



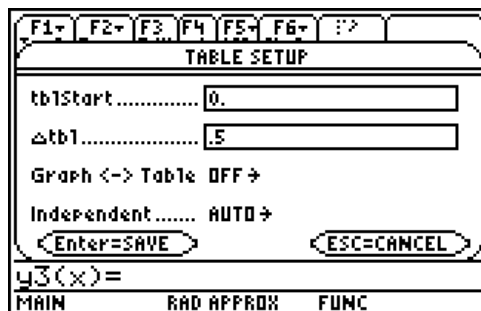
**Part 1 – Warm up**

Use a graph to confirm your answers to the following questions.

1. a. Where is  $f(x) = e^x$  positive?
   
 b. Where is  $f(x) = e^{-x}$  positive?
  
2. What is the domain of  $\ln(x)$ ?
  
3. Where is  $y = \ln(x)$ :
  - a. Positive?
  
  - b. Positive number less than 1?
  
  - c. Greater than 1?

**Part 2 – Investigate the derivative of  $e^x$**

In **y1**, enter  $e^x$ . In **y2**, enter  $d(y1(x),x)$ . Press  $\square + [\text{TblSet}]$  and set the table settings to match the settings to the right. Then, press  $\square + [\text{TABLE}]$  to go to the table. Examine the values of  $e^x$  and its derivative.



4. What is the derivative of  $e^x$ ?

On the HOME screen, enter the following derivatives.

5.  $\frac{d}{dx}(e^{2x}) =$

6.  $\frac{d}{dx}(e^{x^2}) =$

$\frac{d}{dx}(e^{3x}) =$

$\frac{d}{dx}(e^{x^3}) =$

$\frac{d}{dx}(e^{5x}) =$

$\frac{d}{dx}(e^{5x^4}) =$

$\frac{d}{dx}(e^{-7x}) =$

$\frac{d}{dx}(e^{-2x^{10}}) =$

What is the pattern?

What is the pattern?



## Exponentially Fast Derivative

7. Try the follow problems using the Chain Rule. Then use the HOME screen. Record both answers and explain why the two solutions are equivalent.

By Hand	Using Technology	Explanation of Equivalence
a. $\frac{d}{dx}(e^{4x^{0.5}}) =$		
b. $\frac{d}{dx}(e^{e^{3x}}) =$		
c. $\frac{d}{dx}(\sin(e^{-2x})) =$		
d. $\frac{d}{dx}(\cos^3(e^x)) =$		
e. $\frac{d}{dx}(e^{\ln(5x)}) =$		

### Part 3 – Investigate $a^x$

8. On the HOME screen, press  $\square + \text{ENTER}$  to find the decimal approximation for the following numbers.
- a.  $\ln(2)$                       b.  $\ln(e)$                       c.  $\ln(4)$

To understand how the derivative of  $2^x$  is derived, you need to know the derivative of a constant times  $x$ , even when it might not look like a constant at first glance.

9. a.  $\frac{d}{dx}(\pi x) =$                       b.  $\frac{d}{dx}(x \ln(2)) =$



## Exponentially Fast Derivative

Now examine the derivation of  $2^x$  using implicit differentiation:

$$y = 2^x$$

$$\ln(y) = \ln(2^x)$$

$$\ln(y) = x \cdot \ln(2)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x \cdot \ln(2))$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(2)$$

$$\frac{dy}{dx} = y \cdot \ln(2)$$

$$\frac{dy}{dx} = 2^x \cdot \ln(2)$$

10. Differentiate the following functions.

a.  $y = \left(\frac{3}{2}\right)^x$

b.  $y = 3^x$

c.  $y = 4^x$

d.  $y = 0.5^x$

e.  $y = 2 \cdot 5^x$

11. What is the rule applied to exponential functions?

12. Try the following questions on your own first. Then use the HOME screen and explain why the two solutions are equivalent.

By Hand	Using Technology	Explanation of Equivalence
a. $\frac{d}{dx}(3^{x^7}) =$		
b. $\frac{d}{dx}(5^{2x+3}) =$		
c. $\frac{d}{dx}\left(\left(\frac{3}{2}\right)^{2x+2}\right) =$		
d. $\frac{d}{dx}\left(\left(\frac{1}{2}\right)^{3x}\right)$		



## Part 4 – Extension/Homework: Exam practice questions

13. a. When playing a video game, a button is pressed causing a point to move along the  $x$ -axis described by  $x(t) = -2t + 3 + e^{1-2t}$ , where  $t$  is time. Find the acceleration when  $t = 2$ .

b. If the positive  $x$  direction is forward, describe the speed of the particle at  $t = 2$ . Explain your reasoning.

14. Let  $f(x) = \ln(e^{-5x} + 2x - 3)$ . Find  $f'(0)$ .

15. Find  $f'(x)$  if  $f(x) = e^{-\cos(x)}$

16. Let  $f(x) = x \cdot e^{-2x}$ .

a. Solve for  $x$  when  $f'(x) = 0$ .

b. Solve for  $x$  when  $f''(x) = 0$ .