Special Segments in Triangles

ID: 8672

Time required
90 minutes

Activity Overview

In this activity, students explore medians, altitudes, angle bisectors, and perpendicular bisectors of triangles. They then drag the vertices to see where the intersections of the segments lie in relation to the triangle. They see that the intersection of the angle bisectors (incenter) and that of the perpendicular bisectors (circumcenter) can be used, respectively, to draw a circle inscribed in and circumscribed about a triangle.

Topic: Rational Functions & Equations

- *Medians, altitudes, angle bisectors, and perpendicular bisectors of triangles.*
- *Centroid, orthocenter, incenter, and circumcenter*

Teacher Preparation and Notes

- *This activity is designed to be used in a high-school geometry classroom.*
- *Before starting this activity, students should be familiar with the following terms: opposite side (of an angle in a triangle), vertex, midpoint, perpendicular, radius, concurrency, and point of concurrency.*
- *Students will click on the slider in problems 2, 3, 4, and 5 to explore several points of concurrency for a triangle.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8672" in the keyword search box.***

Associated Materials

- *SpecialSegmentsinTriangles_Student.doc*
- *SpecialSegmentsinTriangles.tns*

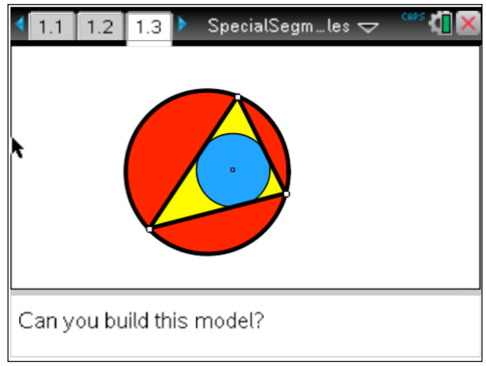
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *The Euler Line and its Ratios (TI-Nspire technology) — 11519*
- *Hey, Ortho! What's your Altitude? (TI-Nspire technology) — 11484*

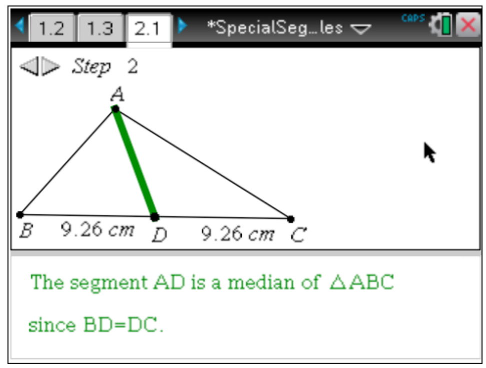
Problem 1 – An Introduction

The diagram on page 1.3 provides a preliminary exploration into some of the concepts that will be encountered within this activity. By dragging the vertices of the triangle, students can simultaneously investigate a circle inscribed in a triangle (constructed using the incenter) and a circle circumscribed about the triangle (constructed using the circumcenter).



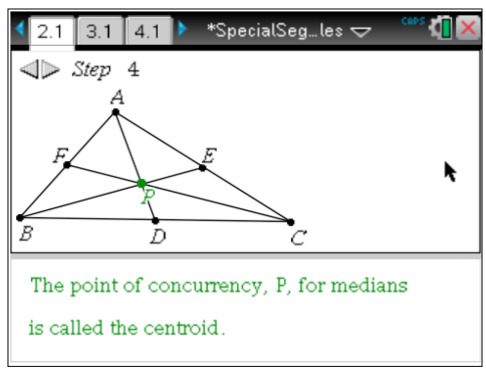
Problem 2 – Medians & the Centroid

Step 1: Students encounter their first "special segment"; the median. The definition of a median is given to the students. The median is defined as the segment from a vertex of the triangle to the midpoint of the opposite side.



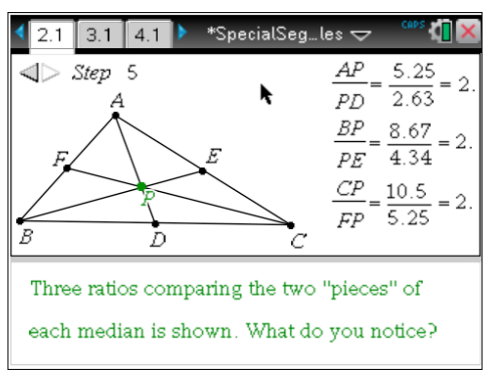
Step 2: Students will verify that $\overline{AD}$ is a median of $\triangle ABC$. The lengths of $\overline{BD}$ and $\overline{DC}$ are shown for the student.

