

Average Orange

ID: 10280

Time required
45 minutes

Activity Overview

This activity presents the normal probability distribution, beginning with the normal curve. Students explore the area under the curve between various x-values and use a model to find what percent of the area lies within 1, 2, and 3 standard deviations of the mean. They then apply this knowledge to answer questions about an orange crop with normally distributed weights. Percentiles are defined and calculated.

Topic: Statistics

- *Determine the probability that a normally distributed variable will take a particular value in a given interval.*
- *Determine the xth percentile for a normally distributed variable given its mean and standard deviation.*

Teacher Preparation

- *This investigation is intended as an introduction to normal distribution for an Algebra 2 class. It could also be used in a precalculus or introductory statistics class.*
- *Students should have experience calculating the probabilities of simple and compound events. Experience calculating the area of compound shapes will also be helpful in completing the activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter “10280” in the quick search box.***

Associated Materials

- *Alg2Week33_AvgOrange_worksheet_TINspire.doc*
- *Alg2Week33_AvgOrange.tns*
- *Alg2Week33_AvgOrange_Soln.tns*

Suggested Related Activities

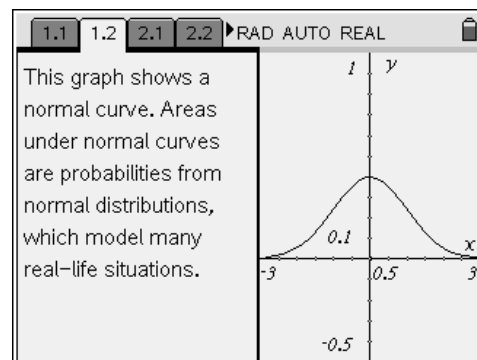
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Percentiles (TI-Nspire technology) — 9539*
- *Percentiles & Z-Scores (TI-Nspire technology) — 11205*
- *What’s Normal, Anyway (TI-Nspire technology) — 8556*

Problems 1 and 2 – Exploring the Normal Curve

Page 1.2 displays and describes a normal curve, also known as a bell curve. Discuss the history of the curve, which was developed by Abraham de Moivre in 1756 as an approximation for the binomial distribution. The normal curve is one the most famous and useful functions in all of mathematics. You may choose to display the formula for the curve:

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

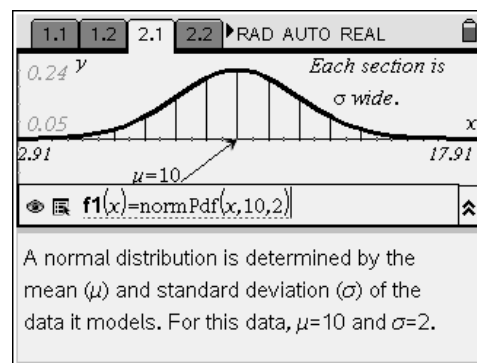


Explain to students that there is no need to use this formula when making calculations: their handheld will do the work for them.

Discuss the shape of the curve. It is symmetrical and extends to infinity in both directions. Its height is greatest in the center and decreases to the right and left, approaching (but never quite reaching) zero.

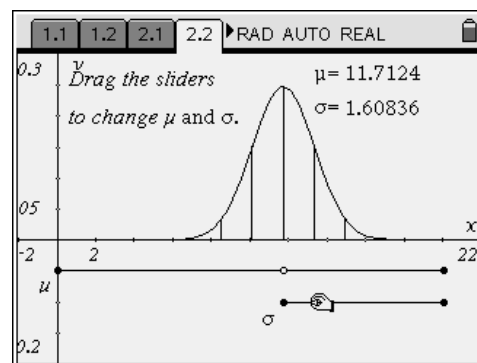
Discuss the significance of the curve. The real-life distribution of many real-life quantities can be approximated by this curve. Also, if you draw many random samples from the same population, their averages will be normally distributed.

On page 2.1, students should press **ctrl** + **G** to display the function **f1(x)**. Note the **normPdf** command and it's arguments: the random variable (x), the mean (10), and the standard deviation (2).



The shape of a normal curve is controlled by two parameters: the mean, which controls the location of the hump of the curve, and the standard deviation, which controls whether the bell is broad and flat (larger standard deviation) or narrow and tall (smaller standard deviation). If the bell curve represents the distribution of some measurement across a population, the mean is the population average and the standard deviation measures the spread of the population measurements.

