NUMB3RS Activity: Tesseract
Episode: "Rampage"

Topic: Tesseracts and Hypercubes **Grade Level:** 9 - 12

Objective: An investigation of the fourth dimension

Time: 30 minutes

Introduction

In "Rampage," Charlie and Amita discuss the idea of dimensions and refer to a tesseract, a four-dimensional cube. This activity explores the idea of dimension and introduces the tesseract. The terms tesseract and hypercube are used by some interchangeably, but the hypercube is the broader name for the objects of any dimension referred to in this activity, and the tesseract is the fourth-dimensional equivalent of a cube.

A beginning discussion of dimension will be helpful for students who have only an intuitive understanding of dimension. Dimension can be thought of as degrees of freedom or movement. A one-dimensional space, which can be shown with a line or line segment, shows one movement which is along the line. A two-dimensional space allows movement in the plane. Think of the *x*-axis and *y*-axis as determining a plane. For example, a point has two coordinates, showing two movements, a vertical one and a horizontal one. To introduce the third dimension, or *z*-axis, draw the *x*- and *y*-axes on the board, and use a ruler starting at the origin and pointing out of the board. The movement of any point in three dimensions includes a horizontal (*x*), vertical (*y*), and an "outward" component (*z*).

More formally, dimension is defined by **www.Mathworld.com** as "…a topological measure of the size of its covering properties." Roughly speaking, it is the number of coordinates needed to specify a point on the object.

In this activity, students explore the development of a tesseract starting with a 0-dimensional point, building a 2-dimensional square and 3-dimensional cube along the way. At each stage, students count the number of vertices, edges, and faces. Writing recursive and explicit formulas that relate these values is left as an extension activity.

Students may struggle with the concept of a fourth dimension. What does it look like? Where is it? Visualizing the construction of the tesseract is a challenge. One of several internet resources are listed at the end of the activity includes a helpful section with references to *Flatland: A Romance in Many Dimensions* (see reference in Extensions), and analogies to lower dimensions.

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Discuss with Students

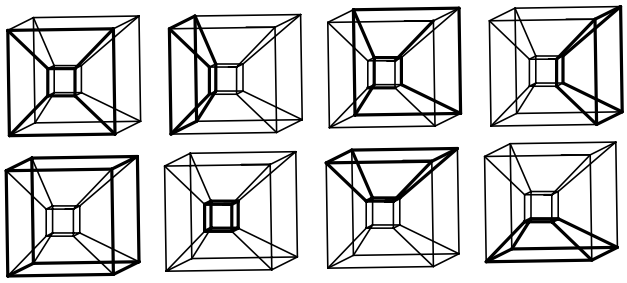
1. How many dimensions does a point have? A line segment? A square? Explain how you know.
2. Name a three-dimensional object. Explain how you know it has three dimensions.
3. In a rectangular prism, you have length, width, and height as your three dimensions. If there was a fourth dimension, what could it be?

Answers to Discuss with Students: 1. *a point only specifies a location but has neither length, width, nor height and is 0 dimensional. A line segment has a length, so it is one-dimensional. A square has length and width, so that it is two-dimensional.* **2.** *Answers will vary; samples include a cube, which has length, width and height.* **3.** *Answers will vary. Note: some students may consider time to be the fourth dimension, but this can be a limiting thought. This question is intended as a brainstorming opportunity for students, and the idea of the fourth dimension will be explored in this activity.*

Student Answers: 1. *point has 1 vertex; segment has 2 vertices* **2.** *point: 0 edges; segment: 1 edge* **3.** *4 vertices, 4 edges* **4.** *8 vertices, 12 edges, 6 faces* **5.** *the complete chart is shown below*

Shape	# of Vertices	# of Edges	# of Faces	# of Cubes
Point	1	0	0	0
Segment	2	1	0	0
Square	4	4	1	0
Cube	8	12	6	1
Tesseract	16	32	24	8

Patterns may include the number of points doubling each dimension; others are possible **6.** *A tesseract has 16 vertices, each of the 8 vertices of the cube doubled.* **7a.** *4 from the original square, 4 from the projection, and 4 from each of the square's vertices sweeping into a new dimension or 12 edges.;* **7b.** *12 from the original cube, 12 from the projection and 8 from the vertices sweeping into a new dimension or 32 edges* **8.** *24 faces: 6 from the original cube, 6 from the projection and 12 from the edges of the cube sweeping into a new dimension.* **9.** *The tesseract has 8 cubes: the original, the copy, and one from each of the cube's 6 faces sweeping into a new direction. A drawing depicting the cubes is shown below.*



10. *16 vertices, 32 edges, 24 faces, 8 cubes* **11.** *Both are cases of trying to understand a dimension one degree higher than the one in which you exist.*

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Episode: "Rampage"

Name _____ Date _____

NUMB3RS Activity: Tesseract

In "Rampage," Charlie is struggling to understand an aspect of the case that he is trying to solve, and he and Amita end up discussing the idea of dimensions and refer to a **tesseract**, a four dimensional cube. Amita helps Charlie look at things from a different point of view or perspective. Seeing the tesseract model reminds Charlie that he needs to look at the big picture and not be so narrowly focused. This activity explores the idea of dimension and introduces the tesseract.

A point is a 0-dimensional object having no length, width, or height. To create a 1-dimensional object, project the point one unit in any direction and connect the point and its image with a segment. This line segment is a 1-dimensional object, having length, for example, 1 unit. In graph theory, points are called vertices and segments are called edges.

