Damped and Driven Harmonic Motion – ID: 9523

Time required
45 minutes

Activity Overview

In this activity, students explore the properties of waveforms representing damped and driven simple harmonic motion. First, they identify the functional form of the damping in a simple harmonic oscillator. Then, they discover the relationship between the driving frequency, the fundamental frequency, and the damping coefficient for a driven simple harmonic oscillator.

Concepts

- *Simple harmonic motion*
- *Damped harmonic motion*
- *Driven harmonic motion*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Lists & Spreadsheet

Teacher Preparation

Before carrying out this activity, review the characteristics of simple harmonic motion and the characteristics of sine and exponential curves. Consider doing a live demonstration of a damped and driven harmonic oscillator.

- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7 and 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9523" in the search box.***

Classroom Management

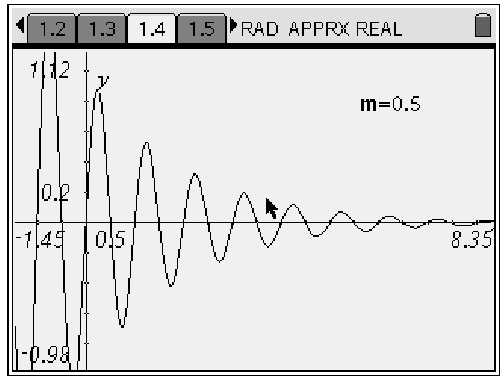
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

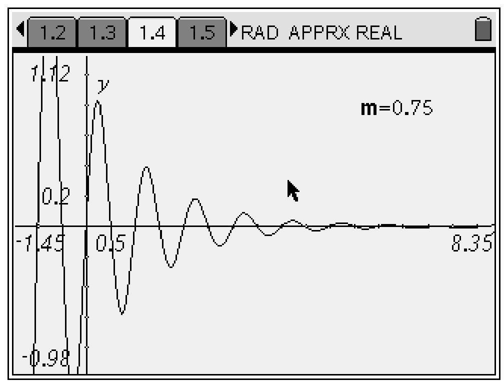
The following questions will guide student exploration in this activity:

- What function describes the motion of a damped simple harmonic oscillator?
- What is the relationship between fundamental frequency, driving frequency, and amplitude for a driven, damped simple harmonic oscillator?

Problem 1 – Damped simple harmonic motion

Step 1: Students should open the file **PhyAct25_DampedSHM_EN.tns** and read the first three pages. Page 1.4 shows a graph of displacement vs. time for a damped simple harmonic oscillator. The variable **m** represents the damping coefficient for the oscillator. Students should vary **m** and observe the effects on the motion of the oscillator. Then, they should answer questions 1 and 2.





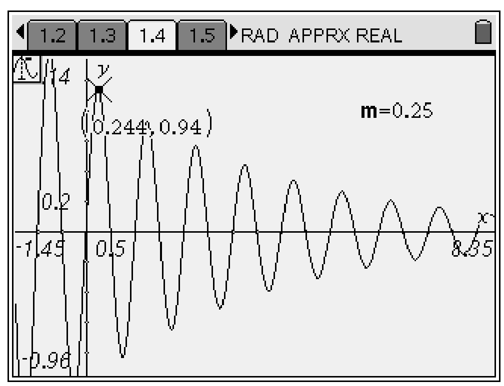
Q1. How does the value of **m** affect the shape of the curve?

A. *The more negative **m** becomes, the more quickly the motion of the oscillator is damped.*

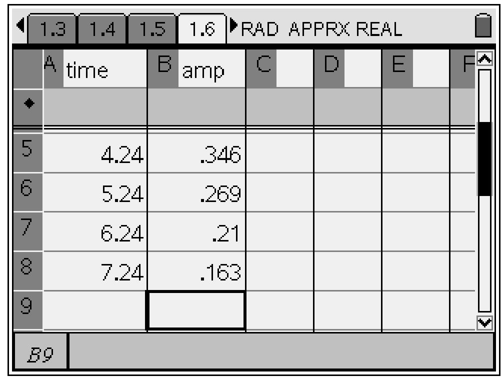
Q2. Imagine a line connecting the peaks of the curve. What form would that line have?

