NUMB3RS Activity: How to Get a Date
Episode: "Bones of Contention"

Topic: Exponential Decay   **Grade Level**: 10 - 12
Objective: To learn about exponential decay and its application to radiocarbon dating.
Time: About 30 minutes
Materials: (For each group) One 9" square baking pan, one 89 g bag of M&M's™, one 113 g box of Reese's Pieces™, and a graphing calculator.

Introduction

For many years, scientists have been using C-14, a radioactive isotope of carbon to find the age of objects that once were parts of carbon-based life forms. When an organism dies, the proportion of C-14 in its tissue declines at a predictable exponential rate. This enables scientists to estimate the age of the tissue mathematically using exponential functions.

In this activity, students use candy to model the time it takes for something to decay. The data collected can be graphed and described using a formula. Students who have learned about logarithms can derive the algebraic formula that gives the age of an object as a function of its C-14 content.

Science Background:

In "Bones of Contention," Charlie explains to Don the mathematics behind radiocarbon dating, a marvelous synthesis of chemistry, biology, and physics that earned Willard F. Libby a Nobel prize in 1960. The technique relies on the fact that cosmic radiation is constantly converting atmospheric nitrogen into a radioactive isotope of carbon called carbon-14. All that is required to accomplish this transformation is for a neutron to hit the nucleus of the nitrogen atom with enough force to kick out a proton, which takes an electron away with it to leave a carbon atom behind. In time, a process called beta decay converts the neutron back to a proton and the C-14 back to a nitrogen atom. What makes this process so useful is that the decaying part of it proceeds quite slowly. In fact, it takes about 5,730 years for half of the C-14 in any sample to decay to nitrogen-14, which is why we say the *half-life* of C-14 is 5,730 years.

Student Page Answers: **2**. *Answers will vary, but they will be close to 50.* **3**. *If n is the answer to question 1, the value entered at T = 1 will be 100 – n. Answers will vary, but the number of atoms removed at T = 2 will be approximately 25. This number is subtracted from the table entry at T = 1 to give the table entry at T = 2.* **4**. *Answers will vary, but each entry in the table should be approximately half the previous entry, e.g.: 100, 50, 25, 12, 6, 3.* **5a**. *The two graphs intersect at the point (1844.65, 4), so the bone is approximately 1,845 years old.* **5b.** $t = 5730 \times \log_{1/2}\left(\frac{y}{A_E}\right)$, *or* $t = 5730\frac{\ln y - \ln A_E}{\ln\ 0.5}$.

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Name: ________________________________ Date: _______________

NUMB3RS Activity: How to Get a Date

In "Bones of Contention," Charlie explains to Don the mathematics of radiocarbon dating, a method that forensic scientists and archeologists use to estimate the age of wood, bone, fabric, or other substances that once were living tissue. The technique relies on the fact that a fairly constant proportion (about a trillionth) of the carbon in all living things is a slightly unstable isotope called C-14 that slowly turns into nitrogen. While an organism is alive, it replaces its carbon faster than the C-14 can decay, so the proportion of C-14 remains about the same as in the environment. After the organism dies and the carbon exchange ceases, the C-14 begins to decay in its slow, predictable way, so that every 5,730 years the amount of C-14 is reduced by half. (We therefore say that the *half*-life of C-14 is 5,730 years.) If you find a bone that has half the proportion of C-14 as the proportion in living tissue, you can conclude that the bone came from a creature that died 5,730 years ago!

Modeling Radioactive Decay

Let's look at the mathematics of the process which converts C-14 to nitrogen. You will need a 9" square baking pan, a "king-size" (89 g) bag of M&M's™ (which will represent C-14 carbon atoms) and a 113 g box of Reese's Pieces™ (which will represent nitrogen atoms).

1. Spill the C-14 atoms into the baking pan and count out exactly 100. Pick ones on which the "M" is easy to see. Enter the number 100 next to time *T* = 0 in the table below.

Time *T*	C-14 Atoms
0	
1	
2	
3	
4	
5	

2. Mix the atoms in the pan thoroughly. Now remove the ones with the "M" side facing up. To find the number of C-14 atoms that remain, subtract the number of atoms you removed from 100 (the number of C-14 atoms at *T* = 0). Record the subtraction result next to time *T* = 1. Then replace the decayed carbon atoms with an equal number of nitrogen atoms. (Remember, use Reese’s Pieces to show nitrogen atoms.)

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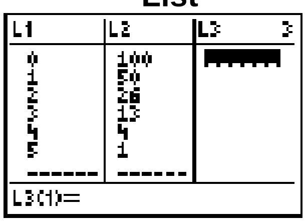
3. Again, mix the atoms thoroughly and remove the ones with the "M" side up. Count the number you removed, and subtract from the number of carbon atoms at $T = 1$. Record this number next to time $T = 2$. Replace the decayed carbon atoms with nitrogen atoms.

