Solids Of Revolution Between Two Curves

ID: 17574

Time required
45 minutes

Activity Overview

Students will investigate 3D visualizations of volumes created by rotating two functions about the x-or y-axis. They will understand the concept and reason for the volume formula in order to be prepared for generalizations. Students will solve the definite integral by hand using the fundamental theorem of calculus and using the definite integral capabilities of the TI-Nspire. Students will also explore find the volume of a shape between two lines rotated about a vertical line.

Topic: Application of Integral

- *Calculate the volume generated by rotating a function about the x- or y-axis.*
- *Set up the definite integral, identify the limits of integration, the radius, and dh.*

Teacher Preparation & Notes

- *This investigation is a follow-up to an introduction to solids of revolution, that focuses on two functions rotated about the x- or y-axis.. Most AP problems extend what is learned in this activity to rotate about lines like y=2 instead y=0 or x=0. Questions are also asked about volumes formed by cross sections with various shapes. See the related activities for other follow up investigations.*
- *This exploration can be done at the students' own pace or with guided instruction.*
- *Many of the pages can be modified and used for other functions. If this is done, changing the window (press* ***menu****) may be necessary.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "17574" in the keyword search box.***

Associated Materials

- *SolidRevOfTwoFunctions_Student.doc*
- *SolidRevOfTwoFunctions.tns*

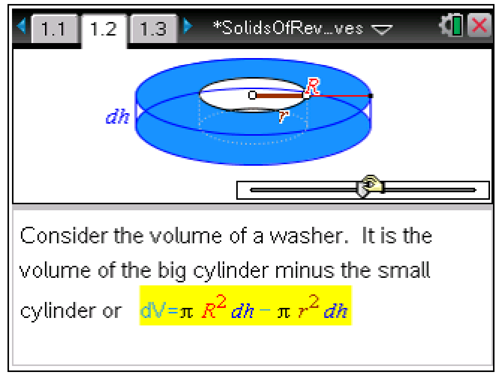
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

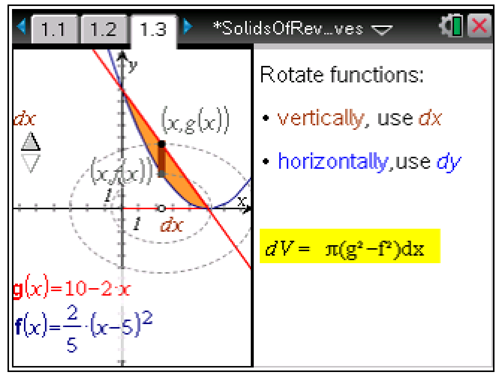
- *Solids of Revolution - Disks (TI-Nspire technology) — 16105*
- *Visualizing Solids of Revolution - Washers (TI-Nspire technology) — 16106*
- *Volume by Cross Sections (TI-Nspire technology) — 12281*

Problem 1 – *dx* vs *dy*

Students will first explore the cross section of a region bounded by two curves rotated about a vertical line. The shape that results is a washer. They can change the outer radius, the inner radius, and the height (or width) of the cross section.



Now students will explore the difference between rotating functions about a horizontal line, also called rotating the function vertically, and rotating functions about a vertical line (rotating the function horizontally).

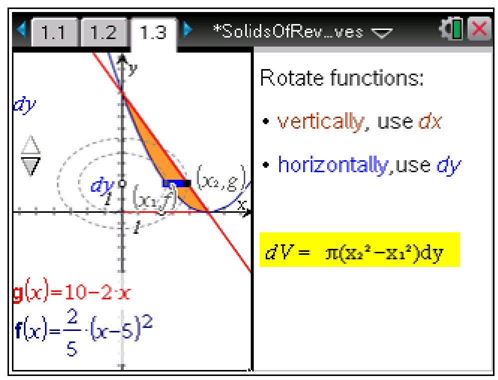


They are to calculate the volume of the solid for *dx* and *dy*. For *dy*, students will need to solve each function for *x*.

