NUMB3RS Activity: Maintaining Balance
Episode: “Blackout”

Topic: Center of Mass, Harmonic Series **Grade Level:** 9 - 12
Objective: Students will calculate the center of mass when stacking blocks and find the maximum sum of overhang lengths when building a tower.
Time: 25 - 30 minutes
Materials: A ruler and 8 identical rectangular prisms (meter sticks work best; a deck of cards, mathematics books, or CD cases will also work). The game of JENGA® is optional. Also, a TI-83 Plus/TI-84 Plus graphing calculator is needed for the “Extensions.”

Introduction

In “Blackout,” there is a series of attacks on the substations that provide electricity to the city of Los Angeles. The network of substations in a city is set up so that, if one substation is knocked out, the other substations provide more electricity to make up for the missing service. Your house or your school is served by several substations. If one substation is taken out of service, you will still have electricity because another substation makes up for the loss. If several substations are taken out, you may lose service if the remaining substations cannot meet the increased demand for electricity.

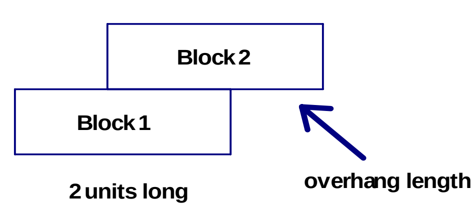
Charlie explains this type of cascading failure using the game of JENGA. He says, “There’s enough redundancy that many of the blocks can be removed or knocked out, without a great affect to the system as a whole...but if the wrong block, or in this case substation, is taken out, especially after the network’s been weakened...then the entire system could come crashing down.” In this activity your students will work with the blocks in the game of JENGA to determine how to avoid bringing down the network.

Discuss with Students

The first two questions involve the game itself. Students can answer them using the actual game or the virtual version. There are many sites that have online versions of the game that can be played for free. Question 1 assumes that, when students remove the wooden blocks, they are not knocking the pieces over on purpose. The virtual version introduces the rules of the game, and the removing of pieces does not seem to be affected by a shaky hand. The game pieces are not needed for questions 3 to 7, and can be answered if questions 1 and 2 are skipped.

Please test stacking the blocks before you ask your student to do questions 3 to 7. Meter sticks are the best blocks to use if they are all the same and have uniform density. They work well because it is easy to measure the overhang lengths. The block lengths are shown to have a length of 2 units, which is to make the pattern of the overhang lengths easier to see. That is, the **harmonic series** is easier to see when the unit on the bottom block is 2 units rather than 1 unit.

Question 3 asks the students to find the maximum overhang length for Block 2 before it tips over. Theoretically this amount should be 1 unit. Practically, this amount will be slightly less than 1 unit depending on the type of block the students are using. The physical blocks may act differently than the theoretical calculations due to some physical variables not accounted for in the model. These variables will make the theoretical overhang lengths greater than is physically possible. Be sure to practice with various blocks before trying this with your students.



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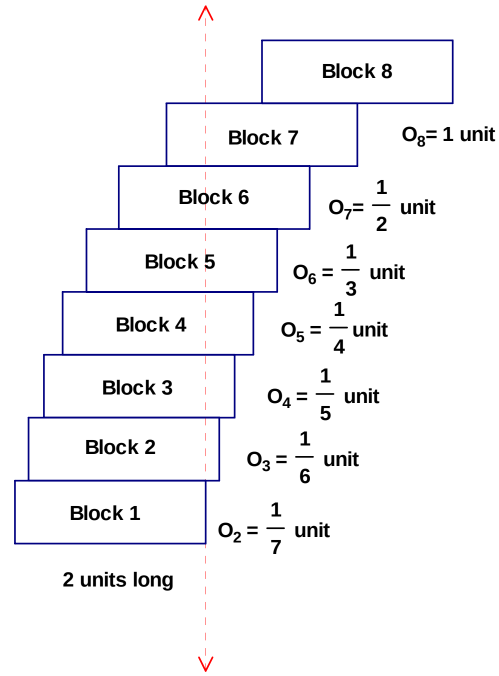
Stacking the blocks as asked in question 4 is difficult to do, however, with practice you can get Block 8 to extend past the edge of Block 1. The picture on the right is drawn proportionally to the theoretical measurements. Each overhang length will be slightly shorter when you do this with a physical model. Remember that the length of each block is 2 units.



