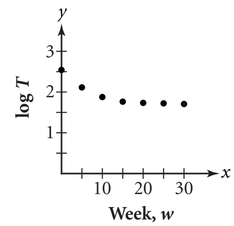
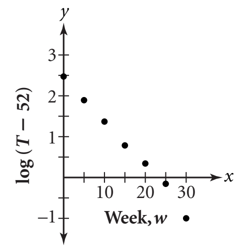
The graph of $(w, \log T)$ shown is not linear. This tells Eva that if the relationship is exponential decay, k is not 0. From the table, it appears that the toxin level may be leveling off at 52 ppm. Subtract 52 from each toxin-level measurement to test again whether $(w, \log T)$ will be linear.



w	0	5	10	15	20	25	30
$T - 52$	297	78.2	23.4	6.1	2.2	0.7	0.1
$\log(T - 52)$	2.47	1.89	1.37	0.79	0.34	−0.15	−1.0

The graph of $(w, \log(T - 52))$ does appear linear, so she can be sure that the relationship is one of exponential decay with a vertical translation of approximately 52. She now knows that the general form of this relationship is $y = 52 + ab^x$.



You could now use the same process as in Lesson 5.4 to solve for a and b, but the work you've done so far allows you to use an alternate method. Start by finding the median-median line for the linear data $(w, \log(T - 52))$.

[▶ See **Calculator Note 3D** to review how to find a median-median line on your calculator. ◀]

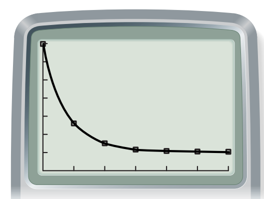
$y = -0.110x + 2.453$	Find the median-median line.
$\log(T - 52) = -0.110w + 2.453$	Substitute $\log(T - 52)$ for y and w for x.
$T - 52 = 10^{-0.110w+2.453}$	Use the definition of logarithm.
$T = 10^{-0.110w+2.453} + 52$	Add 52 to both sides.

This is not yet in the form $y = k + ab^x$, so continue to simplify.

$T = 10^{-0.110w} \cdot 10^{2.453} + 52$	Use the product property of exponents.
$T = 10^{-0.110w} \cdot 283.79 + 52$	Evaluate $10^{2.453}$.
$T = \left(10^{-0.110}\right)^w \cdot 283.79 + 52$	Use the power property of exponents.
$T = (0.776)^w \cdot 283.79 + 52$	Evaluate $10^{-0.110}$.
$T = 52 + 283.79(0.776)^w$	Reorder in the form $y = k + ab^x$.

The equation that models the amount of toxin, T, in the lake after w weeks is $T = 52 + 283.79(0.776)^w$.

If you graph this equation with the original data, you see that it fits quite well.

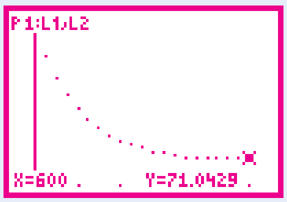


Guiding the Investigation

Step 2 Students may have extra time while data are being collected. You may want to assign exercises or review Example A during this time. If you don't have temperature probes you can use the Cooling Sample Data worksheet.

Step 3 The temperature of the water should approach room temperature. You may want to use an accurate thermometer or an additional temperature probe to measure the room temperature so you have an idea what L should equal.

Step 3 Answers for sample data:



[−60, 660, 120, 67, 93, 50]

The temperature limit for the sample data appears to be 71°.

Step 4 When plotting the data, make sure students use y-values equal to $\log(p - L)$, not $\log p - L$. The data will probably not be perfectly linear. If there are data points for which $(temp - unit)$ equals 0, then the calculator will give an error message. Have students delete those points from their tables and try again. If more than one point with the same temperature appears at the end of the list, students may also delete those points when they graph.

Step 5 Students may use any method for finding a line of fit.

SHARING IDEAS

[Ask] "Can the equation of Example A be solved by undoing?" Work through it together, one step at a time, to get $x = \log_{0.954}\left(\frac{1.47 - 1.25}{0.72}\right) + 10$. Point out the fact that the expression can't be written from left to right because you first write $\frac{1.47 - 1.25}{0.72}$ and then insert log on the left to avoid the potential for errors in placing +10. Explain reasons for using methods such as taking the logarithm of both sides.

