Toying with a Walk

MATH NSPIRED

Math Objectives

- Students will determine the relationship between the number of "steps" a walker takes and the time it takes to make these "steps".
- Students will apply the mathematics they know to solve problems arising in everyday life, society, and the workplace (CCSS Mathematical Practice).
- Students will use functions to model relationships between quantities (CCSS Mathematical Standard).
- Students will construct a function to model a linear relationship between two quantities (CCSS Mathematical Standard).
- Students will determine the rate of change and initial value of the function from a description of a relationship, including reading these from a table or from a graph (CCSS Mathematical Standard).
- Students will interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values (CCSS Mathematical Standard).

Vocabulary

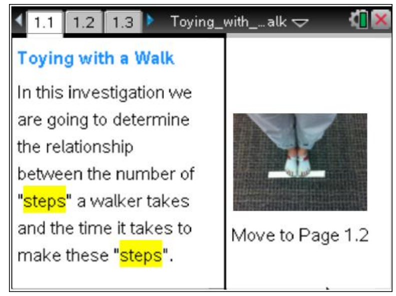
- function
- model
- *y*-intercept
- slope
- table
- mean/average
- speed
- independent variable
- dependent variable

About the Lesson

- This lesson involves collecting time data as a walker moves at a constant pace. As a result, students will:
  - Use a stopwatch built into the TI-Nspire document.
  - Model the event with a function that best fits the time vs. step plot.
  - Relate the real event of a walker moving at a constant speed to a linear function in the form of $y = mx + b$.
  - Optional: Explore how the plot of time vs. distance varies from distance vs. time.

TI-Nspire™ Navigator™

- Transfer and collect documents.
- Use Class Capture with and without Auto-Refresh to examine student work and responses and for Formative Assessment.
- Use Questions and Review Document for Assessment.



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Toying_with_a_Walk_Student.pdf

Toying_with_a_Walk_Student.doc

TI-Nspire document

Toying_with_a_Walk.tns

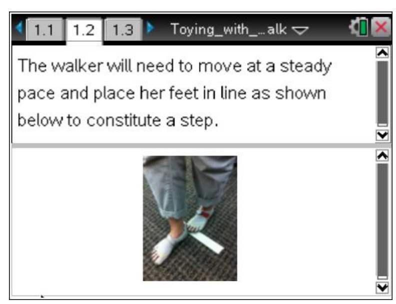
Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software
- Tape to mark the track
- Tape measure or ruler
- Optional: CBR 2™ motion sensor for use with the TI-Nspire handheld, constant-speed battery operated vehicle, or other toy instead of a person (Note: The substituted object should travel in a straight line at a constant speed that is not too fast to collect accurate time data).

Discussion Points and Possible Answers

> **Teacher Tip:** Make sure each class starts with a new version of the TI-Nspire document. If a student is having problems with the document as a result of missteps, just load a new version of the document.
>
> The spreadsheet in the document might need additional rows, or you might want to delete some depending on the length of your track or size of your class. You might want to do this before you send the TI-Nspire document.
>
> You might also want to delete or skip some of the questions depending on the time you have to spend on this topic. In particular, you could easily skip the questions about switching *time* to the independent variable and about the conversion from steps to actual distances.
>
> To save some time, you might want to provide the class with a set of data, rather than actually collecting the data. A short video of an object traveling along a straight path at a constant speed with students taking time data would serve as a quick way to show how the data was obtained.

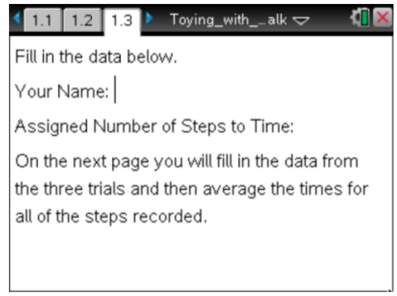
Move to page 1.2.



1. Select one walker for the class, and lay down a track as shown above (your track might be longer).
   - The starting point is at the *leading* edge of the tape (see photographs on handheld).
   - The walker steps heel-to-toe (with feet touching) for the entire walk.
   - The walker should make several test walks to determine if she can cover the track at a fairly constant rate.