Calculating the center of mass for the tower is done using a reference line. The reference line is the dashed line that passes through the right side of Block 1. The tower will not tip if the center of mass is on or to the left of the dashed line. Label the reference line $x = 0$ so that if the center of mass is negative or 0 then the tower will not tip. The center of mass calculation is done by summing the values that each block on top of Block 1 contributes to the tower. If the overhang length for Block 2 is $\frac{1}{7}$ the center of mass for Block 2 located at $-\frac{6}{7}$. The center of mass calculation for the eight block tower is shown below:

$$\begin{aligned}\text{Center of Mass} &= \left(\tfrac{1}{7}-1\right)+\left(\tfrac{1}{7}+\tfrac{1}{6}-1\right)+\left(\tfrac{1}{7}+\tfrac{1}{6}+\tfrac{1}{5}-1\right)+\left(\tfrac{1}{7}+\tfrac{1}{6}+\tfrac{1}{5}+\tfrac{1}{4}-1\right)+\left(\tfrac{1}{7}+\tfrac{1}{6}+\tfrac{1}{5}+\tfrac{1}{4}+\tfrac{1}{3}-1\right)+\\ &\quad\left(\tfrac{1}{7}+\tfrac{1}{6}+\tfrac{1}{5}+\tfrac{1}{4}+\tfrac{1}{3}+\tfrac{1}{2}-1\right)+\left(\tfrac{1}{7}+\tfrac{1}{6}+\tfrac{1}{5}+\tfrac{1}{4}+\tfrac{1}{3}+\tfrac{1}{2}+1-1\right)\\ &= 0\end{aligned}$$

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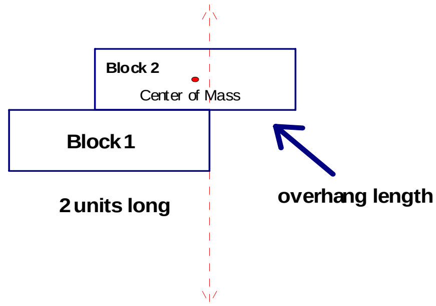
Student Page Answers:

1. *There is enough redundancy in the tower so that removing any one block will not make the tower crash. It is possible to make the tower crash by removing two blocks. These two blocks need to be removed so that the center of mass at each level is not supported by the blocks on the level below.*
2. *The tallest tower on record, according to* ***http://www.hasbro.com/JENGA/pl/page.records/dn/default.cfm,*** *is* $40\frac{2}{3}$ *levels high.* **3.** *The longest overhang length is a little less than 1 unit. Theoretically, Block 2 can be slid over until the center of mass is directly over the edge of Block 1 (red dotted line).*



4. *It is possible. See solution for question 5.* **5a.** *The maximum value for* O_2 *is a little less than* $\frac{1}{2}$ *unit.* ***5b.*** $O_2 = \frac{1}{2}$ *will give a center of mass at x = 0. Center of mass =* $-\frac{1}{2}+\frac{1}{2}$. **6.** *Theoretically the center of mass for the top three blocks is 0 when* $O_2 = \frac{1}{3}$ *units. Center of mass =* $-\frac{2}{3}+\left(-\frac{1}{6}\right)+\frac{5}{6}$. ***7.*** *The maximum sum of the overhangs* ($O_2 + O_3 + O_4 + O_5 + O_6 + O_7 + O_8$) *is* $\left(\frac{1}{7}+\frac{1}{6}+\frac{1}{5}+\frac{1}{4}+\frac{1}{3}+\frac{1}{2}+1\right)$ *units. The center of mass will be at x = 0.*

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Name: ________________________________ Date: _______________

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Playing JENGA

There are 54 wooden pieces in the game of JENGA. All of the pieces are congruent rectangular prisms. The game is played by first stacking the pieces in alternating levels of 3. The starting tower is 18 levels tall. The players take turns removing a block from the tower and placing that removed block on the top of the tower. The person who causes the tower to collapse is the loser. The last player who completed a turn before the tower collapses is the winner.

You may play JENGA using the actual game or you can go to a Web site to play a virtual game. The virtual game may use a different number of blocks.