[Ask] "What kind of equation do you get when you take the logarithm of both sides of an exponential equation?" [linear] **[Alert]** Students may not think of expressions like $\log 0.954$ and $\log 0.3056$ as numbers, constant coefficients in a linear equation. This realization helps lay the groundwork for curve straightening.

After performing curve straightening, students may ask "How do we know we found the right line?" or "What's the best exponential equation for the data?" Point out that mathematical modeling is an art and that the value of a model is tested by the reasonableness of the predictions it makes.

You might use the Fathom demonstration Curve Straightening to summarize the ideas of the investigation.

Investigation
Cooling

You will need

- a cup of hot water (optional)
- a temperature probe
- a data collection device
- a second temperature probe (optional)

In this investigation you will find a relationship between temperature of a cooling object and time.

Step 1 Connect a temperature probe to a data collector and set it up to collect 60 data points over 10 minutes, or 1 data point every 10 seconds. Heat the end of the probe by placing it in hot water or holding it tightly in the palm of your hand. When it is hot, set the probe on a table so that the tip is not touching anything and begin data collection. [▶ See **Calculator Note 5E.** ◀]

Step 2 Let t be the time in seconds, and let p be the temperature of the probe. While you are collecting the data, draw a sketch of what you expect the graph of (t, p) data to look like as the temperature probe cools. Label the axes and mark the scale on your graph. Did everyone in your group draw the same graph? Discuss any differences of opinion.

Step 3 Plot the data in the form (t, p) on an appropriately scaled graph. Your graph should appear to be an exponential function. Study the graph and the data, and guess the temperature limit L. You could also use a second temperature probe to measure the room temperature, L.

Step 4 Subtract this limit from your temperatures and find the logarithm of this new list. Plot data in the form $(t, \log(p - L))$. If the data are not linear, then try a different limit.

Step 5 Find the equation that models the data in Step 4, and use this to find an equation that models the (t, p) data in Step 3. Give real-world meaning to the values in the final equation.

EXERCISES

Practice Your Skills

1. Prove that these statements of equality are true. Take the logarithm of both sides, then use the properties of logarithms to re-express each side until you have two identical expressions.

a. $10^{n+p} = (10^n)(10^p)$ **b.** $\frac{10^d}{10^e} = 10^{d-e}$

2. Solve each equation. Check your answers by substituting your answer for x.

a. $800 = 10^x$ 2.90309 **b.** $2048 = 2^x$ 11 **c.** $16 = 0.5^x$ -4

d. $478 = 18.5(10^x)$ 1.4123 **e.** $155 = 24.0(1.89^x)$ 2.9303 **f.** $0.0047 = 19.1(0.21^x)$ 5.3246

3. Suppose you invest \$3000 at 6.75% annual interest compounded monthly. How long will it take to triple your money? $t = \frac{\log 3}{\log 1.005625} \approx 195.9$; about 195.9 mo or about 16 yr 4 mo

Closing the Lesson

Instead of solving an exponential equation by rewriting both sides to have the same base, you can take the logarithm of both sides (if they're positive).

Taking the logarithm of both sides of the general equation $y = ab^x$ and applying the power property gives the linear equation $\log y = \log a + (\log b)x$. Line-of-fit techniques can be applied to find values of $\log a$ and $\log b$. The **curve-straightening,** or **linearizing,** approach can be extended to get a shifted exponential equation of the form $y - k = ab^x$.

BUILDING UNDERSTANDING

The exercises provide practice in solving exponential equations and in curve straightening.

ASSIGNING HOMEWORK

Essential	2–4
Performance assessment	1, 5–10
Portfolio	8, 9
Journal	4
Group	7
Review	11–14

Helping with the Exercises

If you will be assessing curve straightening, you may want to save an exercise or two for your review unit, because the Chapter Review contains no exercises of this type.

Exercise 1 **[Alert]** Students should use only the properties of logarithms. This is a proof only of the claim that if the logarithm properties are true then the exponent properties are true. Earlier, the logarithm properties were derived from the exponent properties, so this argument would show that the properties are logically equivalent.

