LESSON
7.8

PLANNING

LESSON OUTLINE

One day:

5 min	Introduction
35 min	Activity
10 min	Sharing

MATERIALS

- balls
- metersticks
- motion sensor, *optional*
- soda cans half-filled with water
- string
- Bounce Sample Data (W), *optional*
- Pendulum Sample Data (W), *optional*
- Calculator Note 7D

TEACHING

Exponential decay equations can model processes that slow down.

Guiding the Activity

It is not mandatory to have all of these materials. The motion sensor can be supplanted with careful low-tech data collection. The bouncing ball is needed only for Experiment 1, and the string and can are needed only for Experiment 2. In place of tying a string around the pull tab of a soda can, you might tie the string around the neck of a water bottle.

If materials in general are problematic, try the 100-grid alternative explained in Lesson 7.7.

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7.8

Activity Day

Decreasing Exponential Models and Half-Life

In Lesson 7.7, you learned that data can sometimes be modeled using the exponential equation $y = A(1 - r)^x$. In this lesson you will do an experiment, write an equation that models the decreasing exponential pattern, and find the **half-life**—the amount of time needed for a substance or activity to decrease to one-half its starting value. To find the half-life, approximate the value of x that makes y equal to $\frac{1}{2} \cdot A$.

Technology CONNECTION

You can see simulations of atomic half-life with a link at www.keymath.com/DA .

In the previous investigation, if your plate was marked with a 72° angle and you started with 200 "atoms," a model for the data could be $y = 200(1 - 0.20)^x$. This is because the ratio of the angle to the whole plate is $\frac{72}{360}$, or 0.20. To determine the half-life of your atoms, you would need to find out how many drops you would expect to do before you had 100 atoms remaining. Hence, you could solve the equation $100 = 200(1 - 0.20)^x$ for x using a graph or a calculator table. The x-value in this situation is approximately 3, which means your atoms have a half-life of about 3 years.

Activity
Bouncing and Swinging

You will need

- a motion sensor
- a meterstick
- a ball
- string
- a soda can half-filled with water

There are two experiments described in this activity. Each group should choose at least one, collect and analyze data, and prepare a presentation of results.

LESSON OBJECTIVE

- Write exponential equations that model real-world decay data

NCTM STANDARDS

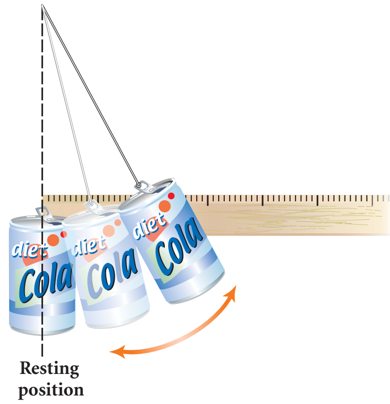
CONTENT		PROCESS	
•	Number		Problem Solving
•	Algebra	•	Reasoning
	Geometry		Communication
•	Measurement	•	Connections
•	Data/Probability	•	Representation

Step 1 Select one of these two experiments.

Experiment 1: Ball Bounce

You will drop a ball from a height of about 1 meter and measure its rebound height for at least 6 bounces. You can collect data "by eye" using a meterstick, or you can use a motion sensor. [▶ See **Calculator Note 7D** for a program to use with your motion sensor. ◀] If you use a motion sensor, hold it $\frac{1}{2}$ meter above the ball and collect data for about 8 seconds; trace the resulting scatter plot of data points to find the maximum rebound heights.

Experiment 2: Pendulum Swing



Make a pendulum with a soda can half-filled with water tied to at least one meter of string—use the pull tab on the can to connect it to the string. Pull the can back about $\frac{1}{2}$ meter from its resting position and then release it. Measure how far the can swings from the resting position for several swings. You can collect data "by eye" using a meterstick (you may have to collect data for every fifth swing in this case), or you can use a motion sensor. [▶ See **Calculator Note 7D** for a program to use with your motion sensor. ◀] If you use a motion sensor, position it 1 meter from the can along the path of the swing; the program will collect the maximum distance from the resting position for 30 swings.

