EXPLORATIONS ™

Activity 3

Objective

- To investigate the properties of a dilation

Cabri® Jr. Tools

Dilations in the Plane

Introduction

A dilation is a transformation that produces a figure with a different size from that of the original figure. In this activity, you will explore the properties of dilations and the relationships between the original and image figures.

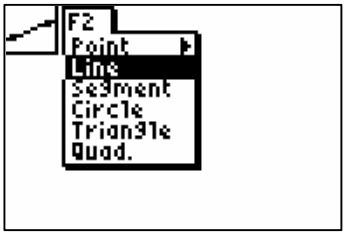
Construction

Part I: Construct a slider tool to generate a number in the range of approximately -4.5 to 4.5 units.

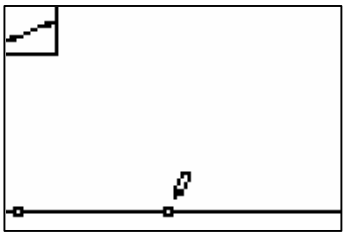
Draw the line $\overleftrightarrow{AC}$ horizontally across the bottom of a new screen with point *A* near the left edge and point *C* near the middle of the screen.

1. Press WINDOW to view the **Drawing Tools Menu** (**F2 Menu**). Highlight **Line**. Press ENTER to select it.

Note: *The tool icon at the top left of the screen indicates that the* **Line** tool *is active.*



2. Move the cursor to the lower left corner of the screen. Press ENTER to anchor one point of a line. Move the cursor to the lower middle of the screen, and press ENTER to anchor the second point of the line.



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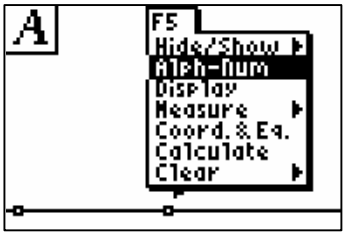
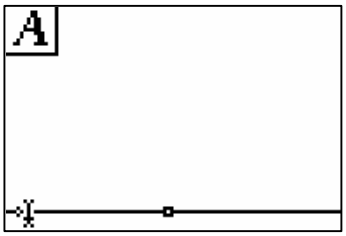
3. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**). Highlight **Alpha-Num**, and press ENTER.

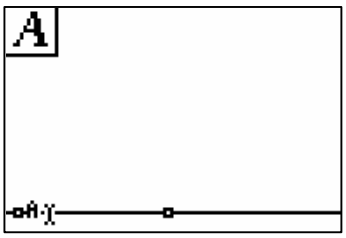
Note: *Alpha lock is on when the* **Alpha-Numeric** *tool is active.*



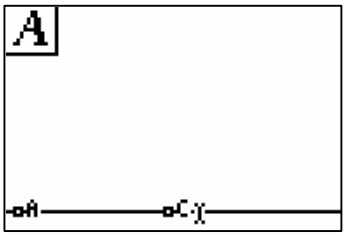
4. Move the cursor to the point in the lower left corner of the screen. The point blinks when the cursor is close enough to the point to select it.



5. Press ENTER to create a label for this point. Press MATH to label that point *A*. Press ENTER to complete the label.

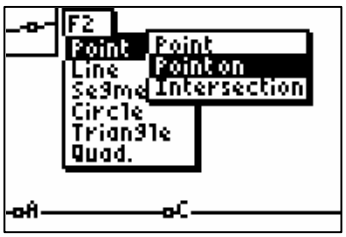


6. Repeat steps **4** and **5** to label the point in the lower middle of the screen as *C* (C is located above PRGM). When complete, press CLEAR to exit the **Alpha-Numeric** tool.



▫ A Construct point *B* on line $\overleftrightarrow{AC}$ to the right of point *C*.

7. Press WINDOW to view the **Drawing Tools Menu** (**F2 Menu**). Highlight **Point**. Press ▶ to view the **Point Menu**. Highlight **Point on**, and then press ENTER.