Exercise 2 In 2c, students may see that they have powers of 2 on both sides and write $2^4 = 2^{-x}$, from which they see that $x = -4$.

As needed in 2d–f, remind students to isolate the exponent by dividing before taking logarithms.

Besides substituting, students can check each answer by graphing both sides of the equation and finding the x-coordinate of the point of intersection.

See page 886 for answers to Steps 4 and 5 and Exercise 1.

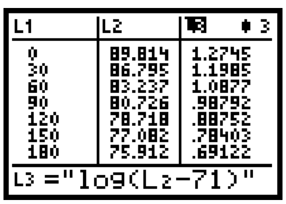
c. Same method and answer as Step 6b.

d. Extend the bottom ruler to the right. Place 5 over 18 and read the number below 1, which is 3.6.

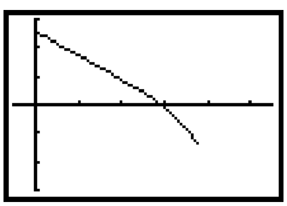
LESSON 5.8, PAGES 289, 291

Step 4 sample data:

Time (s)	Temperature (°F)
0	89.8143
30	86.7953
60	83.2372
90	80.7256
120	78.7182
150	77.0818
180	75.9116
210	74.7286
240	73.8714
270	73.1955
300	72.7045
330	72.2136
360	71.8864
390	71.5571
420	71.3857
450	71.2143
480	71.2143
510	71.2143
540	71.2143
570	71.0429
600	71.0429



Delete repeated values at the end of the list to create a better logarithm graph.



[−60, 600, 120, −1.5, 1.5, 0.5]

Step 5 The median-median line for the sample data in Step 4 is $\hat{y} = -0.004x + 1.364$.

Substitute $\log(y - 71)$ for y and t for x, and solve for y. $y = 71 + 23.121(0.991)^x$. The temperature limit is 71°; the approximate change between the original temperature and the limit is 23.121, and 0.991 is the percentage of this change that occurs each second.

1a. $\log(10^{n+p}) = \log((10^n)(10^p))$

$(n + p)\log 10 = \log 10^n + \log 10^p$

$(n + p)\log 10 = n\log 10 + p\log 10$

$(n + p)\log 10 = (n + p)\log 10$

Because the logarithm of the left side equals the logarithm of the right, the left and right sides are equal. Or, because $\log(10^{n+p}) = \log((10^n)(10^p))$, $10^{n+p} = (10^n)(10^p)$.

1b. $\log\left(\frac{10^d}{10^e}\right) = \log(10^{d-e})$

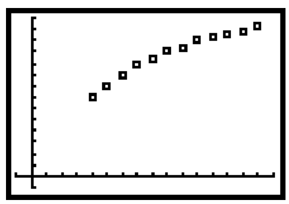
$\log 10^d - \log 10^e = \log(10^{d-e})$

$d\log 10 - e\log 10 = (d - e)\log 10$

$(d - e)\log 10 = (d - e)\log 10$

Because the logarithm of the left side equals the logarithm of the right, the left and right sides are equal. Or, because $\log\left(\frac{10^d}{10^e}\right) = \log(10^{d-e})$, $\frac{10^d}{10^e} = 10^{d-e}$

8a. Let x represent time in minutes, and let y represent tempeature in degrees Fahrenheit.

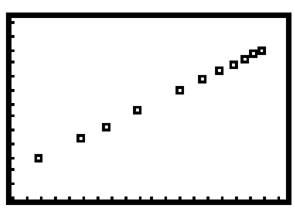


[−2, 32, 2, −5, 70, 5]

8b. Because the curve is both reflected and translated, first graph points in the form $(x, -y)$. Then translate the points up so that the data approach a long-run value of zero. Then graph points in the form $(x, \log(-y + 74))$, which appears to be linear.

The median-median line for these altered data is $\log(-y + 74) = 1.823 - 0.0298x$. Solving for y gives the equation $\hat{y} = 74 - 10^{1.823-0.0298x}$, or $\hat{y} = 74 - 66.53(0.9337)^x$.

9a. The data are the most linear when viewed as (log(*height*), log(*distance*)).



[2.3, 4.2, 0.1, 1.5, 2.8, 0.1]