4. Continue the process until you have filled in the table. After five time periods you should be left with almost all nitrogen atoms, because most of the carbon atoms will have decayed. (This eventually happens with real C-14 as well, which is why this kind of radiocarbon dating is not useful for dating objects more than 50,000 years old.)

Graphing and Showing Data

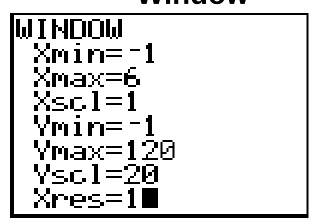
Enter the numbers from the table into lists L_1 and L_2 in your calculator, as shown on the screens below. In the STAT PLOT window, turn Plot 1 On and set up a scatter plot as shown in the second screen. Set up the WINDOW as shown in the third screen.

List

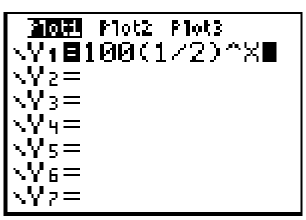


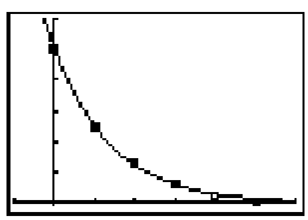
Stat Plot

Window



In the "Y=" window, let $Y_1 = 100(1/2)$^X and press GRAPH. You should see the curve fit the scatter plot very well, as in the example below.





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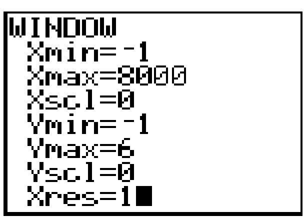
Using the Carbon Dating Principle

The purpose of this simulation was to show you that C-14 decays *exponentially* according to the equation $y = A\left(\frac{1}{2}\right)^t$, where *A* is the amount of C-14 present at time 0. Notice that about half of the original amount remains when $t = 1$, a quarter remains when $t = 2$, and so on. Each time unit must be the half-life of the substance. Because the half-life of C-14 is 5,730 years, we can replace *t* with $\frac{t}{5730}$ to measure time in years. The initial amount *A* can be determined by applying the current environmental proportion to the total carbon content of the sample. The result is the *Carbon-Dating Principle*:

> If an organism has been dead for *t* years (where $0 \le t \le 50{,}000$), the amount *y* of C-14 in a sample of tissue from that organism is approximately
>
> $$y = A_E\left(\frac{1}{2}\right)^{t/5730},$$
>
> where A_E is the amount of C-14 predicted by applying the current environmental proportion to the total carbon content of the sample.

5. Suppose a scientist finds an old bone that contains 4 units of C-14. The scientist believes that a comparable mass of carbon today would contain 5 units of C-14. About how old is the bone? (Hint: Find how many years (*t)* it would take for 5 units of C-14 to decay to 4 units) You can find *t* two ways.

a) Solve for *t* graphically. Graph $Y_1 = 5(1/2)$ ^ $(X/5730)$ and $Y_2 = 4$ in the WINDOW shown below. (Also, be sure to turn off the Stat Plot from earlier in this activity.)



Find the point at which the graphs intersect. What is the approximate age of the bone?

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b) Solve for *t* algebraically. (For those who have studied logarithms) Solve the equation $y = A_E\left(\frac{1}{2}\right)^{t/5730}$ to get *t* in terms of *y*. This equation gives the age *t* of an object (in years) as an explicit function of its C-14 content *y*.

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

- The candy simulation in this activity models exponential decay with base 1/2 since each M&M has a 1/2 probability of landing with its "M" side up. You can model exponential decay with other bases by using a large number of six-sided dice (or number cubes). If, after each time period, you eliminate all dice showing an even number, you will have an alternative model of base-1/2 decay. With what rule could you model base-1/3 decay? Base-2/3 decay? Base-1/6 decay? For a tougher challenge, describe a way to use the dice to model base-1/5 decay.

- Carbon-14 decays exponentially because the rate of decay is proportional to the amount present. (The more radioactive material present, the more quickly the unstable nuclei break down.) Whenever a rate of decay (or growth) is proportional to the amount present, the decay (or growth) is *always* exponential. Give some other examples of exponential decay in the real world.[1]

- Cesium-137 is a radioactive isotope with a half-life of 30.2 years. How long will it take 90% of a sample of Cesium-137 to decay to a stable state?

<u>Additional Resources</u>

There are many explanations of radiocarbon dating on the web. One of the nicest is at http://www.chem.uwec.edu/Chem115_F00/norquicj/project.html.

If you want to learn more about radiocarbon dating, neutron captures, and beta decay through an interactive question-and-answer format, check out http://vcourseware5.calstatela.edu/VirtualDating/files/RC0/RC_0.html.

[1] Some interesting examples of exponential decay in the real world can be found at http://en.wikipedia.org/wiki/Exponential_decay.

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