You Can't Get There From Here

ID: 12098

Time Required
40–45 minutes

Activity Overview

Students explore rational functions graphically and algebraically to identify singularities and asymptotes, both vertical and horizontal.

Topic: Asymptotes

- *Singularities*
- *Vertical Asymptotes*
- *Horizontal Asymptotes*

Teacher Preparation and Notes

- *Problems 1–4 in the TI-Nspire™ document should be done in class as exploration and guided practice. Consider having students work in small groups, which may be particularly helpful for Parts 3 and 4 in the TI-Nspire™ document. Problems 5 and 6 may be done as small group work or homework. Additional problems are provided on the student worksheet for further practice.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12098" in the keyword search box.***

Associated Materials

- *YouCantGetThereFromHere_Student.doc*
- *YouCantGetThereFromHere.tns*
- *YouCantGetThereFromHere_Soln.tns*

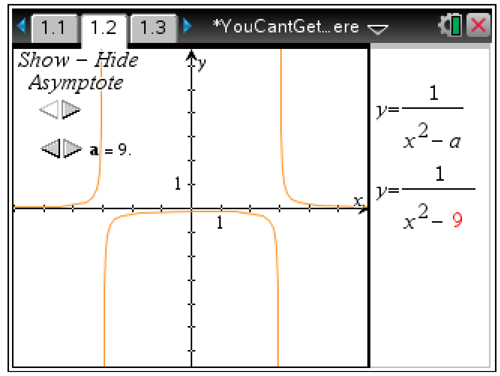
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Exploring Rational Functions (TI-Nspire™ technology) — 8968*
- *Asymptotes & Zeros (TI-Nspire™ technology) — 9286*

Problem 1 – The Basics

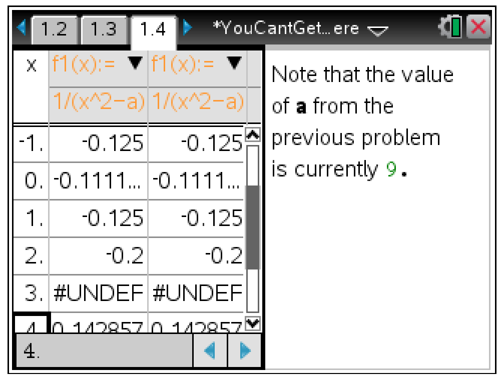
Problem 1 is an exploration of asymptotes involving graphing, exploring a table of values, and algebraic manipulation.



First, a graphic representation is explored. From the graph, students can get reasonable approximations of *x*-values at which the function is undefined.

Students are given a slider to click on and show or hide the vertical asymptotes. Students can also click the ***a***-value slider and explore how changes in the value of *a* affects the asymptotes. The ***a***-values are 1, 4, 9, 16, and 25.

Next, students use a table of values and will observe how undefined function values are represented in the table.



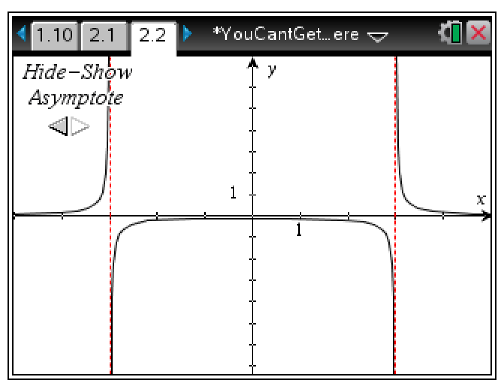
Students are asked several questions relating to pages 1.2 and 1.4 for the remaining of Problem 1.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

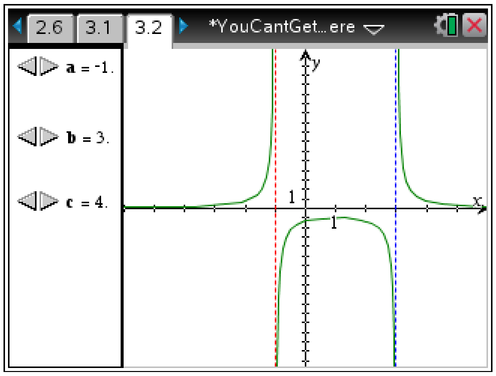
Problem 2 – Asymptotes

Students explore what was learned in Problem 1 to develop an understanding of the patterns involved with asymptotes and rational functions.



Students are asked several questions about asymptotes in Problem 2.

Problem 3 – Asymptotes



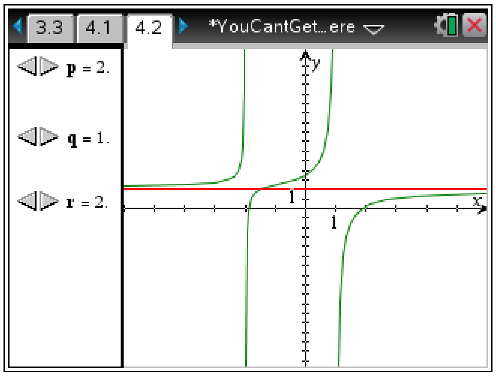
Students will explore the relationships between functions and their vertical and horizontal asymptotes.

On page 3.2, students should change the values of *a*, *b*, and *c* to see how the graph changes for the function $f(x)=\dfrac{c}{(x-a)(x-b)}$.

