Exponential Growth

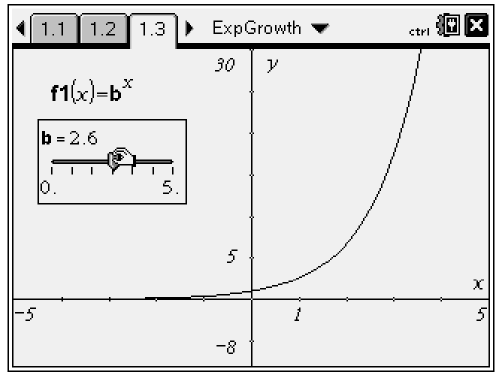
ExpGrowth.tns

Name ___________________________

Class ___________________________

Problem 1

On page 1.3, the graph of the function **f1**(x) = b^x (for b > 0) is displayed. The slider allows the value of b to be changed. Change the value of the slider. Observe how the value of b affects the shape of the graph.



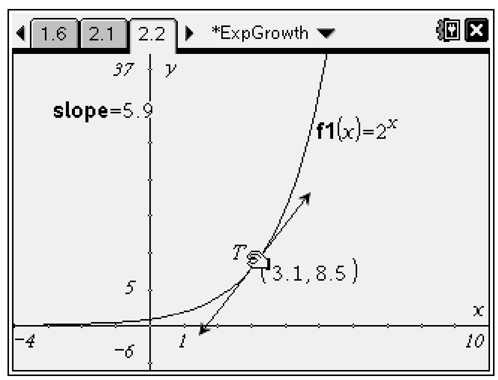
- Write at least three observations about the effect of the value of b on the graph of **f1**(x).

- What value of b results in a constant function? Explain your answer.

- Explain why the value of b cannot be negative.

Problem 2

Advance to page 2.2, which displays the graph of the function **f1**(x) = 2^x, along with a tangent line to the curve at point T. The slope of the tangent line is displayed. Grab and drag point T along the curve, and observe the changing slope.

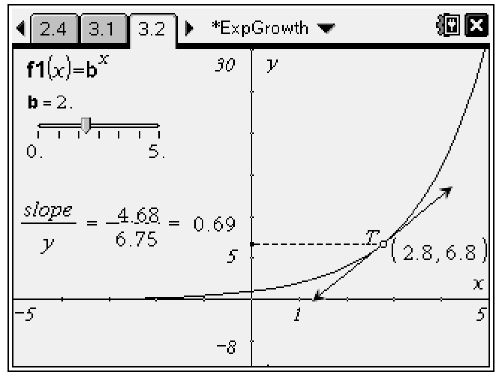


- For any point T, how does the slope of the tangent line at T compare to the value of the function, **f1**(x)? (Remember that the value of f(x) is the same as the value of y.)

- Write at least two observations about the graph and/or the slope of its tangent at T.

Problem 3

On page 3.2, a combination of pages 1.2 and 2.1 is shown. First, use the slider to select a value for *b*. Then explore the relationship between the slope of the tangent line and the value of the function (the slope and corresponding value of *y* are displayed in the upper left corner). Now use the slider to select a new value for *b* and again explore the relationship between the slope and *y*. Continue this exploration for several different values of *b*.



- Slope is a measure of rate of change in a function. In this example, sometimes the slope is ***less than*** *y*, and sometimes it is ***greater than*** *y*. There is only one value of *b* for which the rate of change of the function $y = b^x$ at any point is ***equal to*** the value of the function itself. Can you find an approximate value of this number? (The ratio $\frac{slope}{y}$ displayed on the screen should help.)

  $b \approx$ _________

- This approximation may look familiar. Have you encountered this number before in your study of mathematics?

Applications

The number you found in Problem 3 is an approximation for the mathematical constant *e*. As you discovered, it is unique in that it is the only value of *b* such that $y = b^x$ changes at a rate that is equal to the value of the function itself. It also shows up in a number of functions that model natural phenomena.

Some examples are:

(a) the growth of populations of people, animals, and bacteria;

(b) the value of a bank account in which interest is compounded continuously;

(c) and radioactive decay.

The common feature is that the rate of growth or decay is proportional to the size of the population, account balance, or mass of radioactive material. Growth and decay situations can be modeled by equations of the form $P = P_0e^{kt}$, where *P* is the current amount or population, P_0 is the initial amount, *t* is time, and *k* is a growth constant. An amount is *growing* if $k > 0$ and *declining* if $k < 0$.

The following are examples of exponential growth or decay. For each exercise, write an equation to represent the situation and solve your equation to find the answer.

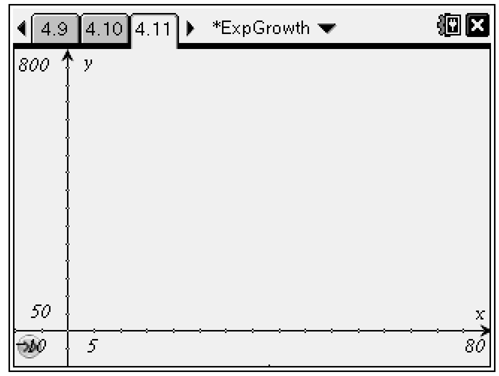
1. Suppose you invest $1,000 in a CD that is compounded continuously at the rate of 5% annually. (Compounded continuously means that the investment is always growing rather than increasing in discrete steps.) What is the value of this investment after one year? Two years? Five years?

2. A colony of bacteria is growing at a rate of 50% per hour. What is the approximate population of the colony after *one day* if the initial population was 500?

3. Suppose a glacier is melting proportionately to its volume at the rate of 15% per year. Approximately what percent of the glacier is left after ten years if the initial volume is one million cubic meters? (This is an example of exponential decay.)

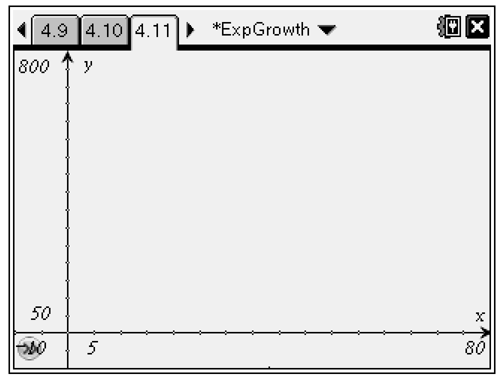
4. A snowball is rolling down a snow covered hill. Suppose that at any time while it is rolling down the hill, its weight is increasing proportionately to its weight at a rate of 10% per second. What is its weight after 10 seconds if its weight initially was 2 pounds? After 20 seconds? After 45 seconds? After 1 minute? What limitations might exist on this problem?

On page 4.11, graph the equation that models the situation in Exercise 4 and sketch a graph of it at the right.



How might you use the graph to answer the snowball questions above?

Now suppose the snowball has reached a weight of 1,000 pounds and come to rest at the bottom of the hill. It now begins to melt at a rate of 20% per hour. Write an equation to represent this situation, graph the equation, and sketch it here.



About how long will it take to melt to half of its weight? To its original weight of 2 pounds? (Again, use the graph to help.)

Think about the weight of the snowball after it has completely melted. What are the limitations of this model? Estimate the number of hours it would take for the snowball to completely melt.