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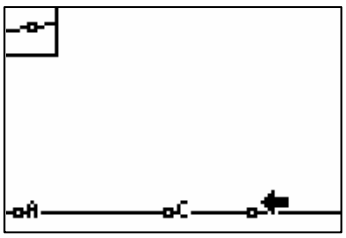
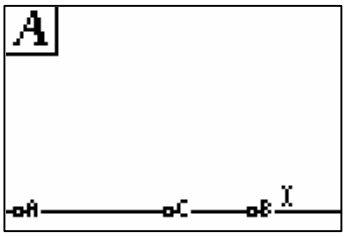
8. Move the cursor to the right of point *C* on line $\overleftrightarrow{AC}$, and press ENTER to place a point on the line.

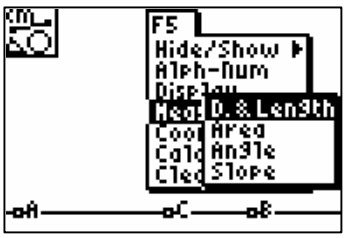


9. Use the **Alpha-Numeric** tool (GRAPH, **F5 Menu**) to label the newly created point as *B* (B is located above APPS). Remember to press ENTER once to create the label, and then again to complete the label. Press CLEAR to exit the **Alpha-Numeric** tool.

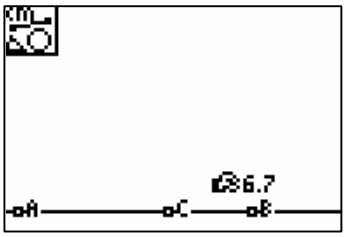


Measure the length of segments $\overline{AB}$ and $\overline{AC}$, and calculate the difference $\overline{AB} - \overline{AC}$.

10. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**). Highlight **Measure**. Press ▶ to view the **Measure Menu**. Highlight **D. & Length**, and press ENTER.



11. Use the **Distance & Length** tool to measure the length of the segment *AB*.



a. Move the cursor to point *A* and press ENTER, and then move the cursor to point *B* and press ENTER.

b. Use the cursor keys (◀ ▶ ▲ ▼) to move the measurement away from the line.

c. Press ENTER a final time to anchor the measurement.

Note: *Make sure that the point itself is blinking, not the entire line.*

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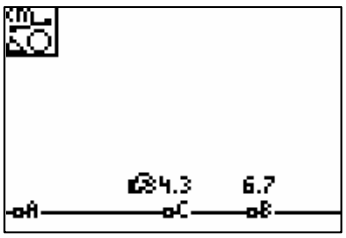
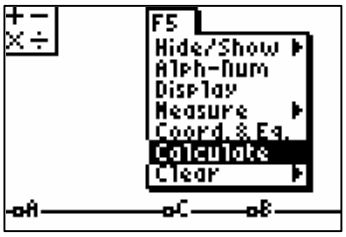
12. Repeat step **11** to measure the length of the segment $\overline{AC}$. Make sure that the **D. & Length** tool is still active.



13. Press [GRAPH] to view the **Display Tools Menu** (**F5 Menu**) and highlight **Calculate**. Press [ENTER].



14. Use the **Calculate** tool to find the difference $\overline{AB} - \overline{AC}$.

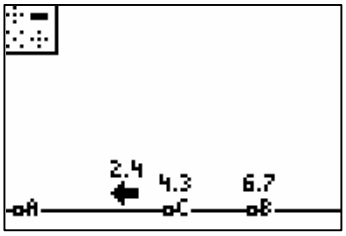
a. Move the cursor to the measurement of $\overline{AB}$ (which should be the larger number). Press [ENTER].

b. Press [−] to subtract.

c. Move the cursor to the measurement of $\overline{AC}$, and press [ENTER].

d. Use the cursor keys to move the calculation to a blank area of the screen and press [ENTER] a final time to anchor the calculation.

e. Press [CLEAR] to exit the **Calculate** tool.

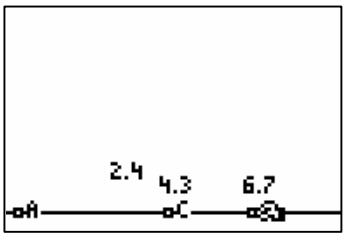


Drag point *B* to change the numeric value, and hide objects no longer needed.