Step 3: Students are given the other two medians of $\triangle ABC$.



Step 4: Students are given the point of concurrency for medians, the centroid.

Step 5: Students are shown three ratios comparing *AP* to *PD*, *BP* to *PE*, and *CP* to *FP*. The students should notice that the point of concurrency, *P*, cuts the median into a 2 to 1 ratio. The length from the vertex of a triangle to the centroid is twice the length from the centroid to the midpoint of the opposite side.

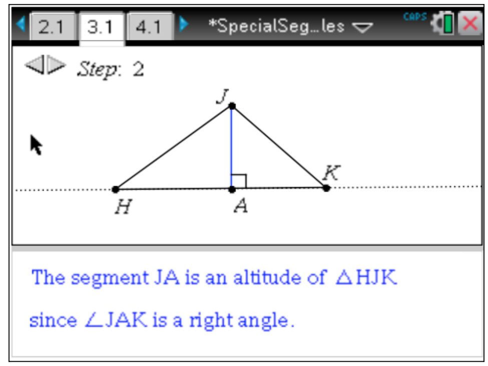


TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

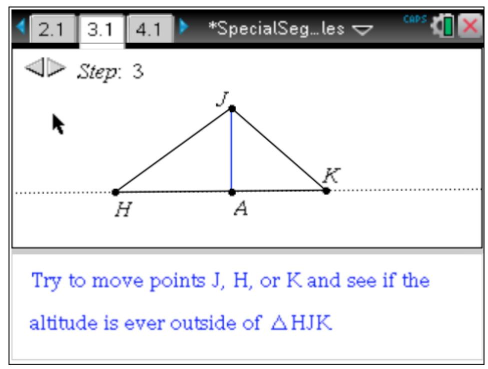
Problem 3 – Altitudes & the Orthocenter

Step 1: Students encounter their second "special segment"; the altitude. The definition of an altitude is given to the students. The altitude is defined as the segment from a vertex of the triangle perpendicular to the opposite side.



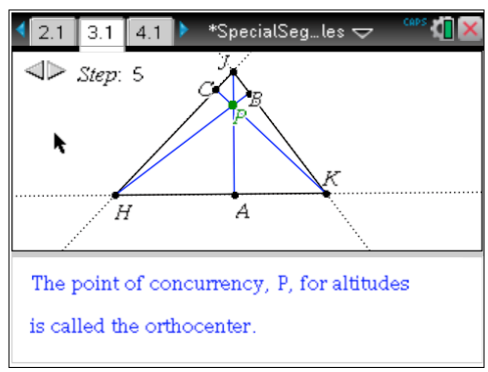
Step 2: Students will verify that $\overline{JA}$ is an altitude of $\triangle HJK$ since $\angle JAK$ is a right angle.

Step 3: Students are asked to move points *J*, *H*, or *K* to see if the altitude is ever outside of $\triangle HJK$.



Step 4: Students are given the other altitudes of $\triangle HJK$.

Step 5: Students are given the point of concurrency for altitudes, the orthocenter.



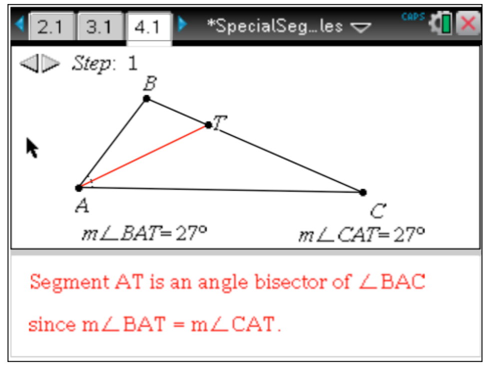
Step 6: Students are given the orthocenter and asked to drag vertices of the triangle to change the shape of $\triangle HJK$ and watch what happens to the orthocenter.

