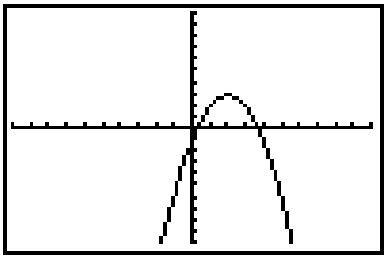
b.



c.



d.

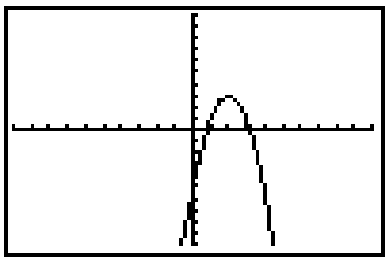


© 2004 TEXAS INSTRUMENTS INCORPORATED

5. $y = 6(x - 2)^2 + 3$

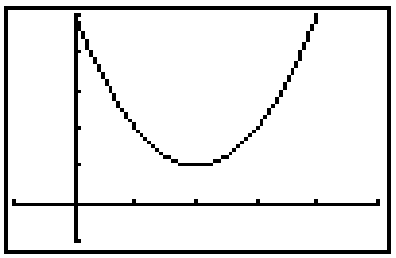
_____

e.



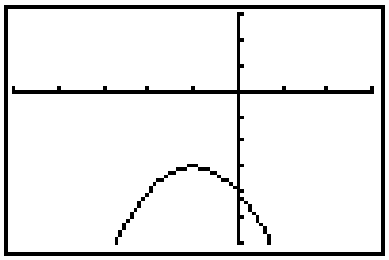
Maximum and Minimum Values Come into Focus

When a parabola opens upward, the vertex will be the lowest point on the curve. Any other point on the parabola will have a larger value for *y*. In the graph shown, the *y*-value of the vertex is 1. This is the lowest value of *y* that is on the parabola, and it is thus called the *minimum* value of the function.



The graph shows a parabola with a minimum value of 1 when $x = 2$.

Likewise, when a parabola opens down, the vertex will be the *maximum* value for *y*. This graph shows a function with a maximum value of $^{-}3$ when $x = {}^{-}1$.



© 2004 TEXAS INSTRUMENTS INCORPORATED

Use Your New Skill

Complete the table.

Equation	Opens up/ down	Function has a maximum/ minimum	Maximum/ Minimum value
$y = 2(x - 3)^2 + 2$	up	minimum	2
$y = -3(x + 1)^2 + 10$			
$y = -16(x - 2)^2 - 100$			
$y = 10(x + 4)^2 - 36$			

A Quick Application

The equation $y = -16(x - 4)^2 + 259$ models the flight of a model rocket where y is the height of the rocket and x is the time since it was launched. What is the maximum height of the rocket? How long after it was launched did it reach its maximum? What does this have to do with this activity?

© 2004 TEXAS INSTRUMENTS INCORPORATED

Student Worksheet

Name _____

Date _____

Look at some equations of linear functions and see how translation applies.

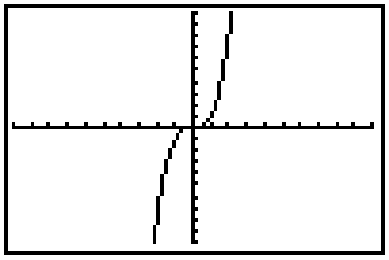
1. Use your graphing handheld to graph $y = x$ and $y = x + 3$ on the same axis. In what two ways is the second equation a translation of the first?

_____

_____

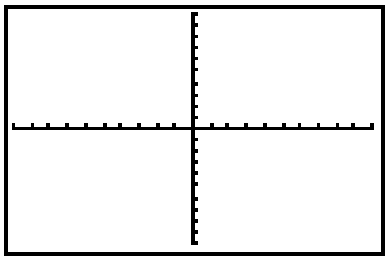
Now look at some functions you might not have already studied and see if you can apply your knowledge in a new situation.

2. The graph of the function $y = x^3$ goes through the origin (0, 0). Look at the graph of $y = x^3$ shown and using the point at the origin as the point you translate (as you did the vertex), sketch the graph of $y = x^3 + 2$. Check your answer by graphing $y = x^3 + 2$ on your graphing handheld.