1. What is the fewest number of removed pieces that could cause the tower to crash? Explain your choice.

2. What do you think is the tallest tower you can make using the 54-piece set?

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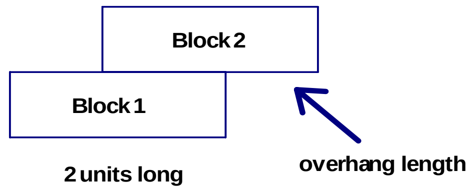
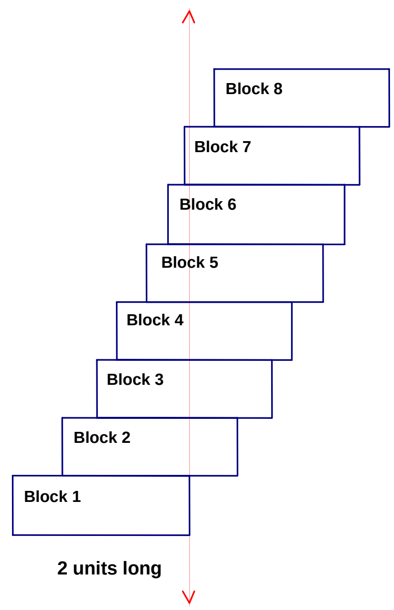
Stacking Blocks

You may have found that the tower stays supported if the center of mass at each level is supported by the levels below. To explore the center of mass, you may use any rectangular prisms as long as they are all identical in size and have uniform density. Meter sticks work best, because you can see the measurements on the sticks. You can also use JENGA pieces, decks of cards, some mathematics textbooks, or CD or DVD cases.

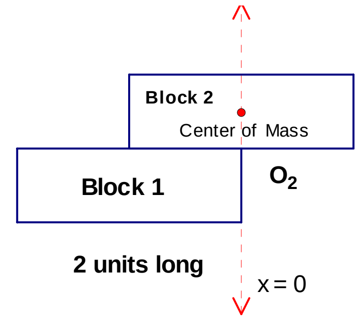
3. Take two blocks and stack them as shown in the side-view picture on the right. Measure the length of the base and label this as 2 units. (For example, if the length of your block is 24 cm, then use 12 cm as your unit.) Push Block 2 over as far as you can before the tower crashes. Determine the longest overhang length (measure in units) possible before the tower crashes.



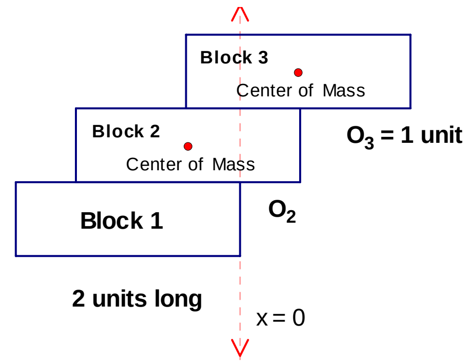
4. The tower shown at right might tip. Notice that no portion of Block 8 is over any part of Block 1 (red line). Is it possible to construct such a tower? Give a reason for your answer.



In Question 3, you found that the greatest **overhang length** for Block 2 was O_2 = 1 unit. This happens because the center of mass is located above the edge of Block 1. Theoretically, Block 2 should not tip over if the center of mass is over Block 1. When you tried this, however, you may have found that the overhang length needs to be just slightly less than 1 unit.



5. a. Use three blocks. Find the greatest experimental value for O_2 if O_3 = 1 unit. (You will probably need to make O_3 just slightly less than 1 unit long to make this work with your blocks.)



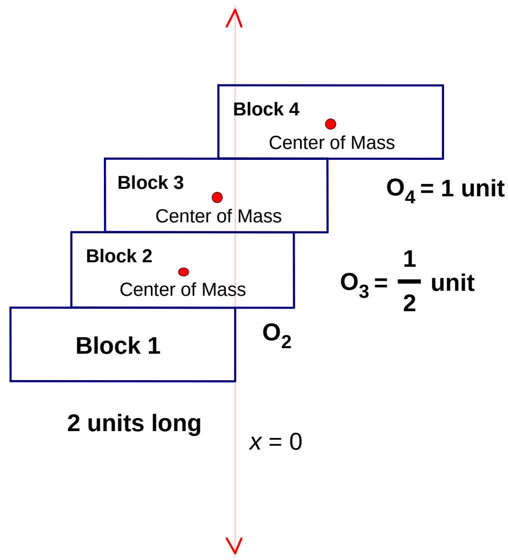
To calculate the theoretical center of mass for Blocks 2 and 3 combined, suppose O_3 = 1 unit and $O_2 = \frac{1}{3}$ unit. The center of mass of Block 3 is at $\frac{1}{3}$ from the reference line. The center of mass of Block 2 is at $-\frac{2}{3}$. The center of mass for both Blocks 2 and 3 combined will be:

$$\textbf{Center of Mass} = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$

If the center of mass is less than or equal to 0, then the blocks will balance on Block 1.