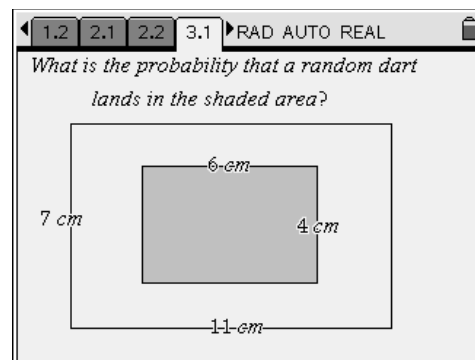
Students can explore the effect of these parameters on the curve by adjusting the sliders on page 2.2. Point out that no matter what the mean and standard deviation, the total area under every normal curve is the same: 1.



Problem 3 – Probability as Area

This idea of representing probability as area can be difficult for many students to grasp, especially when it is introduced simultaneously with the new concept of “area under a curve.” So we begin with a simpler example: the dartboard shown on page 3.1.

Ask students to imagine that they are randomly throwing darts at this board. The darts always hit the board, but where they hit is random; they are equally likely to hit any point on the board. How would you find the probability that a particular dart will land in the shaded area?



In this case, each outcome is represented by a point (where the dart hits) and we can find the number of outcomes by “counting the points,” also known as calculating an area.

What are the possible outcomes?

(All the points on the board, represented by an area of 77 sq. cm.)

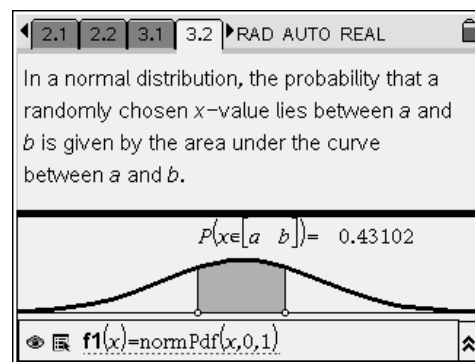
What are the desired outcomes?

(All the points in the shaded area, represented by an area of 24 sq. cm.)

What is the ratio of desired outcomes to possible outcomes? (24/77 ≈ 0.3117)

This is the probability that a dart will land in the shaded area.

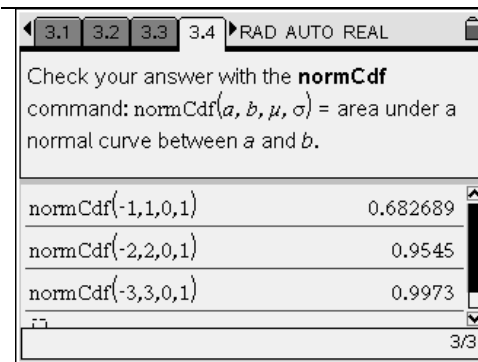
Page 3.2 describes the relationships between the normal curve and the normal distribution. Remind students that a distribution is a function that pairs probabilities and events. In the normal distribution, events are described in terms of x -values. The probability of a randomly selected x -value lying in the interval (a, b) is given by the area under the curve between a and b .



Clarify what is meant by the area under a curve, shown by the shaded area in the screenshot. Have students move the sliders and note how the shaded area changes.

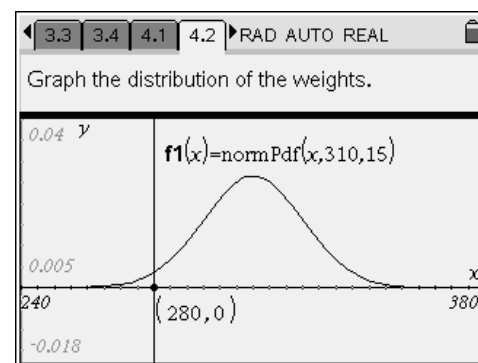
To help students understand why the probability is equal to this area, return to the “darts” analogy for probability: imagine you are randomly throwing darts at a dartboard that is the shape of the normal curve. The probability that a dart lands on a point with an x -coordinate that is between a and b would be equal to the ratio of the shaded area shown and to the total area under the curve. Since the total area of every normal curve is 1, the probability is equal to the shaded area.

Have students follow the directions on pages 3.3 and 3.4 independently to find the area under the curve that is with 1, 2, and 3 standard deviations of the mean. Explain the result in terms of percentages. (68% of the x -values lie within 1 standard deviation of the mean, etc.) This result is a fast way to approximate probabilities from the normal distribution without calculating the area under the curve.



