## Activity Overview

In this activity, students will use polynomial calculations to determine quotients and remainders when performing polynomial division using CAS commands. The Remainder Theorem is introduced and applied to identify roots or zeros and to determine function values. Graphs are incorporated to visually illustrate outcomes of polynomial division.

## Topic: Polynomial Division

- Remainder
- Quotient
- Remainder Theorem
- Evaluating functions
- Roots/Zeros and Factors of Functions


## Teacher Preparation and Notes

- This activity was designed for use with TI-Nspire CAS technology.
- Problems 1 and 2 may be done in class, and Problem 3 could either be done in class or assigned as homework. Questions may be answered on the handheld or the associated worksheet.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11607" in the keyword search box.


## Associated Materials

- PolyDivision_Student.doc
- PolyDivision.tns
- PolyDivision_Soln.tns


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- The Remainder Theorem Using TI-Nspire CAS (TI-Nspire CAS technology) - 9254
- Multiplication \& Division of Functions (TI-Nspire technology) - 10218
- Polynomial Division (TI-89 Titanium) - 5505


## Problem 1 - Introduction

Problem 1 involves a basic review of terminology associated with division. Next, a basic example is provided to introduce the polynomial tools available on the TI-Nspire CAS handheld. This example is first observed graphically to illustrate the result of the degree of the quotient being less than that of the dividend.

After exploring the graph, students then are introduced to the Quotient of Polynomial (polyQuotient) and Remainder of Polynomial (polyRemainder) commands.

Next, students factor the dividend using the Factor command. They are asked about the result of this dividend being divided by $x-1$, a factor.
At this point, encourage students to look back at the graph on page 1.4 of $\mathbf{f 1}(x)$ as compared to $\frac{\mathbf{f}(x)}{\mathbf{f 2}(x)}$. Discuss what happens graphically as a result of the division by the factor $x-1$.

## Problem 2 - Remainders

This section of the activity introduces problems in which the divisor does not divide evenly into the dividend. The Remainder Theorem is introduced and explained.



\section*{| 1.7 | 1.8 | 2.1 |
| :--- | :--- | :--- | <br> }

## Remainder Theorem

A polynomial, $\mathbf{p}(\mathbf{x})$, when divided by a linear factor $\mathbf{x - a}$, where $\mathbf{a}$ is a number, results in a polynomial quotient, $q(x)$, and some remainder, $\mathbf{r}$.

Another way to state this is

$$
\mathbf{p}(x)=(x-a) \cdot q(x)+r .
$$

If $r=0, x-a$ is a factor of $p(x)$

Again, division is performed using the polynomial tools. Students are asked to use the Remainder Theorem to interpret the meaning of their results as they relate to function values and roots or zeros.


| 13.1 | 3.2 | 3.3 |
| :--- | :--- | :--- | :--- |

Use this page for pages 3.2-3.3.
Dividend: $f(x):=6 \cdot x^{2}-5 \cdot x^{2}+4 \cdot x-17$
Divisior: $\mathbf{g}(x):=x-3$
Quotient: $x+7$
Remainder: 4
Factor: $x^{2}+4 x-17$
Value: $\mathbf{f}(3)+4$

\section*{| 3.1 | 3.2 | 3.3 | *PolyDivision $\nabla$ |
| :--- | :--- | :--- | :--- |}

Use this page for pages 3.2-3.3.
Dividend: $\mathbf{f}(x):=x^{5}-23 \cdot x^{3}+6 \cdot x^{2}+112 \cdot x-96$
Divisior: $g(x):=x+4$
Quotient: $x^{4}-4 \cdot x^{3}-7 \cdot x^{2}+34 \cdot x-24$
Remainder: 0
Factor: $(x-4) \cdot(x-2) \cdot(x-1) \cdot(x+3) \cdot(x+4)$
Value: $\mathbf{f ( - 4 ) + 0}$


## Student Solutions

1. a. dividend
b. quotient
c. divisor
2. a. $x^{4}+3 x^{2}-10 x-24$
b. 0
3. $(x-3)(x+2)(x+4)$. Division by $x-1$ will remove the factor $x-1$ from the dividend.
4. a. $x^{2}+4 x+9$
b. 30
5. 30
6. disagree
7. a. $6 x^{2}+13 x+43$
b. 112
8. 112
9. disagree
10. a. $x^{4}-4 x^{3}-7 x^{2}+34 x-24$
b. 0
11. 0
12. agree
13. Dividing by a factor results in the removal of zeros or roots from the graph when comparing the graph of the original function to that of the quotient.