15. Move the cursor to highlight point *B* and press [ALPHA] to grab it.

Note: *Make sure to grab the point itself, not the label* B.

Note: *The pointer changes to a hollow arrow when it is near an available object.*



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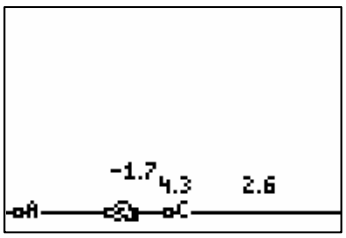
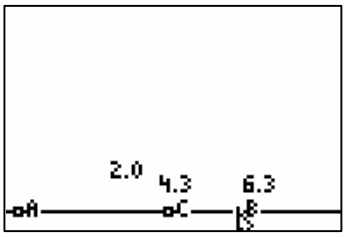
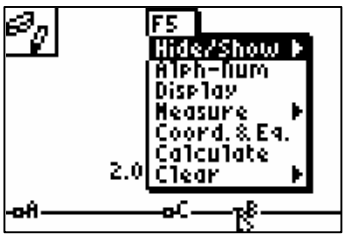
16. Use the cursor keys to drag point *B* along the line. Observe the difference $\overline{AB} - \overline{AC}$ as it changes. When point *B* is to the right of point *C*, the difference is positive. When point *B* is between points *A* and *C*, the difference is negative.



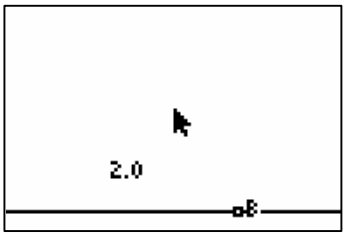
17. Move point *B* so that the difference is 2. Press CLEAR to exit the **Hand** tool.



18. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**). Highlight **Hide/Show**. Press ENTER.



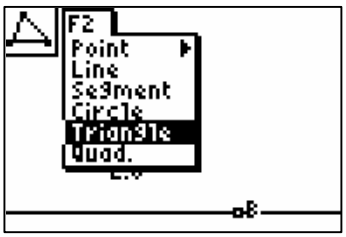
19. Move the cursor to point *A* on the line, and press ENTER to hide point *A*. Repeat this to hide all objects and numbers *except* the line, point *B*, and the calculated difference $\overline{AB} - \overline{AC}$. Press CLEAR to exit the **Hide/Show** tool.



Part II: Construct a triangle, and perform a dilation

Draw a small scalene triangle in the left half of the screen and point *P* in the center of the screen.

1. Press WINDOW to view the **Drawing Tools Menu** (**F2 Menu**). Highlight **Triangle**. Press ENTER.



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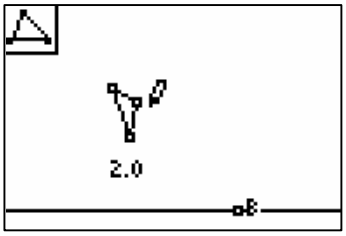
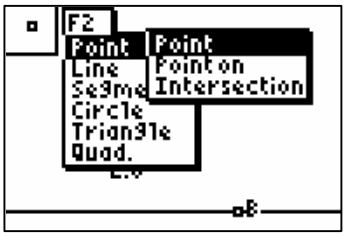
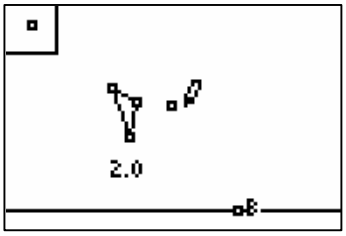
2. Move the cursor to the left half of the screen, and press [ENTER] to anchor one vertex of the triangle. Repeat this two more times to anchor the other two vertices of the triangle. Press [CLEAR] to exit the **Triangle** tool.