2. As she makes her test walks, each student will be assigned a step position. Stand at your assigned position along the track.

Move to page 1.3.



3. Type your name and your assigned step position on this page.
   - Estimate the time it takes the walker to get to your assigned number of steps: __________.
   - How fast do you think the walker is moving? Explain how you determined this.

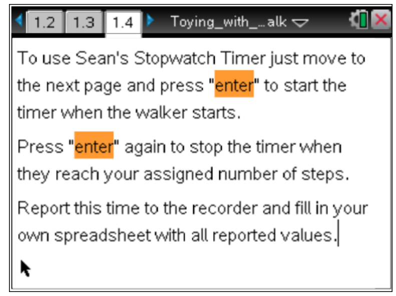
Sample Answers: I just divided the distance by the time.
The students should already see the rate of steps per second as the speed, but will come to see seconds/step as the slope of the line that fits the data. Discuss how hours per kilometer is as revealing as kilometers per hour when discussing how fast you are going, but a speed is defined as distance/time. Additionally, the distance is Step which could be converted to another unit if we knew how long a Step was. That is, how long the walker's foot is. This will be measured later.

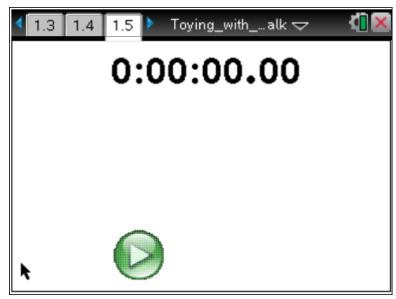
Teacher Tip: Assign individual students step numbers to time {3,6,9,12,….}. For large classes, you can place two individuals at each step, one on the left and one on the right. For classes with large students, place students on opposite sides and alternate as shown above. If you have a small class, you can use the lap-timer capability of the timer and have a student collect the times for two step amounts, say 3 and 15. You probably would want them to have the steps be both even or both odd. Another option is to divide the students into teams of three that would gather their data as a team. Since the TI-Nspire timer can record "lap times" and keeps timing, one student would be the Walker, one student would count the steps of the Walker out loud, and the third student would operate the timer and click on the blue circular arrow each time he hears the counter say three, six, nine, etc. The times are entered automatically into the spreadsheet on the following page and then could be copied to other parts of the document.
Show the students how to determine the step number they have been assigned. It might be that they count all steps or just three steps after the person that just collected data.
The class will need to explore the use of the Stopwatch timer on the TI-Nspire. Have them move to the pages dealing with the timer. This

exploration could be done in the classroom before going to the Track. Ask the students to Start and Stop to collect times that you randomly pick, as you look at your watch or clock. Ask the students to collect 7 seconds. Explore the lap-timer as needed.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

Move to pages 1.4 and 1.5.



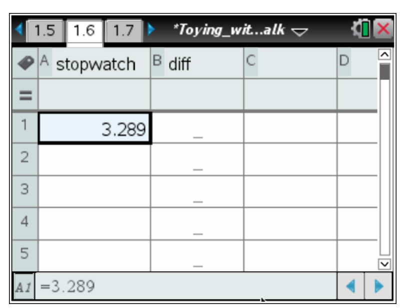


4. Read the directions on Page 1.4, and use the Sean's Stopwatch Timer on Page 1.5 to time the walker.
5. When the walker starts her walk for data collection, press enter to start your timer.
6. Just **after** she takes the step that you have been assigned, press enter again to stop the timer.

Tech Tip: To start the stopwatch, tap the green play button. To stop the stopwatch, tap the red stop button. To collect lap times, tap the blue curved arrow.

Teacher Tip: The next page will show the time in a spreadsheet. To collect lap times, have the students move the cursor to the curved arrow and click to capture a time. Repeat this until you are done timing, and then move the cursor to the red stop button to stop.

Move to page 1.6.

