The Objective:

The goal of this lesson is to introduce students to the concept of finding an optimum solution and to the vocabulary associated with the optimization process called linear programming.

Addresses Sunshine State Standards:
MA.D.1.4.1 Describes… patterns and functions using...words… and graphs.
MA.D.2.4.2 Uses systems of equations and inequalities to solve real world problems graphically and algebraically…

Students will review the prerequisite skills of *graphing systems of linear inequalities* and *solving systems of equations*, and be introduced to the basic vocabulary of linear programming using **StudyCard** application files distributed using the **Navigator System**.

Students will develop an intuitive understanding that solutions to linear programming problems are found at "corner points" of graphs of systems of linear inequalities. This will be accomplished through a small group and whole class activity (a handout that poses a real life situation and relevant questions) that makes use of the **activity center** and **quick poll** features of the **Navigator System**. An assessment will consist of a Learning Check file that is distributed and collected using the **Navigator System**.

The Concept:

Linear Programming is a mathematical procedure for finding a *best solution point* within a *solution region* for a system of linear inequalities. In this context, the words "best solution point" refer to a point whose coordinates produce a maximum or minimum value for a given equation called an **objective function**. In linear programming, the solution region of a system of inequalities is called the **feasible region** and the linear inequalities themselves are called **constraints**.

The Procedure:

1. Transmit StudyCard files to students via Navigator system.
2. In small groups students read information and questions on studycards and then selfcheck their answers.
3. The teacher circulates around the room observing and engaging students in discussion.
4. Teacher passes out "Cookie Dough Kits" lesson. One section at a time. Students work in small groups. At various points the teacher leads a whole class discussion using features of the Activity Center and Quick Poll.
5. After "Cookie Dough Kits" is completed the teacher distributes a LearningCheck file via the Navigator system to assess the students.

Assessment:

LinearProgram LearningCheck file.

Cookie Dough Kits

Your school's band decides to sell cookie dough kits to raise money for a spring field trip. There is a Family Times Cookie Dough kit that you can buy for $7 and sell for $12, and a Baker's Delight kit that you can buy for $15 and sell for $25. The PTA will lend you $2100 to buy supplies. The company selling you the kits informs you that a school your size can expect to sell at most 220 kits. Questions: How many of each type of kit should your band purchase to raise the most money? What is the most money that your band can raise?

Section 1 – (The Objective Function)

A. What would be the profit when you sell one Family Times kit?

B. What would be the profit when you sell one Baker's Delight kit?

C. What equation would represent your overall profit if you sell x (number of) Family Times kits and y (number of) Baker's Delight kits? This is the **objective function** of your linear programming model.

Section 2—(The Constraints)

A. Write an inequality that shows the different number of kit combinations you can purchase (to resell) if x represents the number of Family Time kits and y represents the number of Baker's Delight kits.

B. Write an inequality that reflects the fact that you expect to sell at most 220 kits.

C. Write an inequality that shows you must sell either zero or a positive number of Family Times kits.

D. Write an inequality that demonstrates that you will not sell a negative amount of Baker's Delight kits.

E. List these four inequalities together in four rows. These are the **constraints** of your linear programming model.

F. Graph the first two inequalities and only consider the 1st quadrant of the coordinate plane. Why? (Answer: You must always sell either zero or a positive number of cookie dough kits. You could graph all four inequalities but your calculator screen would become difficult to read). The double-shaded region is the area that contains possible solutions to your problem. In linear programming this area is called the _____________ region. Note: You might want to experiment with different dimensions for your viewing window until you find one that shows the entire feasible region.

Could you enter the inequalities into your calculator so that the unshaded area would represent the solution to all four inequalities? How?

Section 3 – Searching for an Optimum Value

A. Move your cursor around in the feasible region and note the various x and y values for various points within the region. Now, identify a point that has integer values for its x and y coordinates.

B. Evaluate your objective function using these x and y integer values. Show your work.

C. When you evaluate your objective function with these coordinates what information have you obtained?

D. Find another point in the feasible region with coordinates that are integer values.

E. Evaluate the objective function with the x and y coordinates of this new point. Show your work.

F. Compare the result from part B with result from part E. Which pair of coordinates resulted in a greater value when you evaluated the objective function?

G. Continue searching for a pair of coordinates in the feasible region that would produce a greater result when they are used to evaluate the objective function than the coordinates used in part B and part E. What is the greatest value that can be found for the objective function? What are the coordinates that produce this value?

H. Are you 100% certain that you have found the coordinates that produce the greatest value for the objective function? Can you be 100% sure? Explain.

I. What do the results of part G mean within the context of your problem?

J. Consider the context of our linear programming model. Why should we only consider points with coordinates that are whole numbers?

Section 4 – Graphing the Objective Function

In section 3 you sought a maximum value for an objective function (while obeying the constraints of the problem) using a guess and check strategy. Now, we will attempt to find a systematic method for finding the maximum value for an objective function, and to answer the question of whether we can know for sure if we have found a maximum value.

A. Our objective function is 5x + 10 y = P ; where x stands for the number of Family Times kits sold, y stands for the number of Baker's Delight kits sold, and P stands for the profit the band earns from kits sold.

In order to graph this equation on a graphing calculator we would need to solve this equation for y.
Solve the objective function for y.

Y= __________________

B. When this equation is written in y = mx + b form we can see what its slope and y intercept are. What is the equation's slope?

Can the y intercept be expressed as a numerical value or is it expressed as an algebraic expression?

C. Assume the maximum profit for the band is $1000. Graph the objective function on the same coordinate plane as your constraints (system of inequalities) using 1000 for a P value.

D. Assume the maximum value for the objective function is $1100. Graph the objective function using 1100 for a P value.

E. How does the line for the equation from part C compare with the line for the equation from part D?

F. Continue selecting new numbers for P and graphing the resulting equations on the same coordinate plane as the constraints.

Graph equations that have these P values:
P= 1200
P=1300
P= 1400
P = 1500

What do all these lines have in common?

G. At some point (for some value of P) the graph of the objective function will touch the feasible region at just one point. Can you make a conjecture about where in the feasible region this point must be?

H. Once you identify where this point must be, solve a system of equations (or use the intersect feature of your graphing calculator) to identify the coordinates of the point.

I. Evaluate the objective function using these coordinates. Show your work.

This is the maximum value for your objective function. In the context of your linear programming model what does this maximum value and the coordinates that were used to obtain it repesent?

Section 5 – Extensions

A. Change the coefficients of the objective functions so that a maximum value occurs at another point? What would this change of coefficients represent in the real life situation?

B. Go back to the original objective function 5x + 10 y = P. Could changing the number of total kits sold change where your maximum value occurs? If so, give an example?

C. Go back to assuming that no more than 220 kits are sold. If the PTA made more money available to purchase kits (to resell) could that change what your maximum value is? Would that change the number of each type of kit to purchase? Explain.

D. Assume that no more than 150 kits will be sold. What is troublesome about the coordinates of the new corner point (that is, where the new maximum value is found)?

E. What if you assume that not many people will purchase the expensive Baker's Delight kit and so decide that you must purchase four times as many Family Times kits as Baker's Delight kits. How is this represented as an inequality? Does this change where your maximum value is found? Explain.