A. *Students may recognize that the peaks of the curve decrease along approximately an exponential curve. If they do not, you may wish to guide them to this observation.*

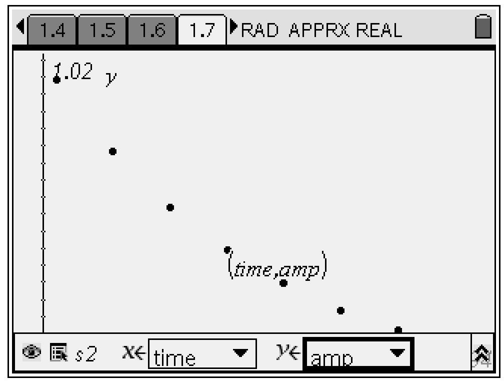
Step 2: Next, students should use the **Graph Trace** tool (**Menu > Trace > Graph Trace**) to identify the coordinates of successive peaks of the curve. After selecting the **Graph Trace** tool, students should move the cursor to the leftmost peak of the curve. The TI-Nspire will display a capital *M* when the cursor is on the peak of the curve. Students should record the *x*- and *y*-coordinates of this peak. They should then move the cursor to the next peak to the right and record its coordinates. They should continue in this way until they have recorded the coordinates of all the peaks that are visible on screen.



Step 3: Next, students should move to the *Lists & Spreadsheet* application on page 1.6. They should enter the *x*- and *y*-coordinates they recorded for the curve peaks into the spreadsheet. They should enter the *x*-values into the **time** column and the *y*-values into the **amp** column.



Step 4: Next, students should make a scatter plot of **amp** vs. **time** on page 1.7. Students should discuss the shape of the curve and attempt to identify its functional form. Encourage students to change the plot to a **Function** plot and plot various functions of *x* in an attempt to fit the data points. Note: When students graph e^x, make sure that they use the ln e^x key or type **exp(–m·x)**, and not $\mathbf{e}^{-m \cdot x}$ where **e** is the (E) key, into the function bar. Then, students should answer questions 3–5.

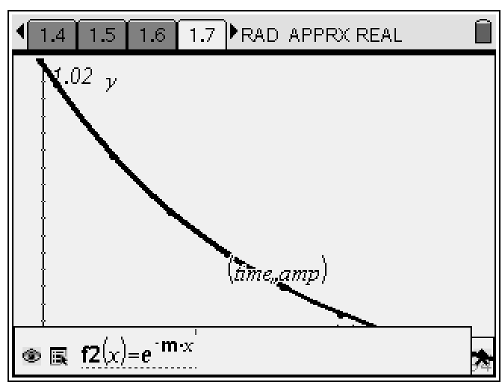


Q3. What function best fits the graph of amplitude vs. time?

A. *Students should realize that the curve is exponential with a negative exponent (i.e., of the form $y = e^{-x}$). You may need to provide guidance to the students to help them realize this.*

Q4. Was your prediction in question 2 correct? If not, explain any errors in your reasoning.

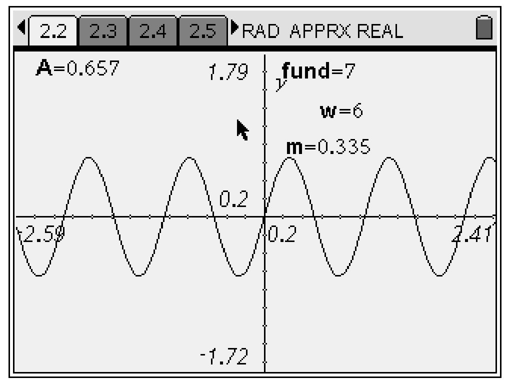
A. *Student answers will vary. Encourage discussion of why the students predicted the shapes they did.*



Q5. Predict the form of the equation that describes damped simple harmonic motion. Explain your answer.