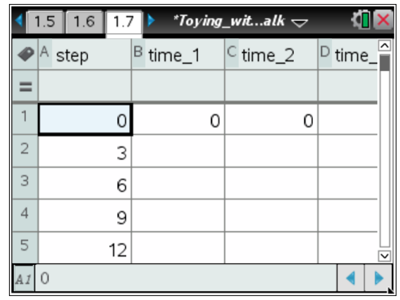


This page gives you the time from the stopwatch up to 3 decimal points. Use this precise number when you are recording your trial times. The number is replaced by a new number every time you turn the stopwatch on and off.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

Teacher Tip: If an individual student misses a time on one of the three trails, you can just have them leave this cell blank. If several students fail, you might want to go back to the earlier steps of getting folks familiar with the event or redo the whole event.

Move to page 1.7.



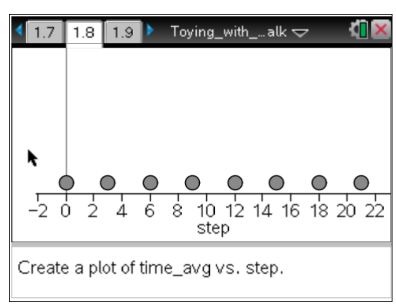
7. On this page, record the time for your step position under Column **time_1**.
8. As other students are called on, record their times for the rest of the positions in the same column.
9. Run 2 more trial walks, collect the time data in the same way, and record it under Columns **time_2** and **time_3**.
10. When you have all of your time data, move to the **time_avg** column.
11. Calculate the average (mean) of the times for the three trials and record the average in this column. If you are unsure how to calculate the mean of three numbers, ask your teacher.

Tech Tip: Have the students move to the first cell of the time-avg column and get the mean of the three time values. Then copy the formula down the column.

Teacher Tip: You might want to have the class just average two times, if it appears that there was a bad run with significantly different times from the other trials. Another technique would be to drop the high and low times and just use the remaining value. You could also run more trials to give bad data a smaller influence on the means.

TI-Nspire Navigator Opportunity: ***Collect from Class***
See Note 3 at the end of this lesson.

Move to page 1.8.



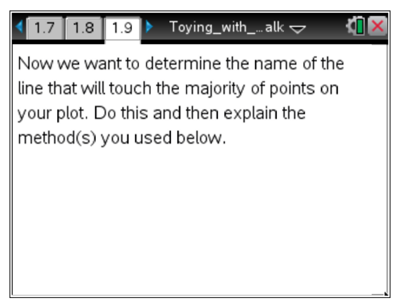
12. On this plot of your time data, select **time_avg** as the *y*-variable.
13. Examine the pattern in the plot, and discuss what you see with your peers and your teacher.

Teacher Tip: You might want to extend this discussion and collect some information from the students. Consider why we want the plot, how it represents the walk, why the *x*-values are spaced evenly, and if there are missing points and why. In addition, look at the any point, the *y*-intercept, the *x*-intercept, or the slope from a conceptual rather than a quantitative point of view.
Note that this is discrete data, but students should be able to visualize any point the walker was on the plot. For example, where was she after the 10th step?

Tech Tip: The students might move their focus to the bottom Notes screen on occasion. Show them how to switch as needed [ctrl tab]. They can always press menu to help clarify which part of the screen they are on, or in most cases move their mouse to the place of interest.
Use tab to get to the *y*-variable choice on the *y*-axis. Repeated tabbing, cycles you through the graph, the *x*-axis, and the *y*-axis.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 4 at the end of this lesson.

Move to page 1.9.



14. Determine the name of the line that will touch the majority of points on your plot. You may use any method that you wish, but you should document your work on your handheld.

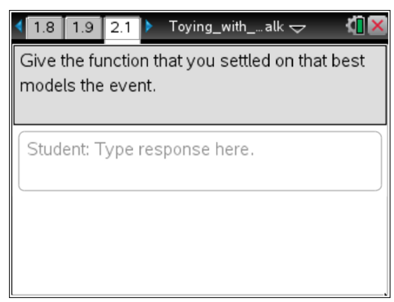
    <u>Sample Answers:</u> 1) I did Add Movable Line to the plot on the Data & Statistics page and then adjusted it until it seemed to be best overall. 2) I used the Linear Regression, and it seemed to fit well with a very good r^2 value. 3) We set up a Scatter Plot in a new Graphs page and then entered a function in *f1*(x) using the $y = mx + b$ form, making $b = 0$ and using a slope calculated from two data pairs, one from 3 steps and the other from the 15th step (rise over run). It seemed to fit well.