TI-Nspire Navigator Opportunity: *Quick Poll*

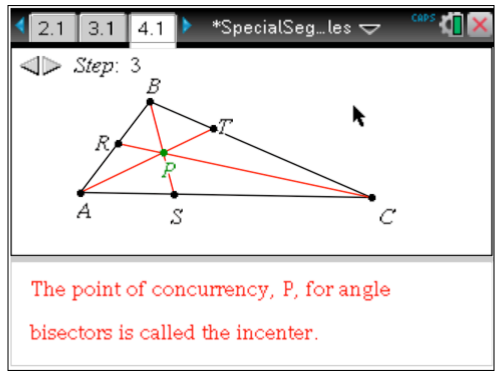
See Note 2 at the end of this lesson.

Problem 4 – Angle Bisectors & the Incenter

Step 1: Students encounter their third "special segment"; the angle bisector. They will verify that $\overline{AT}$ is an angle bisector of $\triangle ABC$ since $m\angle BAT = m\angle CAT$ for all values.

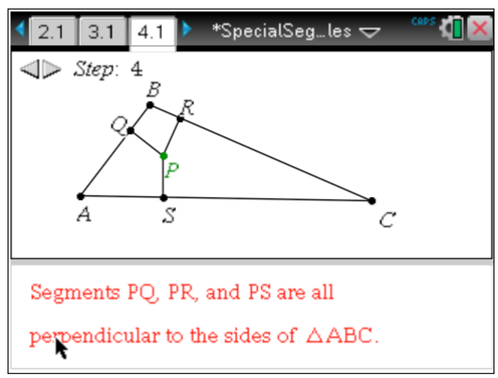


Step 2: Students are given the other angle bisectors of $\triangle ABC$, $\overline{BS}$ and $\overline{CR}$.



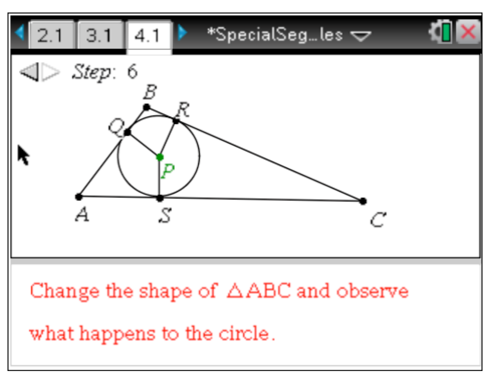
Step 3: Students are given the point of concurrency for angle bisectors, the incenter.

Step 4: Students are given $\overline{PQ}$, $\overline{PR}$, and $\overline{PS}$ which are segments that are from the incenter to a point perpendicular to each side of the triangle.



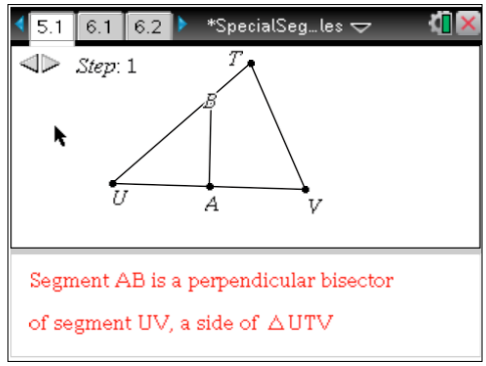
Step 5: Students are given the circle inscribed about $\triangle ABC$

Step 6: Students are given the incenter and asked to drag vertices of the triangle to change the shape of $\triangle ABC$ and observe what happens to the circle.



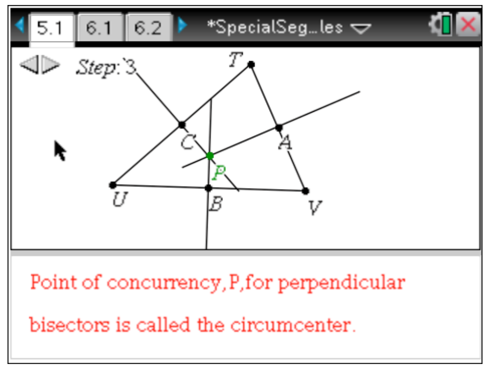
Problem 5 – Perpendicular Bisectors & the Circumcenter

Step 1: Students encounter their fourth "special segment," the perpendicular bisector.

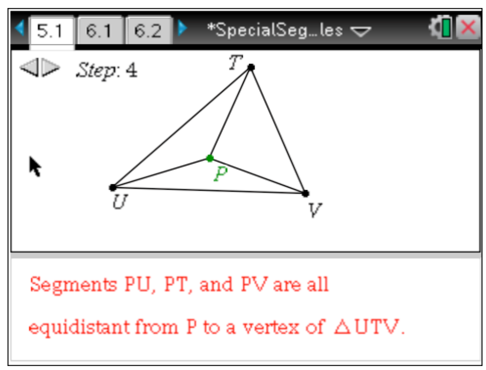


Step 2: Students are given the three perpendicular bisectors of $\triangle UTV$.