A. *The equation will have the form* $y(x) = (e^{-mx}) \sin x$ *. Remind students that undamped simple harmonic motion is described by functions of the form y(x) = A sin x, where* A *is the amplitude of the motion. In this case, the amplitude of the curve decreases according to an exponential curve (*e^{-x}*).*

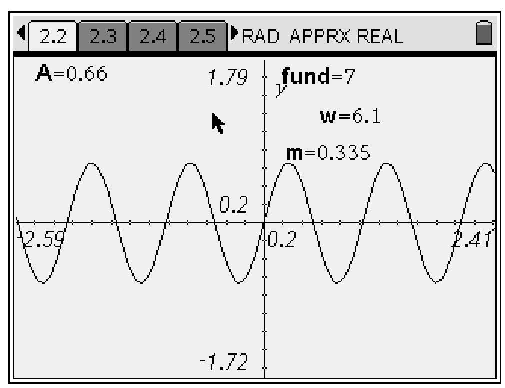
Problem 2 – Driven, damped simple harmonic motion



Step 1: Next, students should read page 2.1. The graph on page 2.2 shows displacement vs. time for a spring that is driven by a motor. The spring's fundamental frequency is represented by the variable **fund,** and the driving frequency of the motor is represented by the variable **w**. The amplitude of the spring's displacement is represented by the variable **A**. Before altering the graph on page 2.2, students should answer question 6.

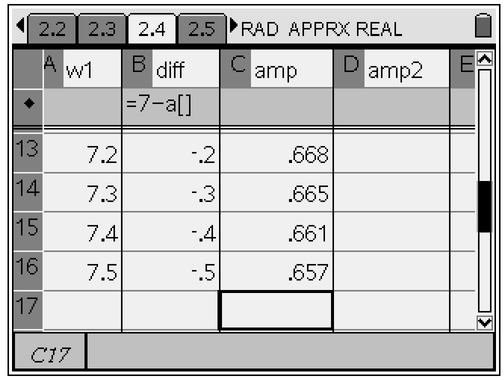
Q6. Predict the value of **w** that will produce the largest amplitude (**A**) for the spring, assuming that **fund** is fixed at 7.

A. *Student answers will vary.*

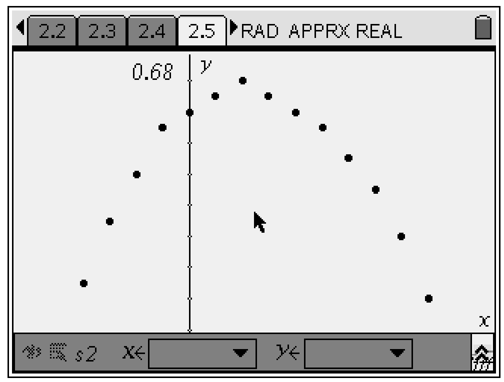


Step 2: Next, students should increase the value of **w** incrementally (increments of 0.1 will yield the best results). They should record the amplitude that each value of **w** yields. Students should increase the value of **w** from 6 to approximately 7.5.

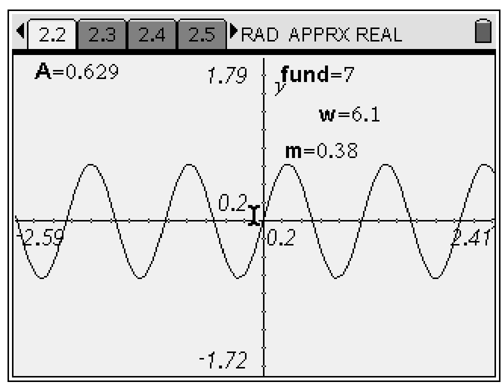
Step 3: Next, students should move to the *Lists & Spreadsheet* application on page 2.4. In column A (variable **w1**), they should enter the incremental values of **w** that they recorded in step 2. In column B (variable **diff**), they should use a formula to calculate the difference between **fund** and the values of **w** they entered (i.e., **diff** = **fund** – **w**). To do this, students should type **=7-a[]** into the formula bar of column B. In column C (variable **amp**), they should enter the amplitude of the motion associated with each frequency difference.