Teacher Tip: The students might not understand what is being asked when we call for the "name" of the line. Depending on past experiences with your students and your goals, you could better define this as the

equation of the line, or you could let each student or team decide what they would do and then lead the group to understand that the equation for the line contains all the information one would need to “name” a line.
As students perform this modeling, you might want to team them in small groups, but each should do the work on their own handheld.
Students will probably introduce new pages into the document, so be careful about referring to a particular page number. Ask the students to move to the page with the timer, or the data from the walker, etc. rather than to page 1.7 or 1.4. This is a good rule with any TI-Nspire document.
Look for a plot of the data and the display of a trend line. They will have the opportunity to name the function later.
It is not required that they use the Step as the *x*-variable, but they should, since it is the Independent variable in this data collection event. This might be a point of discussion, as things progress, and this issue is addressed later in this investigation.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 5 at the end of this lesson.

Move to page 2.1.



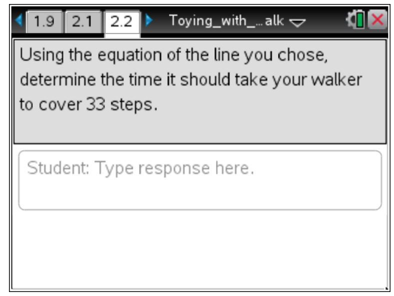
15. On this page, write the function that you settled on.

<u>**Sample Answers:**</u>
$y = 1.4x$
Time = 1.3 * Step
$f1(x) = 1.44444\,x - 0.0000567$
$y = 4/3\ x + 0$

Tech Tip: Change the Documents Toolbox in the Teacher Software and see the suggested responses for the TI-Nspire questions by selecting Document Tools.

TI-Nspire Navigator Opportunity: *Class Capture and Collect from Class*
See Note 6 at the end of this lesson.

Move to page 2.2.

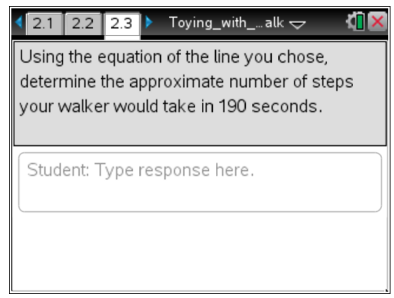


16. Using the equation of the line you chose, determine the time it should take your walker to cover 33 steps.

    <u>Sample Answer:</u> Students should insert 33 into their function as *x* and evaluate it. So for the function y = 1.4*x*, we have: Time = 1.4 * 33 = 46.2 seconds.

> **Teacher Tip:** You might want to recommend the use of the Scratchpad for these calculations or the introduction of a Calculator page. Recall that the introduction of pages will cause the numbering to change.

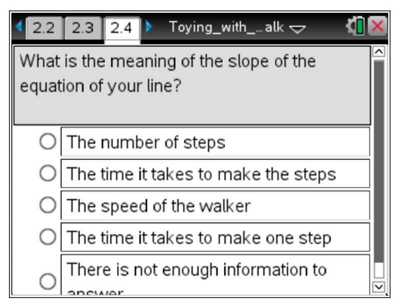
Move to page 2.3.



17. Using the equation of the line you chose, determine the approximate number of steps your walker would take in 190 seconds.

    <u>Sample Answer:</u> Students should place 190 seconds in their function for y and then solve. So as before with y = 1.4*x*, we have 190 = 1.4*x* and the *x* = Step = 190/1.4 = 135.714 which would be rounded to approximately 136 steps. Note the need to round and the variance in the data as it was demonstrated in the plot.

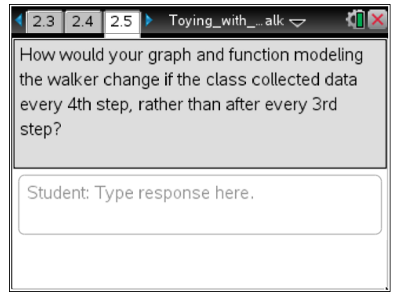
Move to page 2.4.



