# Average Value of a Function

**MATH NSPIRED**

**TEACHER NOTES**

## Math Objectives

- Students will examine areas as integrals and as rectangles for given functions in order to determine the properties of functions that allow the areas to be equal.

## Activity Types

- Student Exploration
- Group Activity

## About the Lesson

- Students will find a rectangle whose area is the same as the area under a curve within a defined interval.
- Students will examine several examples and then try to generalize conditions that allowed the area of the rectangle and the area under the curve to be equal, leading to the Mean Value Theorem for Integrals.

## Directions

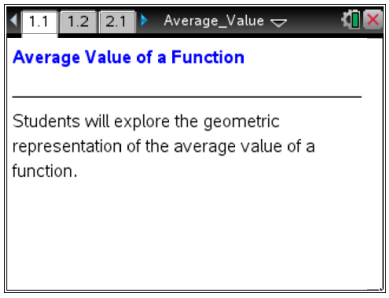
- For each of the problems, move the open circle until the area of the rectangle matches the area under the curve. When the area matches, record the function, limits of integration, and the *c* value **(cval)** for each problem.
- Students can double-click on the **cval** and type in a specific value to attempt to make the areas equal.
- The five problems for this activity are shown on the following pages.

## TI-Nspire™ Navigator™

- Send a document.
- Use Class Capture to formerly assess students' understanding.
- Use Live Presenter to demonstrate and provide a means for students to share their thinking.
- Use Quick Poll to assess students' understanding.

## Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software




**Tech Tips:**

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**

*Student Activity*

- Average_Value_of_a_Function_Student.pdf
- Average_Value_of_a_Function_Student.doc

*TI-Nspire document*

- Average_Value.tns

## Discussion Points and Possible Answers

**Move to page 1.2.**

Record the function, limits of integration and the *c* value **(cval)** for each problem.

> **Tech Tip:** *Cvals* are approximate due to rounding on the TI-Nspire calculator application.

| | |
|---|---|
| **Move to page 2.1.**<br>Function: $-x^2+7x-10$<br>Limits of Integration: $a = 2$, $b = 5$ | *cval:* 4.367, 2.633 |
| **Move to page 3.1.**<br>Function: $x^3 - x^2 - 2x$<br>Limits of Integration: $a = -1$, $b = 0$ | *cval:* –0.821, –0.246 |
| **Move to page 4.1.**<br>Function: $\begin{cases} 3, 0 < x < 3 \\ 3x-6, 3 < x \le 6 \end{cases}$<br>Limits of Integration: $a = 0$, $b = 5$ | *cval:* 3.525 |
| **Move to page 5.1.**<br>Function: $4 - x^2$<br>Limits of Integration: $a = 0$, $b = 3$ | *cval:* 1.728 |
| **Move to page 6.1.**<br>Function: $\begin{cases} 1, 0 < x < 2 \\ 2, 2 < x < 4 \\ 3, 4 < x < 6 \end{cases}$<br>Limits of Integration: $a = 0$, $b = 6$ | *cval:* None |

**Classroom Discussion:** Students should work on answering these questions if they complete problem 6 before other students. These questions can be part of a class discussion to move the class to writing the average value of a function.

1. Was the *c* value always between the limits of integration?

   **Answer:** Yes, the *c* value was always between the limits of integration.

2. What is the relationship between the area of the rectangle and the integral area?

   **Answer:** The area of the rectangle and integral were equal.

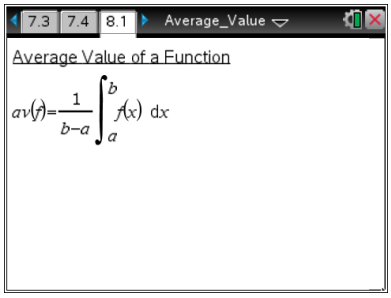
3. What property of a function held when the areas were equal?

   **Answer:** The function was continuous on the interval.

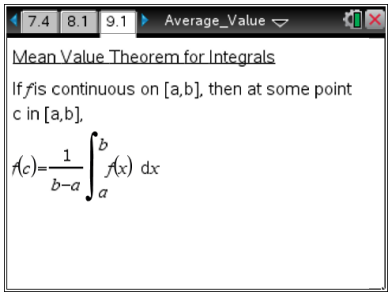
4. Can this relationship be written using calculus notation?

   **Answer:** Yes, as the Mean Value Theorem for Integrals.
   (See the Notes page of the TI-Nspire document for the theorem.)

**Notes for Students:**
Formula is given for the average value of a function.




**Notes for Students:**
Teacher Note: Problem 6 is a counterexample, but this statement can be true if *f* is not continuous. Have students rewrite the function in problem 6 to make the statement true, even if *f* is not continuous.



**Exploration:** Have students think about how the two equations are the same. Students can manipulate the second expression to get the first using algebra and calculus.