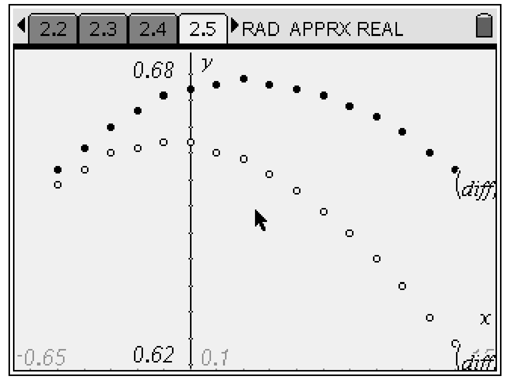
Step 4: Next, students should create a scatter plot of **amp** vs. **diff** on page 2.5. They should resize the graph (**Menu > Window > Zoom - Data**).



Step 5: Next, students should move back to page 2.2 and change the value of **m,** the damping coefficient, to 0.38. They should then reset **w** to 6 and repeat step 2, using the same values of **w** that they used in step 2. Then, they should move to page 2.4 and enter the amplitudes they recorded for each value of **w** into column D (variable **amp2**).



Step 6: Next, students should move to page 2.5 and plot **amp2** vs. **diff** on the same scatter plot with the graph of **amp** vs. **diff**. They should compare the two plots and then answer questions 7–9.



Q7. What value of **diff** produced the maximum value of **A** in each case?

A. *For **m** = 0.335, the maximum amplitude occurs when **w** is approximately 6.8. For **m** = 0.36, the maximum amplitude occurs when **w** is between 6.9 and 7.*

Q8. Was the prediction you made in question 6 correct? If not, explain any errors in your reasoning.

A. *Student answers will vary. Most students will probably have predicted that the maximum amplitude would occur at **fund** = **w**. The graph will show that the maximum amplitude actually occurs when **w** is slightly less than **fund**. Encourage students to discuss why they made the predictions they made.*

Q9. What do you think causes the maximum amplitude to occur when **fund** and **w** are not equal?

A. *Student answers will vary. The maximum amplitude occurs when **fund** and **w** are not equal because of the effects of damping on the oscillator. If the oscillator were not damped, the amplitude would approach infinity when **fund** = **w**.*

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(Student)TI-Nspire File: *PhyAct25_DampedSHM_EN.tns*

1.1

DAMPED AND DRIVEN HARMONIC MOTION

Physics

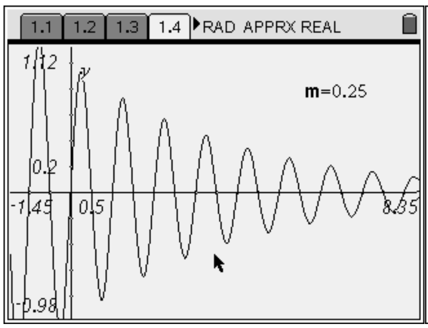
Simple Harmonic Motion

1.2

The displacement of an undamped simple harmonic oscillator can be modeled by a simple sine curve. However, truly undamped simple harmonic motion is impossible in a world with friction. If no additional energy is added to the oscillator, it will eventually stop moving.

1.3

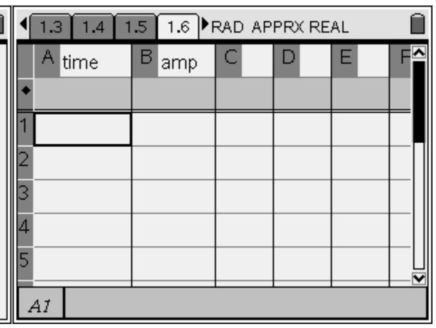
The graph on the next page shows the displacement of a spring as a function of time. Notice that the displacement decreases with time. The variable **m** represents the damping coefficient. Vary the value of **m** and observe the effects on the curve. **NOTE:** For this simulation, **m** must be a positive number.