1. How many vertices does a point have? A segment?

2. How many edges does a point have? A segment?

To create a 2-dimensional object, slide the unit segment in a direction along a perpendicular to the segment and with the length one unit. Connect corresponding vertices with edges. The resulting 2-dimensional object is a square, having both length and width of 1 unit.

3. How many vertices and edges does a square have?

4. To create a 3-dimensional object, slide the square 1 unit along a perpendicular to the plane of the square. Connect corresponding vertices with edges forming a cube, which has length, width, and height of 1 unit. The resulting squares formed as a part of the cube are faces of the cube. How many vertices, edges, and faces does a cube have?

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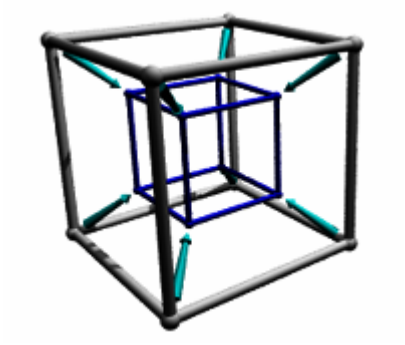
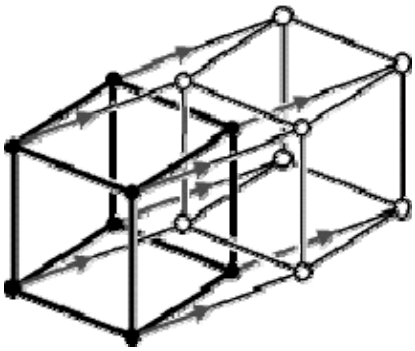
5. Fill in the chart below with the values determined so far (some values will be 0). Make a note of any patterns you see.

Dimension (Shape)	**# of Vertices**	**# of Edges**	**# of Faces**	**# of Cubes**
0-dimensional (Point)				
1-dimensional (Segment)				
2-dimensional (Square)				
3-dimensional (Cube)				

A tesseract is built by projecting a cube 1 unit into a fourth dimension. Two different illustrations of this projection are shown below. When the vertices of the cube and its projection are connected with edges (the arrows in the drawings), a tesseract is formed. Visualizing this object presents quite a challenge, since we do not have a fourth dimension to use in making the drawing.



http://tetraspace.alkaline.org/introduction.htm



http://www.math.union.edu/~dpvc/math/4D/basics/cube-move.html

6. Remember that a point was projected to create a segment with 2 vertices. Those 2 vertices and the segment they determined were projected to create a square with 4 vertices. Follow this pattern to determine the number of vertices in a tesseract.

7. A square has four edges. The original segment and its image are two of the four. The other two edges were created when the corresponding vertices were connected (giving the new dimension).

a. Use the same logic to explain the cube's 12 edges.

b. Determine how many edges a tesseract has.

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8. A cube has 6 faces, one from the original square, one from the image, and 4 created when the vertices of the segments of the original square are connected to their corresponding parts in the image (giving the new dimension). How many faces does a tesseract have?

9. How many cubes does a tesseract have? Your answer should be verified by the sketch below of a tesseract (the sketch is actually a "net of a tesseract "unfolded into 3 dimensions, the same way a box can be unfolded into 2 dimensions).

Cube net unfolds into 2 dimensions Tesseract net unfolds into 3 dimensions

10. In summary, a tesseract is a four-dimensional object with _____ vertices, _____ edges, _____ faces and _____ cubes. Visit some of the websites suggested by your teacher to learn more about what a tesseract looks like and how to build a model.

11. In the book *Flatland: A Romance of Many Dimensions (*New York: Dover Publications, 1952), Edwin A. Abbott tells the tale of a group of people who exists in two dimensions and are visited by a sphere. How does their struggle to understand the sphere relate to our struggle to understand the tesseract?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

The table developed in this activity is shown below. Develop recursive and explicit formulas that relate the values.

Shape	# of Vertices	# of Edges	# of Faces	# of Cubes
Point	1	0	0	0
Segment	2	1	0	0
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Other Resources

- This Web site provides a summary of this activity.
  http://www.math.union.edu/~dpvc/math/4D/basics/connections.html
- This Web site has an easily understood paper on viewing four-dimensional objects in three dimensions.
  http://www.geom.uiuc.edu/docs/forum/polytope
- This paper by Alan B Scrivener is about visualizing a tesseract which refers to Flatland and using analogies to lower dimensions
  http://www.well.com/~abs/SIGGRAPH96/4Dtess.html
- This Web site has a variety of activities on dimension by Thomas F. Banchoff, a professor at Brown University.
  http://www.geom.uiuc.edu/~banchoff/projects.html
- A presentation about the fourth dimension by Thomas F. Banchoff.
  http://www.faculty.fairfield.edu/jmac/cl/tb4d.htm
- This Web site contains an applet to rotate and view a tesseract
  http://www.maa.org/editorial/knot/tesseract.html
- The two Web sites below contain nice visuals of tesseracts.
  http://www.howtoadapt.com/ncubes/ncubes/sld009.htm
  http://dogfeathers.com/java/hyprcube.html
- This Web site describes a group of students who built a model of a tesseract
  http://howtoadapt.com/ncubes/MakingTesseracts/MakingTesseracts.shtml
- In the book *A Wrinkle in Time by* Madeline L'Engle, tesseracts are used to allow characters to move through time.

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