18. What is the meaning of the slope of the equation of your line?

    <u>Answer:</u> The time it takes to make one step.

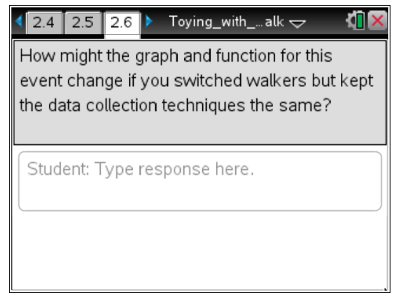
Move to page 2.5.



19. How would the graph and the function modeling the walker change if the class collected data every 4th step, rather than after every 3rd step?

    __Sample Answer:__ The individual points plotted would land every 4th step (4,8,12,..) rather than every 3rd, but the line and equation would look the same if the walker kept the same pace.

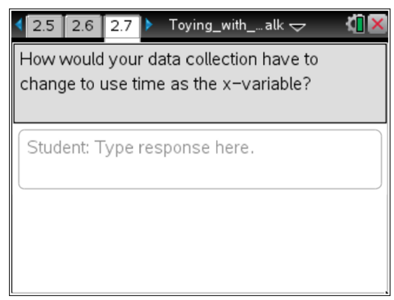
Move to page 2.6.



20. How might the graph and function for this event change if you switched walkers but kept the data collection techniques the same?

    __Sample Answer:__ If the walker had a different pace, the slope of the plot and function would change.

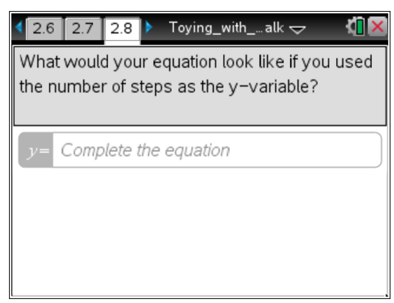
Move to page 2.7.



21. How would your data collection have to change to use time as the *x*-variable?

    __Sample Answer:__ Collect the number of steps taken at set times such as 5, 10, 15 seconds and report fractions of a step, probably to the nearest half-step.

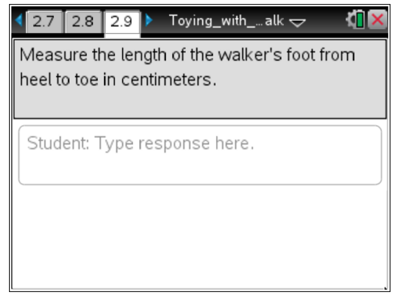
Move to page 2.8.



22. What would your equation look like if you used the number of steps as the *y*-variable?

    __Sample Answers:__ This will just be the inverse of the original function. Using the example from before, $y = 1.4x$ becomes time = 1.4 * step and then time/1.4 = step, or $y = 5/7 * x$ or $y = 0.7x$. The equation would look like you took the reciprocal of the slope so that 14/10 now becomes 10/14.

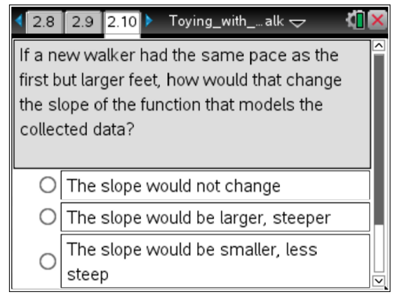
Move to page 2.9.



23. Measure the length of the walker's foot from heel to toe in centimeters.

    Sample Answer: 30 cm.

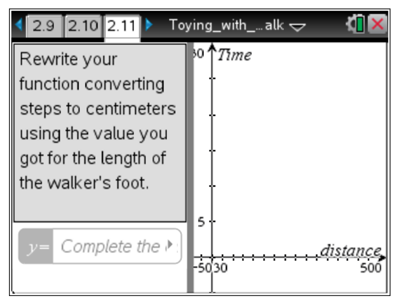
Move to page 2.10.



24. If a new walker had the same pace as the first but larger feet, how would that change the slope of the function that models the collected data?

    Sample Answer: The slope would not change.

Move to page 2.11.