Step 2 Set up your experiment and collect data. Based on your results, you might want to modify your setup and repeat your data collection.

Step 3 Define variables and make a scatter plot of your data on your calculator. (If you used a motion sensor, you should have this already.) Sketch the plot on paper. Does the graph show an exponential pattern?

Step 4 Find an equation of the form $y = A(1 - r)^x$ that models your data. Graph this equation with your scatter plot and adjust the values if a better fit is needed.

Step 5 Find the half-life of your data. Explain what the half-life means for the situation in your experiment. (Read p. 414 to review the calculation of half-life.)

Step 6 Find the y-value after 1 half-life, 2 half-lives, and 3 half-lives. How do these values compare? With each consecutive half-life, the value of y will be $\frac{1}{2}$ the previous value of y.

Step 7 Write a summary of your results. Include descriptions of how you found your exponential model, what the rate r means in your equation, and how you found the half-life. You might want to include ways you could improve your setup and data collection.

Step 3 The scatter plot should show an exponential pattern.

Step 5 One method is to graph $Y_2 = \frac{1}{2} \cdot A$ and find the intersection with $Y_1 = A(1 - r)^x$. Using sample data half-lives are:
Exp. 1: about 2 bounces
Exp. 2: about 17 swings

In the real world, eventually your ball will stop bouncing or your pendulum will stop swinging. Your exponential model, however, will never reach a y-value of zero. Remember that any mathematical model is, at best, an approximation and will therefore have limitations.

SHARING IDEAS

Groups can share the summaries from Step 7. The class can discuss the reasons for the differences in data and equations that model the data.

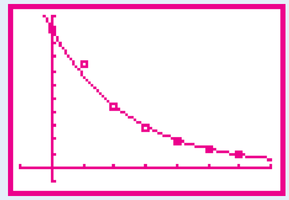
Closing the Lesson

As needed, point out that the term *exponential decay* refers to slowing processes other than radioactive decay.

Step 3 The location of the pendulum bob is harmonic, but its maximum distance from center is roughly exponential.

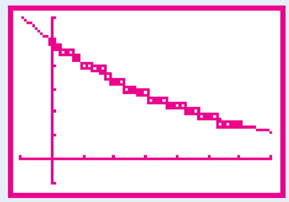
Step 4 Students who collect pendulum data by eye will need to account for collecting data every fifth swing. One option is $y = A(1 + r)^{\frac{x}{5}}$, where x is the number of swings.

Step 4 For sample data:
Exp. 1: $y = 1(1 - 0.33)^x$



$[-1, 7, 1, -0.1, 1.1, 0.1]$

Exp. 2: $y = 0.50(1 - 0.04)^x$



$[-5, 35, 5, -0.1, 0.6, 0.1]$

Step 6 You might want to introduce the equation $y = A\left(\frac{1}{2}\right)^{\frac{x}{t}}$ where t is the half-life. Students can see that the graph of this equation is similar to that of their equation in the form $y = A(1 - r)^x$. Ask them to think about why the graphs are the same. [Looking at special cases, when $x = t$, the quantity A is multiplied by $\frac{1}{2}$. When $x = 2t$, $\left(\frac{1}{2}\right)A$ is multiplied by $\frac{1}{2}$, to make $\frac{1}{4}A$. And so on. Symbolically, by definition of half-life, $b^t = \frac{1}{2}$, so raising both sides to the $\frac{x}{t}$ power gives $b^x = \left(\frac{1}{2}\right)^{\frac{x}{t}}$.]

ASSESSING PROGRESS

Watch for ability to collect data systematically, to define variables, to make a scatter plot, to find common ratios and write an appropriate exponential decay equation, and to find the half-life.