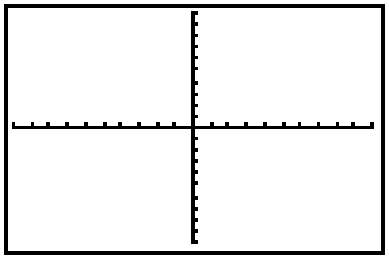


Note: *You can either use* [^] **3** *for the power of three or* [MATH] **3** *to select* **3:**$\sqrt[3]{}$.

3. Sketch $y = (x - 2)^3$, and check your answer.



4. Sketch $y = (x + 1)^3 - 5$, and check your answer.



© 2004 TEXAS INSTRUMENTS INCORPORATED

© 2004 TEXAS INSTRUMENTS INCORPORATED

Teacher Notes

Activity 3

Exploring the Vertex Form of the Quadratic Function

Objectives

- Explore the effects of changes in parameters on the graph of a parabola that is given in vertex form
- Predict the location of the vertex based on the quadratic function

Materials

- TI-84 Plus/TI-83 Plus

Time

- 60 minutes

The purpose of this activity is to allow the students to explore the vertex form of the parabola and discover how the vertex, direction, and width of the parabola can be determined by looking at the parameters.

Students will be able to make predictions about the location of the vertex of a parabola expressed in vertex form. They will not have had as much practice understanding the effect of *A*. They will be able to make statements about location based on the quadratic function.

The concept of translation is basic to the general study of functions, and the activity ends with a look at translation in general. These closing questions on translation are not really part of the study of the vertex form; you can easily leave them out. They serve as a overview of translation for those students who encounter translation as part of their study of function.

The work with translation at the end of the activity encourages students to make hypotheses and to verify them with their graphing handhelds.

Answers

The Graph of a Quadratic Function in Vertex Form

3. The vertex is at (2, 1). The parabola opens up.

Studying the Effect of B*: Questions for Discussion*

1. The curve moves in the *x*-direction. This moving of the curve is called a translation in the *x*-direction, or horizontal translation.

2. If **B=3**, the vertex is at (3, *y*). If **B=5**, the vertex is (5, *y*). If **B=⁻1**, the vertex is at (⁻1, *y*).

3. The value of *B* indicates the *x*-coordinate of the vertex of the parabola.

© 2004 TEXAS INSTRUMENTS INCORPORATED

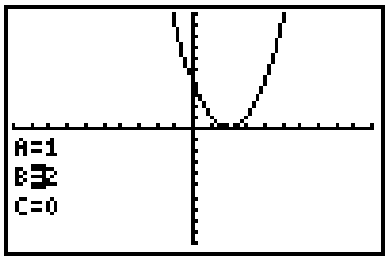
Studying the Effect of C

3. Changes in the value of *C* create a vertical translation of the curve. When the value of *C* increases, the curve moves up. When it decreases, the curve moves down. The value of *C* is the *y*-coordinate of the vertex.

Check Your Understanding So Far

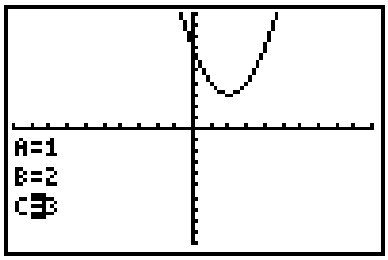
$y = (x - 2)^2$

Vertex is at (2, 0).



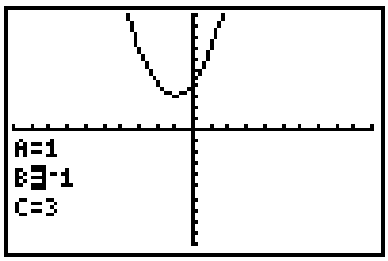
$y = (x - 2)^2 + 3$

Vertex is at (2, 3).



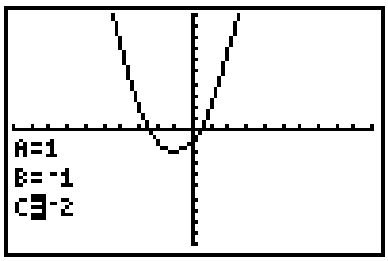
$y = (x + 1)^2 + 3$

Vertex is at (−1, 3).



