What's the Difference?

ID: 12557

Time Required
15–20 minutes

Activity Overview

Students explore the angle difference formula for cosine. Students will apply the formula and compare their results to interactive unit circle diagrams that assist the student in visualizing the problems involved. The derivations of the angle difference and sum formulas for cosine are optional extensions included with this activity.

Topic: Cosine Difference Identity

- *Angle Sum and Difference Identity Derivation (optional extensions)*
- *Unit Circle*
- *Sine and Cosine values*
- *Verification of Equivalence by Graphing*

Teacher Preparation and Notes

- *This activity was designed for use with TI-Nspire technology. If the extensions are used during class, the activity will take approximately 30–45 minutes to complete.*
- *The first problem and second problems engage students in an exploration of the difference formula for cosine. Problem 1 is devoted to unit circle review and developing an understanding of the angle difference diagram included in the activity.*
- *Problem 2 engages students in the application of the angle difference formula for cosine. Students find the cosine for angles such as 15° from well-known angles on the unit circle, such as 45° and 60°.*
- *The extensions of this activity have students derive the angle sum and difference formulas for cosine.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12557" in the quick search box.***

Associated Materials

- *WhatsTheDifference_Student.doc*
- *WhatsTheDifference.tns*
- *WhatsTheDifference_Soln.tns*

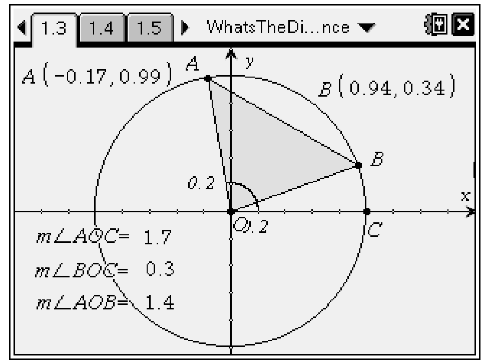
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Proof of Identity (TI-Nspire technology) — 9847*
- *Round and Round She Goes… (TI-Nspire technology) — 12386*

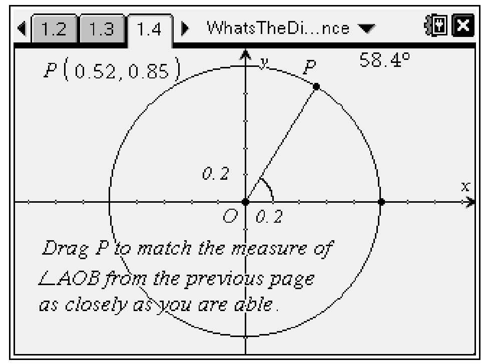
Problem 1 – Exploring the Angle Difference Formula for Cosine

Students explore the measures of angles and values of sine and cosine related to a diagram useful in the derivation of the cosine difference formula. On page 1.3, points *A* and *B* may be dragged to adjust angle measures so that students may explore angle difference relationships.



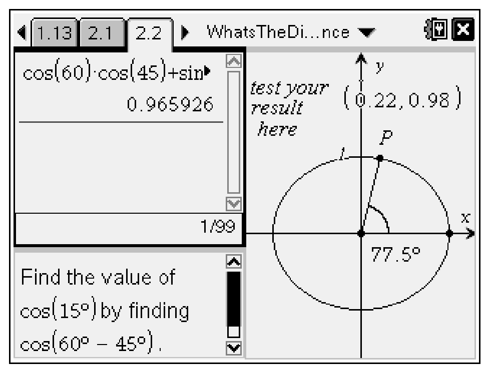
Page 1.4 is useful in further exploration to obtain sine and cosine values for a single angle, in this case, ∠*AOB*.

Students answer a variety of questions related to these angle difference diagrams.

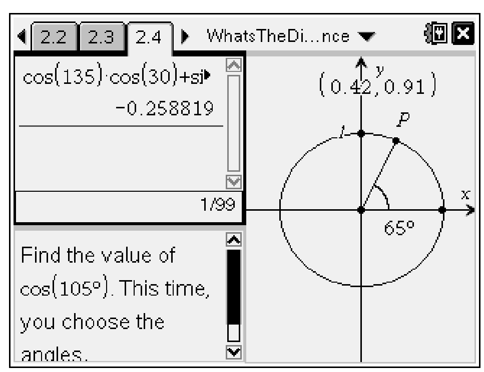


Problem 2 – Applying the Angle Difference Formula

Students find cosine values for angle measures such as 15° and 105°, which take advantage of angles with well known values (for many students) of sine and cosine. CAS users will find that exact values are obtained and decimal approximations will be needed to test results by comparing to the unit circle on the right side of pages 2.2 through 2.4.