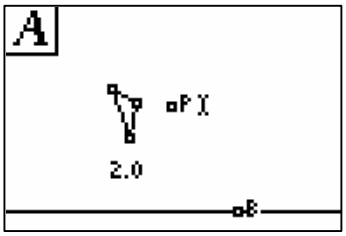
3. Press [WINDOW] to view the **Drawing Tools Menu** (**F2 Menu**), and highlight **Point**. Press [▶] to view the **Point Menu**. Highlight **Point**, and press [ENTER].



4. Move the cursor to the center of the screen. Press [ENTER] to anchor a point there.



5. Use the **Alpha-Numeric** tool ([GRAPH], **F5 Menu**) to label the newly created point as *P* (P is above the [8] button). Remember to press [ENTER] once to create the label and then again to complete the label.

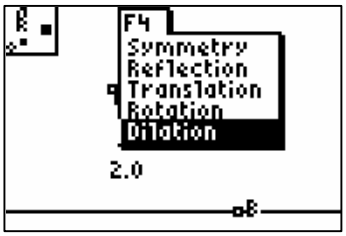


Dilate the triangle.

To use the **Dilation** tool, you must do the following:

- Select what is to be dilated.
- Select a point to be the center of dilation.
- Select the numeric value to be used as a dilation factor.

6. Press [TRACE] to view the **Transformation Tools Menu** (**F4 Menu**), and highlight **Dilation**. Press [ENTER].



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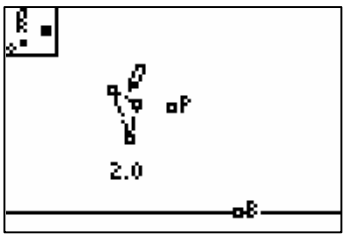
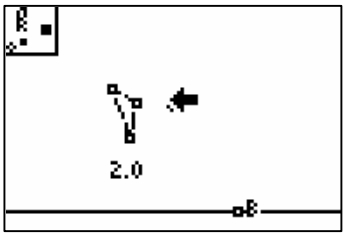
7. Move the cursor to highlight the triangle, and then press ENTER to select it.

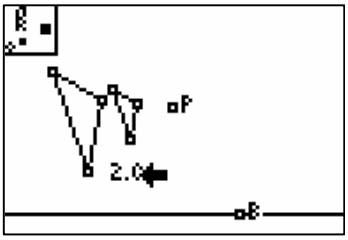
Note: *Make sure to highlight the whole triangle, not just one side.*



8. Move the cursor to highlight point *P*, and then press ENTER to select it.



9. Move the cursor to the numeric difference $\overline{AB} - \overline{AC}$, and press ENTER to select it. A new triangle (the image) is created. Press CLEAR to exit the **Dilation** tool.



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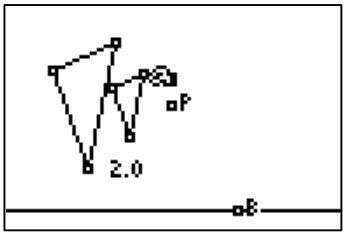
Exploration

Investigate the relationship between the original triangle (the pre-image) and the new triangle (the image).

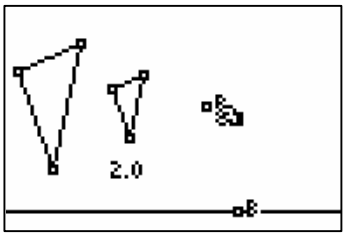
Drag a vertex of the pre-image.

1. Move the cursor to highlight the desired vertex, and press [ALPHA] to grab it. Use the cursor keys to drag the vertex around the screen. Observe what happens to the image. When complete, press [CLEAR] to exit the tool.



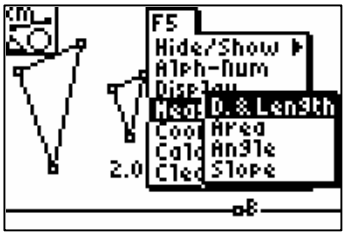
Drag the center of dilation.

2. Move the cursor to highlight point *P*, and press [ALPHA] to grab it. Use the cursor keys to drag the point around the screen. Observe what happens to the image. When complete, press [CLEAR] to exit the tool.



Measure the lengths of corresponding sides, angles, and areas of the pre-image and image.

