Definite Integrals and Area Under A Curve

by

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Textbook Correlation: Key Topic

- Definite Integrals
- Applications of Definite Integrals

NCTM Principles and Standards:

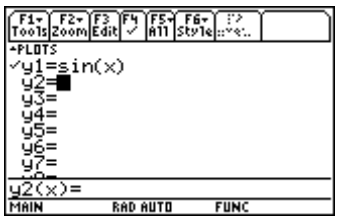
- Process Standard
  - Representation
  - Connections

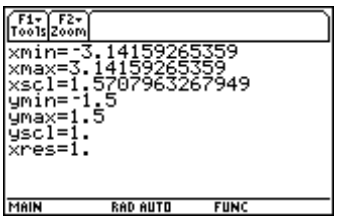
Exercise 1.

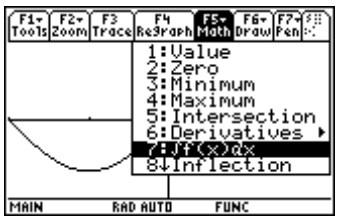
Evaluate the area bounded by the curve $y = \sin(x)$ and the x-axis between
a) $x = -\pi/2$ and $x = 0$.
b) $x = 0$ and $x = \pi/2$.
c) $x = -\pi/2$ and $x = \pi/2$.

Solution:

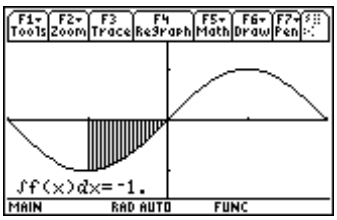
Graphical Interpretation:
Enter the equation in the Y= editor. In the **Window** editor set the window size to $[-\pi, \pi] \times [-1.5, 1.5]$. **GRAPH** (♦, **F3**). Choose **F5**, **7:(∫ *f*(*x*) *dx*)**.





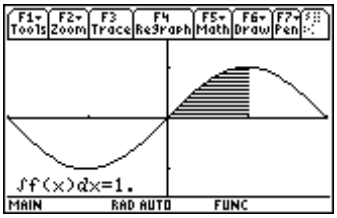


a) Set the limits of integration as: Lower Limit = - π/2 and Upper Limit = 0.



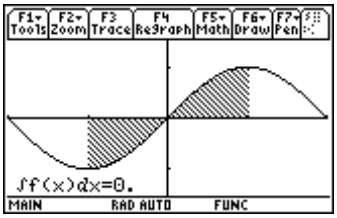
Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = 0$ is –1.

b) Set the limits of integration as: Lower Limit = 0 and Upper Limit = π/2.



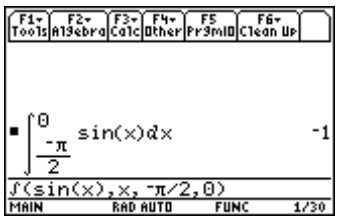
Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = 0$ and $x = \pi/2$ is 1.

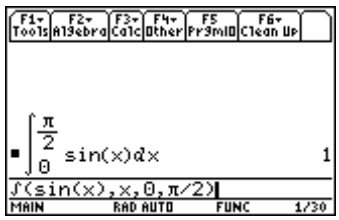
c) Set the limits of integration as: Lower Limit = - π/2 and Upper Limit = π/2.

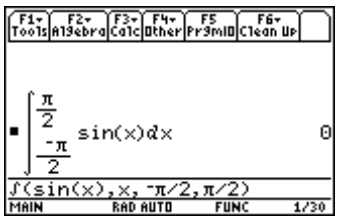


Answer:
The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 0.

Analytical Method:







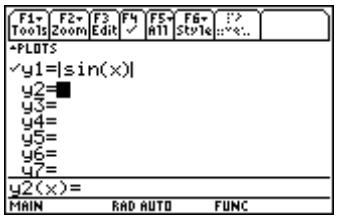
Answer:
a) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = 0$ is –1.
b) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = 0$ and $x = \pi/2$ is 1.
c) The area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 0.

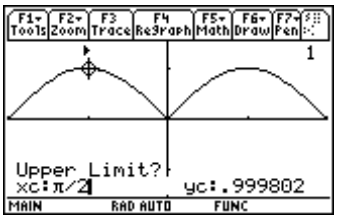
Exercise 2:

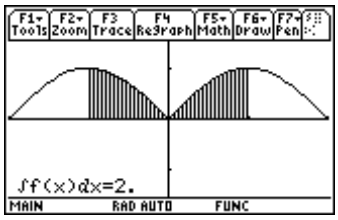
Compute the **total area** bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$.

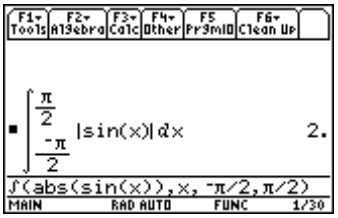
Solution:

To compute the total area, use the absolute value function as illustrated below.









Answer:
The total area bounded by the curve $y = \sin(x)$ and the x-axis between $x = -\pi/2$ and $x = \pi/2$ is 2.

Additional Exercises:

1. To evaluate the area bounded by the curve $y = x\sin(\pi x)$ and the x-axis between $x = -1$ and $x = 1$
   a) graph the function on the relevant interval and interpret the value of the integral as an area or as the negative of an area, and
   b) compute the definite integral.

2. Calculate the total area bounded by the curve $y = x\sin(\pi x)$ and the x-axis between $x = -1$ and $x = 1$.

3. To evaluate the area bounded by the curve $y = 15x^3(x-1)$ and the x-axis between $x = 0$ and $x = 1$
   c) graph the function on the relevant interval and interpret the value of the integral as an area or as the negative of an area, and
   d) compute the definite integral.

4. Calculate the total area bounded by the curve $y = 15x^3(x-1)$ and the x-axis between $x = 0$ and $x = 1$.