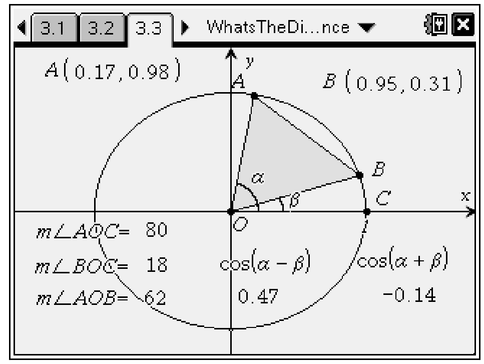


For non-CAS users, you may wish to take some additional time with this activity to work with students in obtaining exact values for the angles involved in this problem.

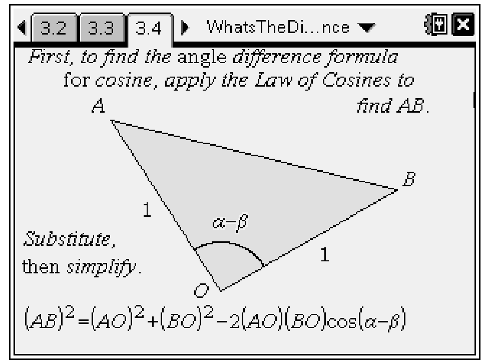


Extension – Deriving the Angle Difference Formula for Cosine

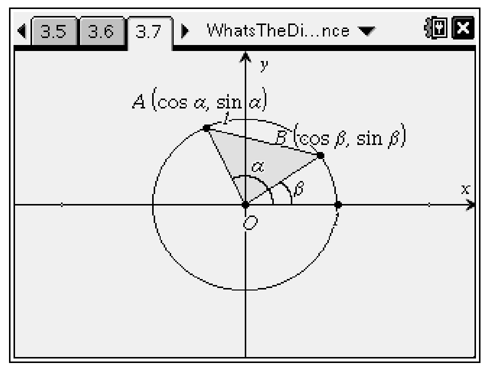
Students use the Law of Cosines to derive the angle difference formula for cosine. A unit circle representation is provided to help students visualize the problem and to provide the necessary background to set up the derivation.



CAS users may use the included *Calculator* application to assist with the simplification. Since the variables α and β are linked to values, it will be necessary for students to use other variables, such as *x* and *y*, on the *Calculator* application.



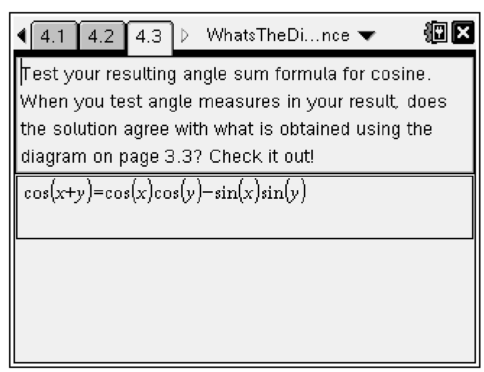
Guidance regarding how to begin the derivation will be helpful to students. Show students how to set up their work for the first derivation and students should be able to follow that example for the remaining derivations in this activity.



Extension – Angle Sum Formula for Cosine

Students using CAS will be able to quickly derive the angle sum formula for cosine by simply substituting $-\beta$ (or $-y$) in for β (or y).

It is important to take some time here to discuss with students why $\cos(-y) = \cos(y)$ and $\sin(-y) = -\sin(y)$. Consider having students insert a *Geometry* page and construct a model to help illustrate and explain these two situations involved in this formula derivation.



Student Solutions

1. cosine

2. sine

3. 0.98

4. –0.17

5. 0.34

6. 0.94

7. 0.98

8. 0.17

9. answers may vary—relationship is not easy to quickly obtain from the interactive graph page

10. 0.97

11. 0.26

12. –0.26

13.
$$\begin{aligned}(AB)^2 &= AO^2 + BO^2 - 2 \cdot AO \cdot BO \cdot \cos(AOB)\\ &= 1 + 1 - 2\cos(\alpha - \beta)\\ &= 2 - 2\cos(\alpha - \beta)\end{aligned}$$

14.
$$\begin{aligned}(AB)^2 &= (\cos(\alpha) - \cos(\beta))^2 + (\sin(\alpha) - \sin(\beta))^2\\ &= \cos^2(\alpha) - 2\cos(\alpha)\cos(\beta) + \cos^2(\beta) + \sin^2(\alpha) - 2\sin(\alpha)\sin(\beta) + \sin^2(\beta)\\ &= 1 - 2\cos(\alpha)\cos(\beta) + 1 - 2\sin(\alpha)\sin(\beta)\\ &= 2 - 2\cos(\alpha)\cos(\beta) - 2\sin(\alpha)\sin(\beta)\end{aligned}$$

15. $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

16. $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$