3. Press [GRAPH] to view the **Display Tools Menu** (**F5 Menu**), and highlight **Measure**. Press [▶] to view the **Measure Menu**. Highlight **D. & Length**. Press [ENTER].



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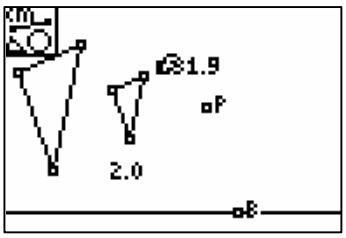
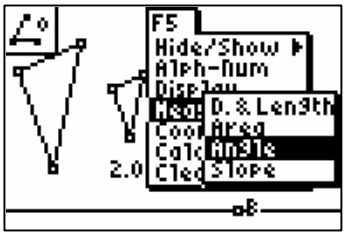
4. Move the cursor to highlight the desired side of the triangle. Press ENTER to measure the side, and press ENTER again to anchor the measurement on the screen. Repeat this for all desired length measurements. When complete, press CLEAR to exit the **Distance & Length** tool.



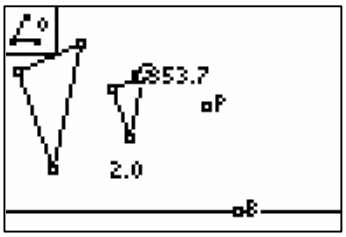
Note: *Be careful to highlight the side of the triangle and not the whole triangle. Wait for only the side to highlight, or press* 2nd *to switch between full object and side only.*

Note: *You could also measure the side length as a point-to-point distance. Highlight one endpoint, and press* ENTER*. Highlight a second endpoint, and press* ENTER *again.*

5. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**), and highlight **Measure**. Press ▶ to view the **Measure Menu**. Highlight **Angle**. Press ENTER.



6. Angles are measured by selecting three points. Select the vertex point of the angle *second*.



a. Move the cursor to highlight one point on the side of the desired angle, and press ENTER.

b. Move the cursor to the vertex of the angle, and press ENTER.

c. Move the cursor to the third point on the angle, and press ENTER.

d. Finally, move the measurement to a desired location, and press ENTER to anchor it on the screen.

Repeat this for all desired angle measurements. When complete, press CLEAR to exit the **Angle Measure** tool.

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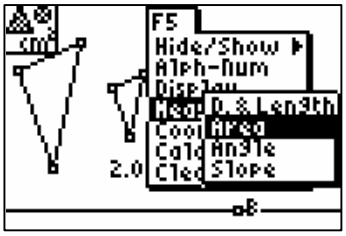
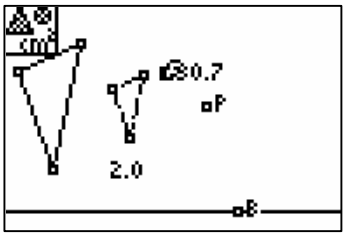
7. Press [GRAPH] to view the **Display Tools Menu** (**F5 Menu**), and highlight **Measure**. Press [▶] to view the **Measure Menu**. Highlight **Area**. Press [ENTER].



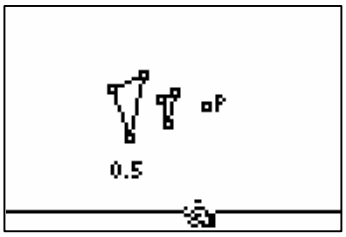
8. Move the cursor to highlight the desired triangle. Press [ENTER] to measure the area, and press [ENTER] again to anchor the measurement on the screen. Repeat this for all desired area measurements. When complete, press [CLEAR] to exit the **Area Measure** tool.



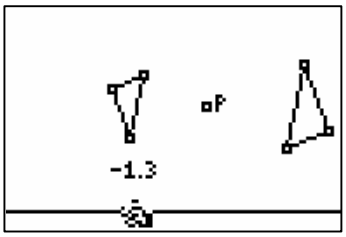
Drag point *B* to explore different dilation factors.

9. Move the cursor to highlight point *B*, and press [ALPHA] to grab it.

Note: *Make sure to grab the point itself, not the label* B.