TI-Nspire Navigator Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

Problem 4 – Asymptotes



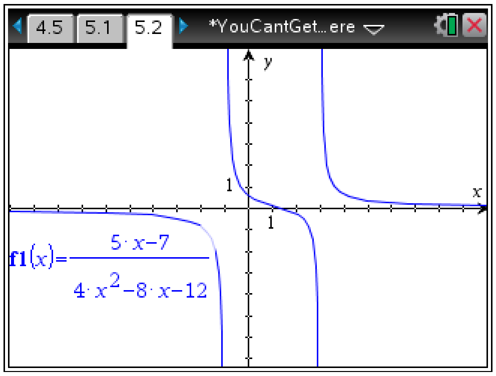
Next, students will explore the relationships between functions and their asymptotes.

Students explore what happens to the horizontal asymptotes as values for *p*, *q*, and *r* change in the function $f(x)=\dfrac{px^r-7}{qx^2+x-2}$

Students will also explore the effect of the degree of the numerator and denominator in identifying horizontal asymptotes.

Point out to students that a horizontal asymptote may cross a function graph at the middle, but as $x\to\pm\infty$, the function graph approaches the horizontal asymptote.

Problem 5 – Practice



Students apply what was learned in Problems 1 and 2 to finding singularities and asymptotes for the function $f(x)=\dfrac{5x-7}{4x^2-8x-12}$.

TI-Nspire Navigator Opportunities

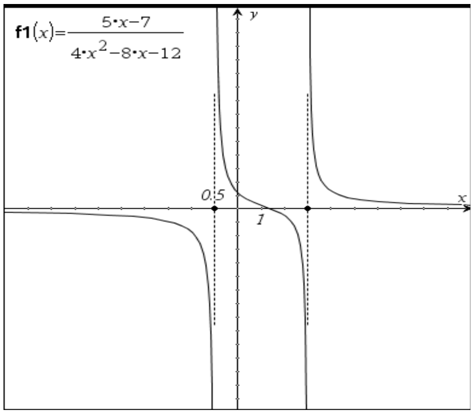
Note 1
Use Quick Poll to assess student understanding during all problems. The worksheet questions can be used as a guide for possible questions to ask.

Note 2
Problem 3, *Class Capture*
Use Class Capture to verify that students are able to click on the sliders.

Student Worksheet Solutions

1. No; for certain values of x, the function is undefined.
2. The table displays #undef for the x-values at which the function is undefined.
3. ±3
4. $(x – 3)(x + 3)$
5. The factors match the “skipped” x-values. $(x – 3)$ matches the skipped value of $x = 3$, $(x + 3)$ matches the skipped value of $x = –3$.
6. These values make the denominator equal to zero, causing the function to be undefined at these values of x.
7. Yes, $y = 0$
8. At $x = a$ and $x = b$
9. Horizontal asymptotes may be present whenever the degree of the numerator is less than or equal to the degree of the denominator.
10. $y = \frac{p}{q}$, or at y = (ratio of leading coefficient of numerator to leading coefficient of denominator)
11. $y = 0$
12. **a.** Singularity: an x-value that makes the denominator of a rational function equal to zero. At that x-value, the function is undefined
    b. Asymptote: a “line” which the graph of a function approaches, getting closer and closer, but never reaches
13. $4(x – 3)(x + 1)$; the function is undefined at $x = 3$ and at $x = –1$.
14. Yes; $y = 0$

15.



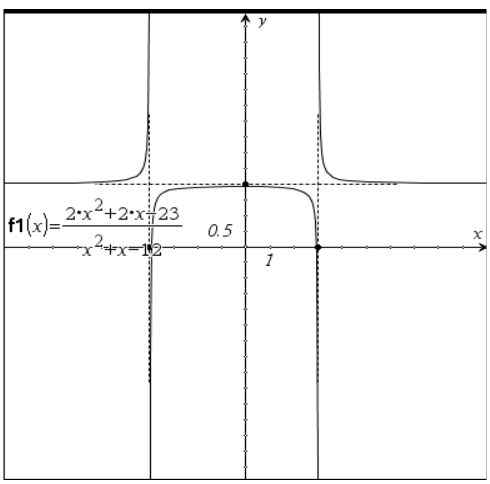
16. (x – 3)(x + 4)

17. –4, 3

18. Yes, y = 2

19. The +2 at the end indicates that the function was shifted up two units. Without the +2, the horizontal asymptote would be at y = 0. Yes, it does agree with page 2.4. The ratio of the coefficients of the leading terms of the numerator and denominator is 2/1, and the degrees of numerator and denominator are equal, so y = 2 is the horizontal asymptote.

20.



Function	Singularities	Vertical Asymptotes	Horizontal Asymptotes
21. $f(x)=\frac{1}{x^2-16}$	±4	x = –4, x = 4	y = 0
22. $f(x)=\frac{-7x-11}{x^2+4x+4}$	–2	x = –2	y = 0
23. $f(x)=\frac{x^3}{x^2+2x-8}$	–4, 2	x = –4, x = 2	none
24. $f(x)=\frac{2x^2+42}{x^2+2x-24}$	–6, 4	x = –6, x = 4	y = 2