

Teacher Notes



Activity 13

Investigating One Definition of Derivative

Problem

Students will numerically and graphically investigate the definition of the derivative $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x}$. Students will complete a table where the h -values approach zero. Then students will graph the function f ; $t(x)$, the line tangent to f at $(1, f(1))$; and a series of secant lines, $g(x)$, that pass through $(1, f(1))$ and $(1 + h, f(1 + h))$.

Numerical Exploration

3. $m = 3$
5. $ms = 3$
6. When $h = 1$, $(2, f(2))$ and $(1 + h, f(1 + h))$ are the same point.
7. $f(1) = 2, f(1 + h) = 5$ and $ms = 3$

Objective

- ◆ Students will investigate the

$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x}$ definition of the derivative.

Applicable TI InterActive! Functions

- ◆ Define `variable:= value`
- ◆ Graph

9.

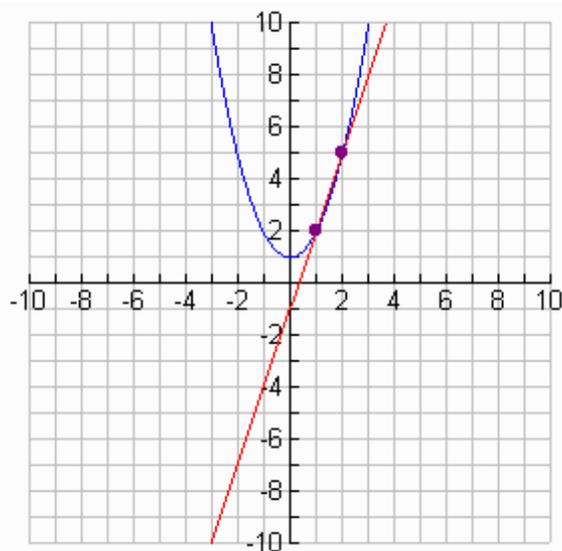
	Point 1		Point 2		
h	x	$f(x)$	$x + h$	$f(x + h)$	ms
1	1	2	2	5	3
0.5	1	2	1.5	3.25	2.5
0.1	1	2	1.1	2.21	2.1
0.01	1	2	1.01	2.0201	2.01
\vdots					
-0.01	1	2	0.99	1.9801	1.99
-0.1	1	2	0.9	1.81	1.9
-0.5	1	2	0.5	1.25	1.5
-1	1	2	0	1	1

Numerical Analysis

1. As $x + h$ gets closer to x , h is approaching 0.
2. As $x + h$ gets closer to x , the slope is approaching 2.
3. As $h \rightarrow 0$, the slope appears to be approaching 2.

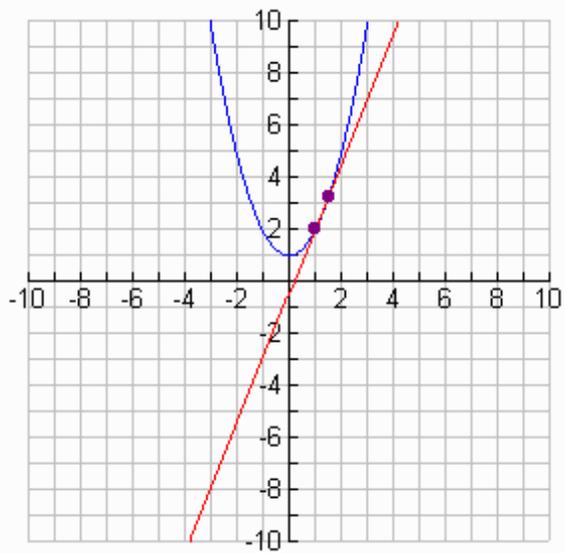
Graphical Exploration

5.

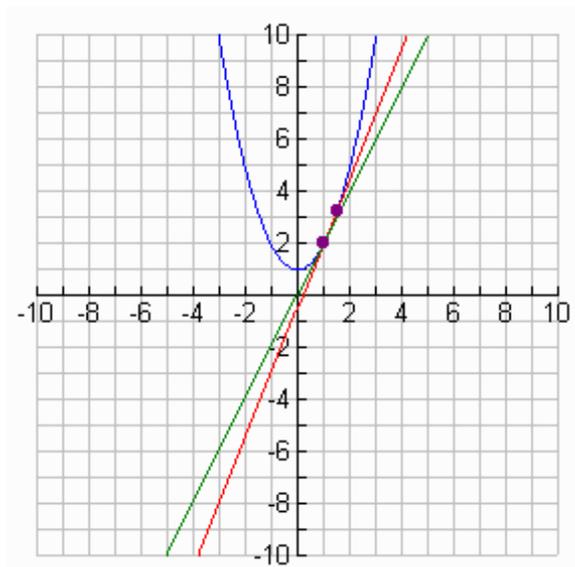
**Numerical Analysis**

1. $(1, f(1 + h))$ is getting closer to $(1, f(1))$ and the secant line changes.

2.



3.



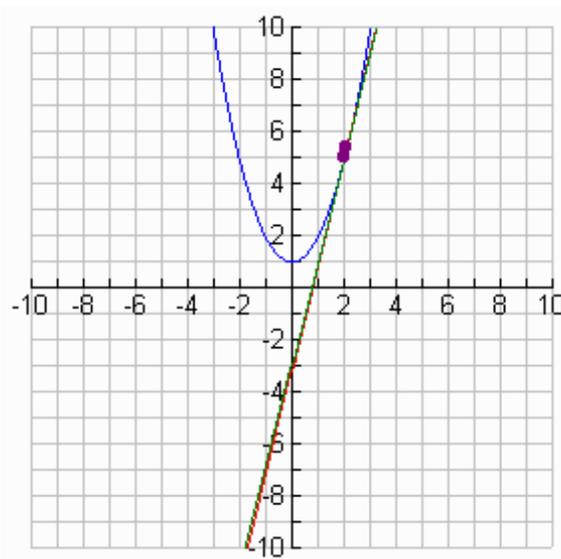
4. As $h \rightarrow 0$, the secant lines are approaching the tangent line.
5. When h is very small, $g(x)$, the secant line, and $t(x)$, the tangent line, appear to be the same line.