Problem 4 – Application of the Normal Distribution

Page 4.1 presents a real-life variable that is normally distributed: the weight of each orange in a farmer’s crop. Students should graph the distribution on page 4.2 using what they have learned about the **normPdf** command. They will need to adjust the window in order to see the curve clearly.



Page 4.3 asks for the percent of the oranges that weight 280 grams or less. Students should realize that 280 is 2 standard deviations away from the mean ($310 - 15 - 15$). Since 95% of the area lies within 2 standard deviations of the mean, the total area under the curve is 1, and the curve is symmetrical, the percent of oranges weighing less than 280 grams is

$$P(x < 280) = \frac{1 - 0.9545}{2} = 0.02275 = 2.275\%$$

TI-nspire calculator screen showing text instructions for the problem. The text reads:

The farmer sells all of the oranges weighing 280 grams or less to a juicer.

On the previous page, draw a vertical line to show this area on the graph.

What percent of his oranges will be sold to the juicer?

(Hint: Use your results from page 3.3.)

Once students have used geometric logic and their results from pages 3.2 and 3.3 to solve this problem, demonstrate another way. Use the **normCdf(a, b, μ , σ)** command, which calculates the area under a normal curve with mean μ and standard deviation σ between a and b . In this case, the command **normCdf(-9E99, 280, 310, 15)** yields the same result.

The number -9E99 represents negative infinity. If you prefer not to use this approximation, calculate the area between 280 and 310 (**normCdf(280, 310, 310, 15)**) and subtract it from 0.5, the area to the left of 310.

Students should use the **normCdf** command to find the percent of oranges weighing 300 grams or more on page 4.4. Two methods for doing so are shown in the screen shot.

Once students have used the **normCdf** command to solve this problem, demonstrate another way. Use the **Integral** command to find the area under the curve. On page 4.2, place a point at (300, 0). Then go to **MENU > Measurement > Integral**, click on the curve, click on (0, 300) to set the lower bound, then click on a point at the far right of the screen to set the upper bound.

Remind students that because the curve extends to infinity, this is not the exact area, but because the height of the curve is so close to 0 this far away from the mean, it is very close.

Page 4.5 defines percentile and page 4.6 asks students to find the percentile of particular orange and find the weight of an orange at a particular percentile.

Students should use either the **normCdf** or integral methods for calculating the area under the curve to the left of 320 grams.

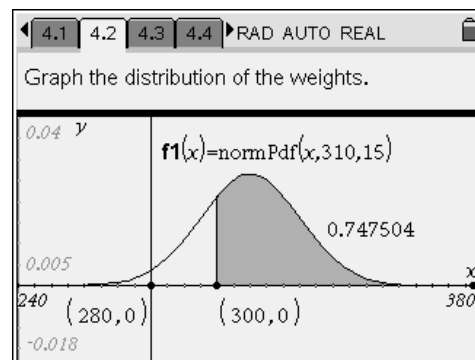
To find the weight of an orange at the 84th percentile, student should recall that 68% of the area under a normal curve is within 1 standard deviation of the mean. Half of 68% is 34%, so 34% of the area is within one standard deviation and to the right of the mean. Half of the curve, or 50%, is to the left of the mean, so the area from negative infinity to one standard deviation above the mean is 84%. An orange weighing $310+15 = 325$ g is at the 84th percentile. Students should use the **normCdf** or integral commands to check their answers.

4.1 4.2 4.3 4.4 RAD AUTO REAL

The farmer sells the oranges weighing 300 g or more to a commercial buyer. What percent of his oranges will be sold to the buyer?

$\text{normCdf}(300,310,310,15)+0.5$	0.747508
$\text{normCdf}(300,1.E99,310,15)$	0.747508

2/99



4.3 4.4 4.5 4.6 RAD AUTO REAL

What is the percentile rank of an orange weighing 320 grams?

$\text{normCdf}(-1.E99,320,310,15)$	0.747508
$0.5+\text{normCdf}(310,320,310,15)$	0.747508

2/99

4.4 4.5 4.6 4.7 RAD AUTO REAL

What is the weight of an orange at the 84th percentile?

$310+15$	325
$0.5+\text{normCdf}(310,325,310,15)$	0.841345

2/99