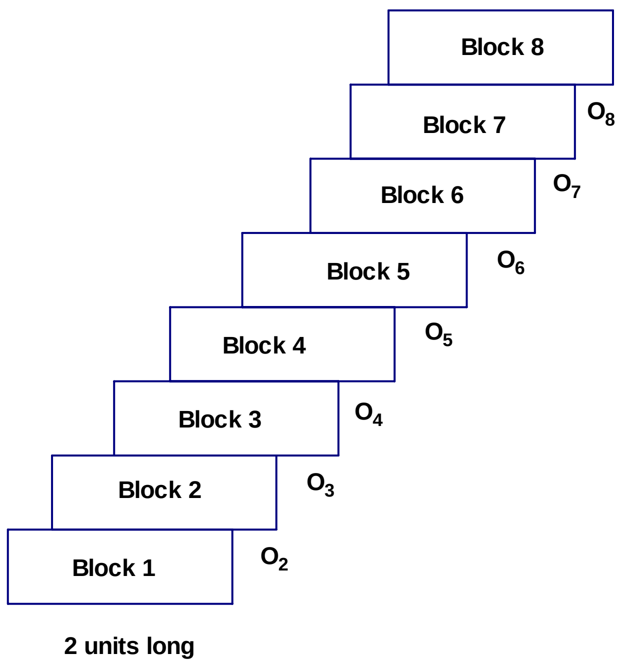
b. Find the value of O_2 so that the combined center of mass will equal 0. This means that the center of mass of Blocks 2 and 3 together will be at the edge of Block 1. Use O_3 = 1 unit.

6. Find the theoretical value of O_2 that would put the center of mass of Blocks 2, 3, and 4 at $x = 0$ if $O_4 = 1$ unit and $O_3 = \frac{1}{2}$ unit.



7. Create a tower using 8 blocks that maximizes the sum of the overhang lengths (that is, maximize the sum $O_2 + O_3 + O_4 + O_5 + O_6 + O_7 + O_8$). If actual blocks were used, the figure shown at the right would probably tip over. Theoretically, what is the maximum overhang sum?

Use your results to make such a tower with 8 blocks. Remember that your calculations are theoretical and that when you construct this arrangement with actual blocks, you will have to shorten each overhang a little bit to overcome some physical limitations.



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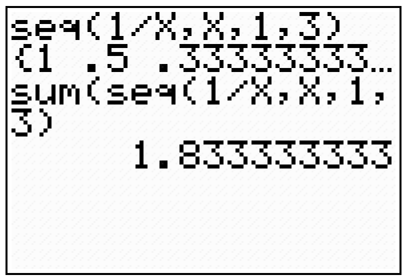
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Challenge Problems

- Find the maximum sum of the overhang lengths if twenty CD cases are stacked using the harmonic series for the overhang lengths. CD cases are approximately 14 cm long.

- Determine the number of CD cases that are needed to make a tower that has a sum of overhang lengths equal to 100 cm. The TI-84 Plus can be used to find the sum of sequences as shown in the window below. To find the **seq(** and **sum(** commands, press [2nd] [LIST]; the command **seq(** is listed under the **OPS** menu, and the command **sum(** is listed under the **MATH** menu. Determine the height of the tower. A CD case is about 1 cm tall.



JENGA®

- For a mathematical description of JENGA moves, go to:
http://www.maths.tcd.ie/~icecube/maths/a-JENGA-probability-distribution/

- For some fun facts about JENGA, go to:
http://www.hasbro.com/JENGA/pl/page.fun/dn/default.cfm

Harmonic Series

- For more information about the harmonic series, go to:
http://mathworld.wolfram.com/HarmonicSeries.html

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