10. Use the cursor keys to drag point *B* along the line and observe the dilation factor as it changes. When point *B* is to the right of the original point *C*, the difference is positive. When point *B* is between the original points *A* and *C*, the difference is negative. Observe what happens to the image triangle when the dilation factor changes. When complete, press [CLEAR] to exit the tool.



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Data Collection and Analysis

Name _____

Date _____

Questions and Conjectures

1. Did the new triangle change shape compared with the original triangle?

_____

2. How are the pre-image triangle and the image triangle related?

_____

3. What happens when the center of dilation is moved?

_____

4. What is the relationship between lengths of corresponding sides in the triangles?

_____

5. What is the relationship between measures of corresponding angles in the triangles?

_____

_____

6. What is the relationship between the areas of the triangles?

_____

_____

7. Explain what happens when point *B* is dragged to make a larger dilation factor.

_____

_____

8. Explain what happens when the dilation factor is a negative number.

_____

_____

9. Did the triangle change orientation with a positive or negative dilation?

_____

_____

10. Is there another transformation that is equivalent to a dilation by a factor of -1?

_____

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Teacher Notes

Activity 3

Objective

- To investigate the properties of a dilation

Cabri® Jr. Tools

Dilations in the Plane

Answers to Questions and Conjectures

1. Did the new triangle change shape compared with the original triangle?

The new triangle and the old triangle have the same shape, but different sizes.

2. How are the pre-image triangle and the image triangle related?

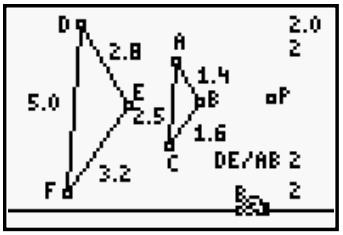
The pre-image and image triangles are similar to each other. Their corresponding angles will be congruent and their corresponding sides will be in proportion. The ratio of the sides will equal the dilation factor.

3. What happens when the center of dilation is moved?

The distance from the image triangle to the center of dilation and the distance from the pre-image triangle to the center of dilation will be in proportion, and their ratio will equal the dilation factor. For example, if the dilation factor is 2, then the image will be twice as far away from the center of dilation as the pre-image. Therefore, if the center of dilation is moved, the image will also move. If the center moves toward the pre-image, then the image will move toward the pre-image. If the center moves away from the pre-image, then so will the image.

4. What is the relationship between lengths of corresponding sides in the triangles?

Corresponding sides have the same ratio as the dilation factor.



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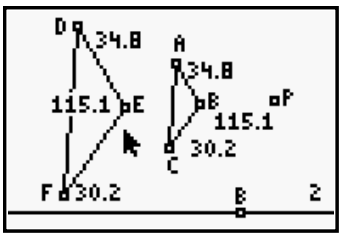
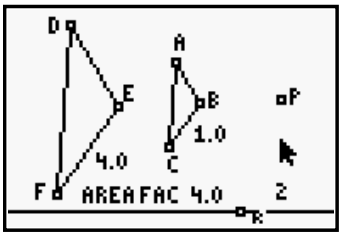
5. What is the relationship between measures of corresponding angles in the triangles?



Measures of corresponding angles will be equal; their ratio will be 1:1.

6. What is the relationship between the areas of the triangles?



The ratio of corresponding areas will equal the square of the dilation factor. For example, if the dilation factor is 2, then the ratios of areas will be 4:1.

7. Explain what happens when point *B* is dragged to make a larger dilation factor.

The image triangle gets larger and moves farther away from the center of dilation.

8. Explain what happens when the dilation factor is a negative number.

A negative dilation factor causes the image to be rotated 180° around the center of dilation and then dilated by the absolute value of the dilation factor.

9. Did the triangle change orientation with a positive or negative dilation?

No; dilations preserve orientation whether the dilation factor is positive or negative.

10. Is there another transformation that is equivalent to a dilation by a factor of -1?

A dilation by a factor of -1 is equivalent to a rotation of 180° around the center of dilation.

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