Step 3: Students are given the point of concurrency for perpendicular bisectors, the circumcenter.

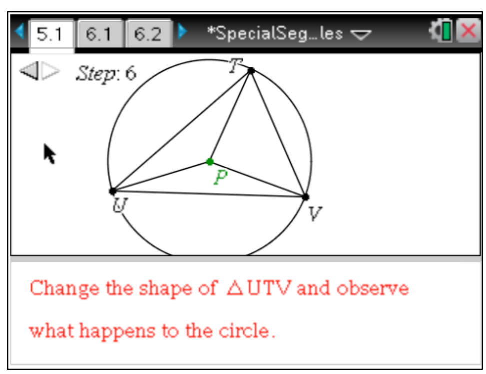


Step 4: Students are given $\overline{PU}$, $\overline{PT}$, and $\overline{PV}$ which are segments that are from the circumcenter to the vertices of the triangle.



Step 5: Students are given the circle circumscribed about $\triangle UTV$.

Step 6: Students are given the cirumcenter and asked to drag vertices of the triangle to change the shape of $\triangle UTV$ and observe what happens to the circle.

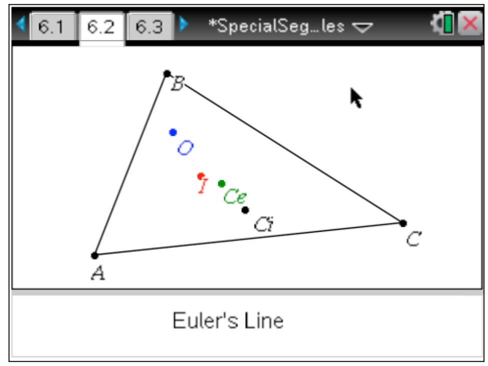


Problem 6 –Extension – Euler's Line

For ΔABC, the four points of concurrency explored in this activity are shown.

Three of the four points constructed are collinear, even as the shape of the triangle is changed. Which three points?

Students are to draw the line through these three points, known as Euler's line. Then they should drag the vertices once again to verify.



TI-Nspire Navigator Opportunities

Note 1
Problem 2, *Screen Capture*
This would be a good place to do a screen capture to verify students are clicking on the slider and moving points to investigate the properties. Screen Captures should be taken several times throughout this exercise to make sure all students are working through the activity.

Note 2
Problem 3, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask. Use Quick Poll throughout the activity to ask questions from the worksheet to check for understanding.

Sample Student Solutions

Problem 1 – An Introduction

- One stays outside and one stays inside the triangle. The outer circle contains the vertices of the triangle. The inner circle is tangent to each side of the triangle.

Problem 2 – Medians & the Centroid

- *BD* and *DC* are equidistant.
- Centroid always located inside of the triangle.
- The ratio is 2 to 1. From vertices to centroid is twice the length from centroid to midpoint of the opposite side.

Problem 3 – Altitudes & the Orthocenter

- $\angle JAK$ is always a right triangle.
- The altitude is outside of an obtuse triangle. The altitude is a side of a right triangle. The altitude is inside of an acute triangle.
- When the orthocenter is located inside the triangle? Acute Triangle
- When the orthocenter is located on the triangle? Right Triangle
- When the orthocenter is located outside the triangle? Obtuse Triangle

Problem 4 – Angle Bisectors & the Incenter

- $m\angle CAT$ and $m\angle BAT$ are always equal.
- The incenter is always located inside of the triangle.
- The lengths of $\overline{PQ}$, $\overline{PR}$, and $\overline{PS}$ are all radii of the inscribed circle. Therefore, they are all congruent.
- Always inside of the circle and tangent to the sides of the triangle.

Problem 5 – Perpendicular Bisectors & the Incenter

- The circumcenter is always inside of the triangle.
- The lengths of $\overline{PU}$, $\overline{PT}$, and $\overline{PV}$ are all radii of the inscribed circle. Therefore, they are all congruent.
- The circle is always outside of the triangle and always contains the vertices of the triangle.

Problem 6 –Extension – Euler's Line

- The orthocenter, the circumcenter, and the centroid are collinear.