25. Rewrite your function converting steps to centimeters using the value you got for the length of the walker's foot.

    Sample Answers: If your foot was 30 cm long, each step would represent 30 cm. The slope in the function *y* = 1.4*x* has units of seconds per step. We need a slope in seconds per cm. Since 1 step is 30 cm, we get

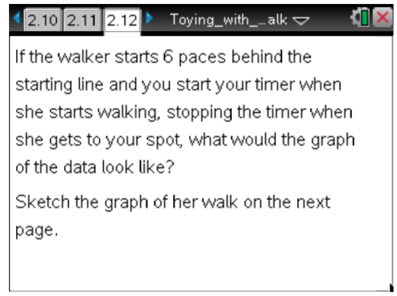
    $\frac{14*\sec}{10*step} * \frac{1*step}{30*cm} = \frac{7*\sec}{150*cm}$ which gives

    Time = 0.047 * Distance or y = 0.047*x*.

Teacher Tip: Consider focusing more or less on this item since the determination of this new function requires additional skills beyond the topic at hand. Point out that the data that we collected uses different "rulers" if the size of the each walker's feet is different. That is, the word 'step' could be changed to yard, or decimeter, or kilometer, so the rate would be seconds per yard, or seconds per meter. We have just created a new unit of length called the 'step' (or better yet, Sally's Foot).
Note that if we changed walkers and used the actual distances, two walkers with the same pace but different foot lengths would create different slopes with the function that models the individual walks.

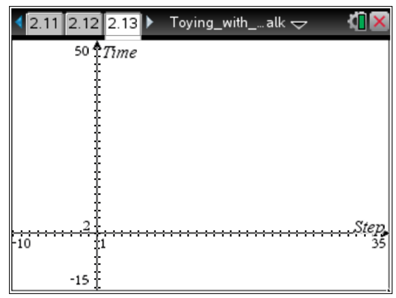
Ask the students how the slope would change if the feet were smaller/larger.

Move to page 2.12.



26. If the walker starts 6 paces behind the starting line and you start your time when she starts walking, stopping the timer when she gets to your spot, what would the graph of the data look like? Sketch the graph of her walk on Page 1.13.

    <u>Sample Answer:</u> See figure to the right.



Teacher Tip: Suggest the use of the Segment tool under Points & Lines in the Menu.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to collect data and report it in the form of a table, plot, and function that models the pattern.
- The relationship between the slope of a linear function and the event that the function models.
- How to determine the rate of change and initial value of a function from a description of a linear-type situation or from data collected from an actual event that has a linear relationship between two variables.

Assessment

1. What kind of plot would you get if the walker did not move at a steady pace? What choices are there for a walk (speed up, slow down, alternate, stand still)?
2. Is it possible to have a vertical plot (with distance on the y-axis and time on the x-axis)?
3. Is it possible to have a horizontal plot (with distance on the y-axis and time on the x-axis)?
4. Can you have a negative distance?
5. What is the difference between total distance traveled and displacement?

TI-Nspire Navigator

Note 1

Question 3, Class Capture

Use Class Capture to determine how well the class is doing in these tests of the timer usage.

Note 2

Class Capture

If you are in class for this investigation, you can collect screenshots of the individual times and have the students direct you to arrange the times in order. Using the Zoom feature and Print Screen button, copy and paste the Class Captures into Word document (or another appropriate application) to compare with additional trials or save for later use.

Note 3

Question 11, Collect from Class

Collect data from this page and send it to the class as another document that they can copy and paste onto the spreadsheet in their own document. This can be done to get a quick average, or as a demonstration for the class on computing the average. You could also send the completed spreadsheet to individual students who have missed the data or failed to key it in for some reason.

Note 4

Question 13, Class Capture

Collect screenshots of the students' plots to help identify those who got the data wrong or who have been unable to make the plot.

Note 5

Question 14, Class Capture

Collect screenshots of the students periodically to show others how their peers are solving this problem. Use the Auto-Refresh option.

Note 6

Question 15, Class Capture and Collect from Class

Formatively assess students' understanding by collecting screenshots along the way, and provide assistance as necessary.

After the students have had an opportunity to answer all of the questions, collect the document and report the answers to the class in the Review Workspace.