# Exploring Asymptotes

## Student Worksheet

**Name** ______________________

**Class** ______________________

### Rational Functions

1. On page 1.3 of the *CollegeAlg_Asymptotes.tns* file is a graph of the function $f(x) = \dfrac{1}{x^2 - 9}$.

   **a.** Is the function defined over all values of $x$? Sketch your graph below and explain.

   **b.** On the graph on page 1.3, insert a table of values for the function. While on the graphing screen, press ctrl + T and a function table will be inserted. How does the table of values indicate that a value is undefined for a function? What $x$-values have no corresponding $y$-value?

   **c.** Go back to the graph on page 1.3 and construct a vertical line perpendicular to the $x$-axis. Select **MENU > Construction > Perpendicular**, move the pencil to the $x$-axis, and press enter. Drag this line horizontally to identify the $x$-values for which there is no corresponding $y$-value.

   **d.** In order to understand why certain $x$-values are undefined, it is helpful to look at the function in factored form. Factor the denominator of the function. You can use the *Calculator* page on the bottom of page 1.7. How does the factored denominator relate to the undefined $x$-values found on the graph?

   **e.** When the undefined $x$-values are substituted into the function, what happens to the denominator? What effect does this have on the function at these values?

These undefined $x$-values that result in a denominator becoming equal to zero are referred to as **singularities**. In the case of the given function, the singularities are $-3$ and 3 because these values make the denominator equal to zero.

**f.** Look back over your graph on page 1.3. Notice that the function value approaches $\pm\infty$ to the left and right of the values for which the function is undefined. In this particular case, this means that the locations of the $x$-values where the function is undefined are also locations of **asymptotes**, or lines on the graph that the function approaches, getting closer and closer, but never reaches. Do you think $x$-values where the function is undefined will always give asymptotes? Explain your reasoning.

**g.** On the *Calculator* page at the bottom of page 1.11, factor the denominator of the function $f(x) = \frac{x-3}{x^2-9}$. For what values of $x$ would you expect the graph of $f(x)$ to have asymptotes?

**h.** Graph $f(x) = \frac{x-3}{x^2-9}$ on page 1.13 of the TI-Nspire document. From the graph of the function $f(x) = \frac{x-3}{x^2-9}$, what is (are) the vertical asymptote(s)?

**i.** Explain why the number of asymptotes as seen on the graph does not match the number expected by looking at the factored form of the denominator.

  

### Horizontal Asymptotes

**2.** Asymptotes are generally represented with dashed lines. On the graphing page on page 2.2, insert the vertical asymptotes (lines) at $x = 3$ and $x = -3$. Select a line and select **MENU > Actions > Attributes** and press [enter] to change attributes for the line. The second choice in the attributes menu will allow you to arrow left or right to select **dashed**.

When the degree of the numerator is less than or equal to the degree of the denominator of a rational function, a **horizontal asymptote** may exist. Horizontal asymptotes are represented by dashed lines. In this case, since the horizontal asymptote is on the *x*-axis, it will not be constructed as a dashed line on the graph.

When the degree of the numerator equals the degree of the denominator, a horizontal asymptote exists at $y =$ (ratio of leading coefficients of numerator and denominator). If the degree of the numerator is less than that of the denominator, a horizontal asymptote exists at $y = 0$.

**a.** Does the function graphed on page 2.2 have a horizontal asymptote? If yes, what is the equation of the horizontal asymptote?

**b.** By thinking about the values of the denominator as $x$ gets closer to the vertical asymptote(s), explain why the graph would behave as it does.

### Functions and Relations to Vertical & Horizontal Asymptotes

**3.** Next, we will explore the relationships between functions and their vertical and horizontal asymptotes. On page 3.2 of the TI-Nspire document, explore what happens to the vertical asymptotes as the values for $a$ and $b$ change. You can change the values of these parameters by clicking on the up and down arrows. Describe how the location of vertical asymptote(s) is related to $a$ and $b$ for the equation.

**Lead Coefficients and Relations to Asymptotes**

**4.** On page 4.2 of the TI-Nspire document, you will explore how the values of $p$, $q$, and $r$ impact horizontal asymptotes. First, change only $r$ to see how the degree of the polynomial impacts the situation.

**a.** When only the value of $r$ is changed, based on visual inspection, horizontal asymptotes may not be present when...

**b.** On page 4.2, let $r = 2$ so the degree of the numerator and denominator are equal, and manipulate the values for $p$ and $q$. When the degree of the numerator and denominator are equal, the equation has a horizontal asymptote at...

**c.** On page 4.2, let $r = 1$, and then let $r = 0$. Vary $p$ and $q$ in both situations and observe where horizontal asymptotes exist for the resulting graphs. When the degree of the denominator is greater than the degree of the numerator, a horizontal asymptote exists at...

**5.** Consider the function $f(x) = \frac{5x - 7}{4x^2 - 8x - 12}$. Graph the function on page 5.2 and identify values for which the function is undefined.

**a.** Factor the denominator to algebraically identify the singularities for the given function.

**b.** Go back to the graph on page 5.2 and construct the vertical asymptotes for the rational function using dashed lines. Do any horizontal asymptotes exist for this function?

  

**6.** Consider the function $f(x) = \dfrac{2x^2 + 2x - 23}{x^2 + x - 12}$. Graph the function on page 6.2 and identify values for which the function is undefined.

**a.** Factor the denominator to algebraically identify the singularities for the given function.

**b.** Go back to the graph on page 6.2 and construct all vertical and horizontal asymptotes for the given rational function using dashed lines.

**c.** The function $f(x) = \dfrac{2x^2 + 2x - 23}{x^2 + x - 12}$ may be rewritten as $f(x) = \dfrac{1}{x^2 + x - 12}$. What information does the second representation yield about the horizontal asymptote of the function? Does this agree with the information on page 2.4?

  

# Notes