Using the *x*-values of the intersection points for the integration limits:

$$dV = \pi \cdot \int_0^5 \left((g(x))^2 - (f(x))^2\right)dx$$

$$= \pi \cdot \int_0^5 \left((10-2x)^2 - \left(\frac{2}{5}(x-5)^2\right)^2\right)dx = \frac{200\pi}{3}$$

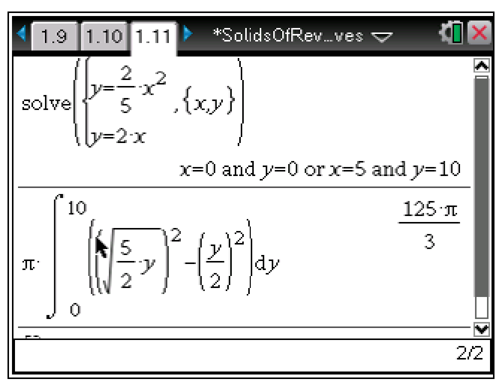


Using the *y*-values of the intersection points for the integration limits:

$$dV = \pi \cdot \int_0^{10} \left(\left(5 - \frac{y}{2}\right)^2 - \left(5 - \sqrt{\frac{5}{2}y}\right)^2\right)dy = \frac{125\pi}{3}$$

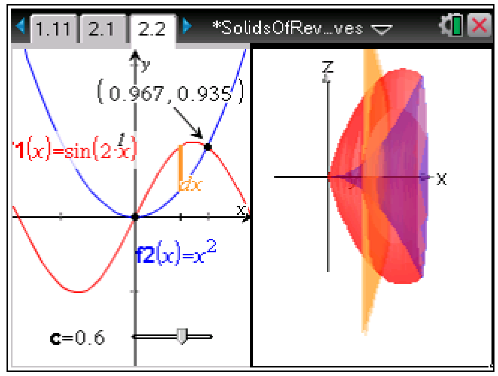
On page 1.9, students are shown a 3D rendering of a solid formed by rotating two functions around the *y*-axis. They are to use the 2D graph to determine the function that defines the outside radius of the solid.

The intersection points of this region are given on page 1.11. Students are to calculate the volume on this page.



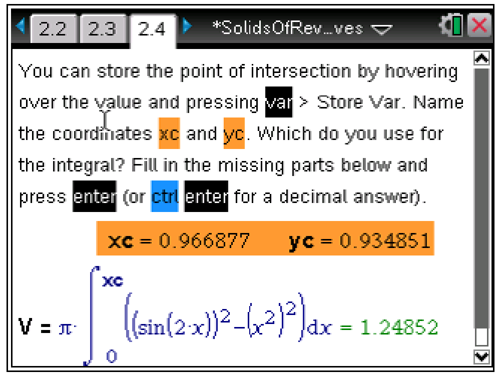
Problem 2 – Region bounded by sin(*x*)

In this problem, students will calculate the volume of a solid in which the function sin(2*x*) and another function have been rotated about the *x*-axis. They can use the slider at the bottom of the page to change the value of *c*, the location of the slice.



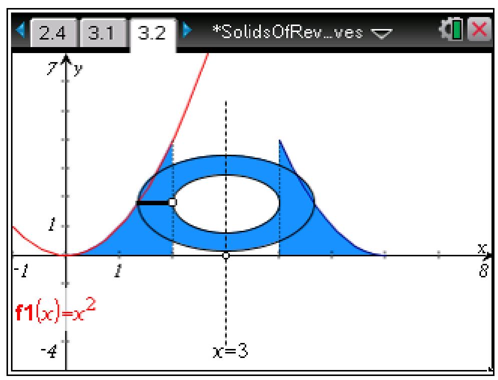
Students need to store the values of the integration limits, or the *x*-value of the intersection point. One of the intersection point's coordinates have already been placed on the screen. Hover over the x-value, press var **> Store Var** and enter **xc**. Repeat for the *y*-value. The other intersection point (0, 0) will not need to be stored because it is easily remembered.

The values of **xc** and **yc** will appear automatically on page 2.4. Students are to use the integral on this page to calculate the volume.



Problem 3 – Rotation around a vertical line

In this problem, students will explore two functions rotated about a vertical line that is not the *y*-axis. An interactive page on page 3.2 allows students to change the line of rotation and the inner radius. They will not be able to change the function f1(x) = x^2. However, students can use the **Geometry Trace** tool to create an approximate 3D shape of the solid.



Students should determine dh (dx vs dy) and the inner and outer radius. Ask students what they inner and outer radius have in common (they both subtract a function or line from the line of rotation).

dh: *dy*

Outer: line of rotation – ($y = x^2$)

Inner: line of rotation – (x = 2)

They are to use page 3.4 to find the volume of the solid. Remind students that the limits of integration are the intersection point of the 2D region.