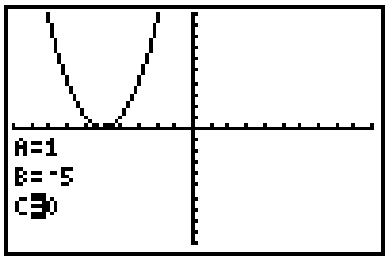
$y = (x + 1)^2 - 2$

Vertex is at (−1, −2).



$y = (x + 5)^2$

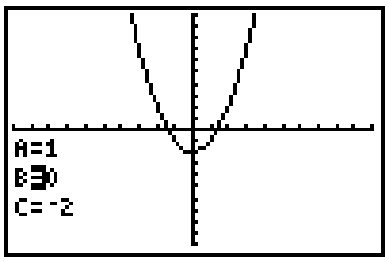
Vertex is at (−5, 0).



© 2004 TEXAS INSTRUMENTS INCORPORATED

$y = x^2 - 2$

Vertex is at (0, ⁻2).



What is the equation of the parabola (quadratic function)?

$y = (x - 4)^2 + 2$

Studying the Effect of A*: Question for Discussion*

1. The value of *A* determines the direction of the parabola and its width. The larger the magnitude of *A*, the narrower the curve. The smaller the magnitude of *A*, the wider the curve. A positive sign means that the parabola is opening up. A negative sign means that the parabola is opening down.

Check Your Understanding

1. b

2. d

3. a

4. e

5. c

Use Your New Skill

Equation	Opens up/ down	Function has a maximum/ minimum	Maximum/ minimum value
$y = 2(x - 3)^2 + 2$	up	minimum	2
$y = -3(x + 1)^2 + 10$	down	maximum	10
$y = -16(x - 2)^2 - 100$	down	maximum	⁻100
$y = 10(x + 4)^2 - 36$	up	minimum	⁻36

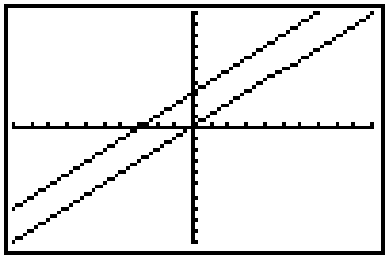
A Quick Application

The maximum height is 259 feet. It takes 4 seconds to reach the maximum. The vertex is the maximum or minimum value of the function. Because *A* is negative, the parabola opens down and has a maximum.

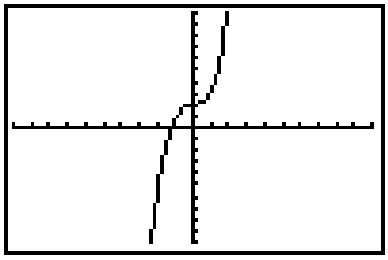
© 2004 TEXAS INSTRUMENTS INCORPORATED

Student Worksheet

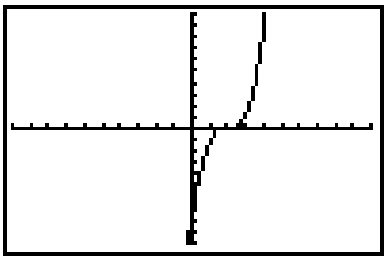
1. The second function ($y = x + 3$) can be viewed as a translation either 3 units up or 3 units to the left. If it is viewed as a translation 3 units up, you would be viewing the equation as $y = x + 3$. If you view it as a translation 3 units to the left (x-direction), you would be seeing the equation as $y = (x + 3)$.



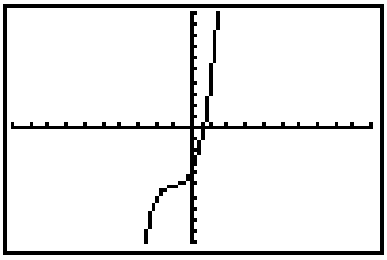
2. The graph of $y = x^3 + 2$ is a vertical translation of $y = x^3$ two units up. The point at the origin should be moved to (0, 2).



3.



4.



© 2004 TEXAS INSTRUMENTS INCORPORATED