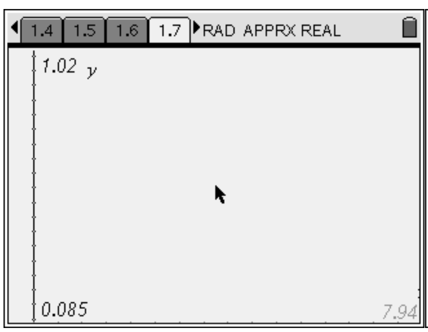


1.5

1. How does the value of **m** affect the shape of the curve?

2. Imagine a line connecting the peaks of the curve. What form would that line have?





1.8

3. What function best fits the graph of amplitude vs. time?

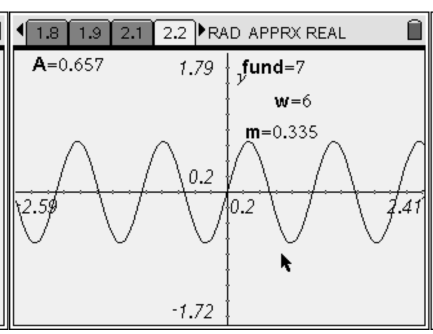
4. Was your prediction in question 2 correct? If not, explain any errors in your reasoning.

1.9

5. Predict the form of the equation that describes damped simple harmonic motion. Explain your answer.

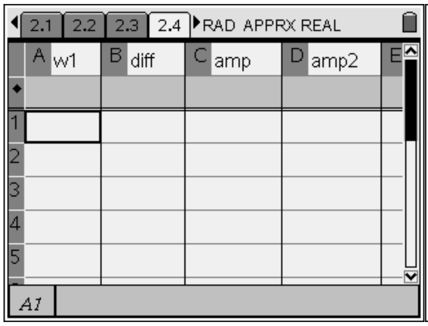
2.1

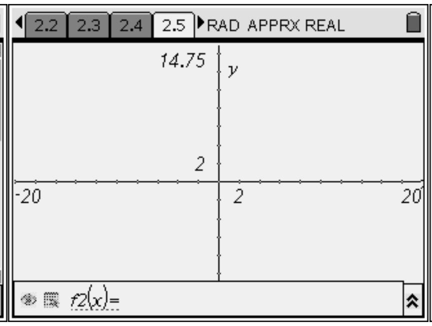
The graph on the next page shows the displacement of a spring that is driven by a motor. The motor drives the spring with frequency **w**. The spring's fundamental frequency is **fund**, the damping coefficient is **m**, and the amplitude of the curve is **A**.

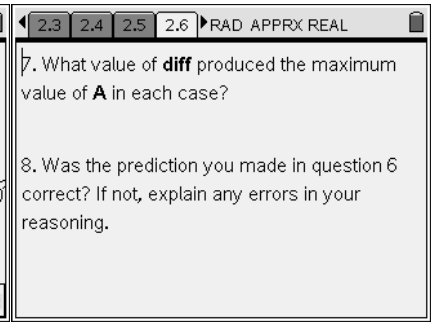


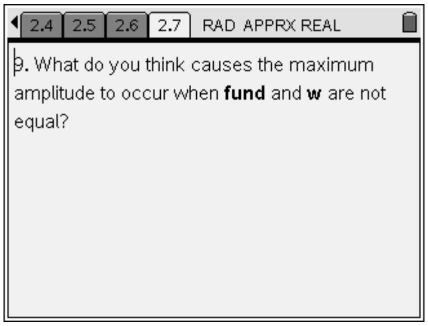
2.3

6. Predict the value of **w** that will produce the largest amplitude (**A**) for the spring, assuming that **fund** is fixed at 7.


2.1 2.2 2.3 2.4 RAD APPRX REAL
A w1
B diff
C amp
D amp2
E
A1


2.2 2.3 2.4 2.5 RAD APPRX REAL
14.75
y
2
-20
2
20
f2(x)=


2.3 2.4 2.5 2.6 RAD APPRX REAL
7. What value of **diff** produced the maximum value of **A** in each case?
8. Was the prediction you made in question 6 correct? If not, explain any errors in your reasoning.


2.4 2.5 2.6 2.7 RAD APPRX REAL
9. What do you think causes the maximum amplitude to occur when **fund** and **w** are not equal?