Additional Exercises

Use steps 1 through 27 to investigate each of the following.

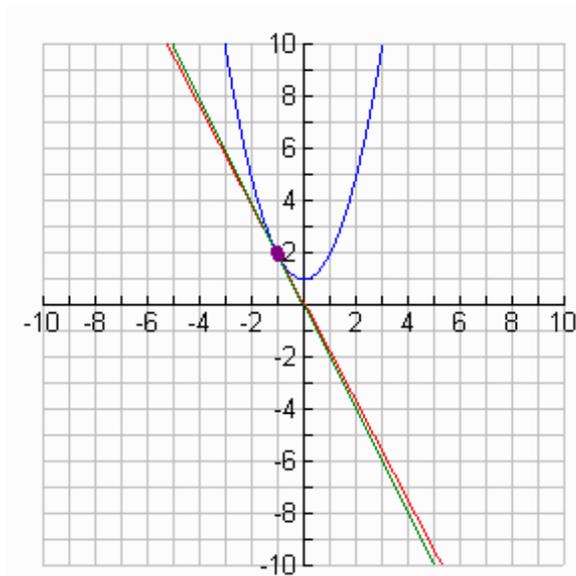
- $f(x) = x^2 + 1$ at $x = 2$. As $h \rightarrow 0$, $ms \rightarrow 4$, which implies $mt = 4$.

	Point 1		Point 2		
h	x	$f(x)$	$x + h$	$f(x + h)$	ms
1	2	5	3	10	5
0.5	2	5	2.5	7.25	4.5
0.1	2	5	2.1	5.41	4.1
0.01	2	5	2.01	5.0401	4.01
\vdots					
-0.01	2	5	1.99	4.9601	3.99
-0.1	2	5	1.9	4.61	3.9
-0.5	2	5	1.5	3.25	3.5
-1	2	5	1	5	2



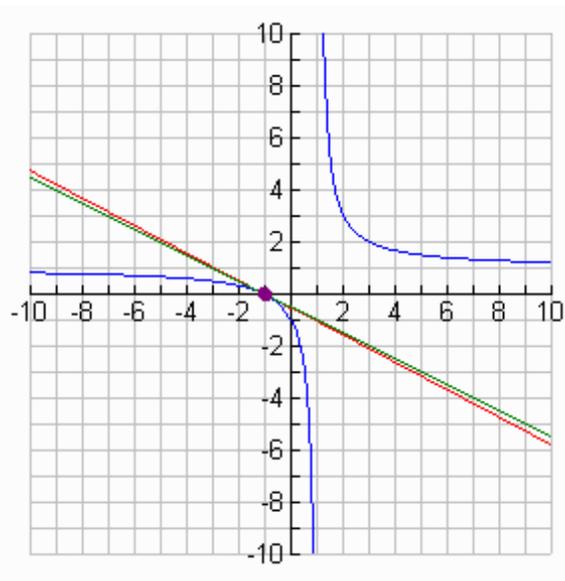
2. $f(x) = x^2 + 1$ at $x = -1$. As $h \rightarrow 0$, $ms \rightarrow 2$, which implies $mt = -2$.

	Point 1		Point 2		
h	x	$f(x)$	$x + h$	$f(x + h)$	ms
1	-1	2	0	1	-1
0.5	-1	2	-0.5	1.25	-1.5
0.1	-1	2	-0.9	1.81	-1.9
0.01	-1	2	-0.99	1.9801	-1.99
\vdots					
-0.01	-1	2	-1.01	2.0201	-2.01
-0.1	-1	2	-1.1	2.21	-2.1
-0.5	-1	2	-1.5	3.25	-2.5
-1	-1	2	-2	5	-3



3. $f(x) = \frac{x+1}{x-1}$ at $x = -1$. As $h \rightarrow 0$, $ms \rightarrow -0.5$, which implies $mt = -0.5$.

h	Point 1		Point 2		ms
	x	$f(x)$	$x + h$	$f(x + h)$	
1	-1	0	0	-1	-1
0.5	-1	0	-0.5	-0.33333	-0.66667
0.1	-1	0	-0.9	-0.05263	-0.526316
0.01	-1	0	-0.99	-0.00503	-0.502513
⋮					
-0.01	-1	0	-1.01	0.004975	-0.497512
-0.1	-1	0	-1.1	0.047619	-0.476191
-0.5	-1	0	-1.5	0.2	-0.4
-1	-1	0	-2	0.333333	-0.333333



4. $f(x) = \frac{x-2}{x^2-4}$ at $x = -1$. As $h \rightarrow 0$, $ms \rightarrow -1$, which implies $mt = -1$.

	Point 1		Point 2		
h	x	$f(x)$	$x + h$	$f(x + h)$	ms
1	-1	1	0	0.5	-0.5
0.5	-1	1	-0.5	0.666666	-0.666667
0.1	-1	1	-0.9	0.909090	-0.909090
0.01	-1	1	-0.99	0.990099	-0.990099
⋮					
-0.01	-1	1	-1.01	1.010101	-1.001010
-0.1	-1	1	-1.1	1.111111	-1.111111
-0.5	-1	1	-1.5	2	-2
-1	-1	1	-2	Undef	Undef

