Optimization

ID: 9609

Time required
45 minutes

Activity Overview

Students will learn how to use the second derivative test to find maxima and minima in word problems and solve optimization problems in parametric functions.

Topic: Application of Derivatives

- *Find the maximum or minimum value of a function in an optimization problem by finding its critical points and applying the second derivative test. Use* ***Solve*** *(in the Algebra menu) to check the solution to* $f'(x) = 0$.
- *Use the command* ***fMin*** *or* ***fMax*** *to verify a manually computed extremum.*
- *Solve optimization problems involving parametric functions.*

Teacher Preparation and Notes

- *This investigation uses* ***FMax*** *and* ***fMin*** *to answer questions. Students will have to take derivatives and solve on their own.*
- *Before starting this activity, students should go to the home screen and select* ***F6:Clean Up > 2:NewProb****, then press* ß *. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- *This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9609" in the quick search box.***

Associated Materials

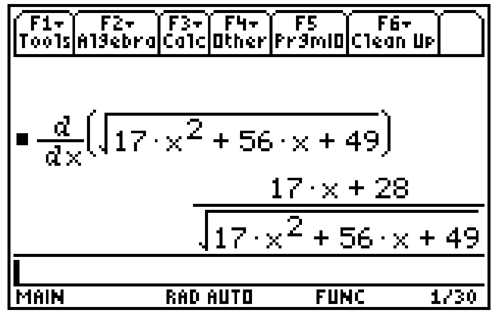
- *CalcWeek33_Optimization_Worksheet_TI89.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange *and enter the number in the quick search box.*

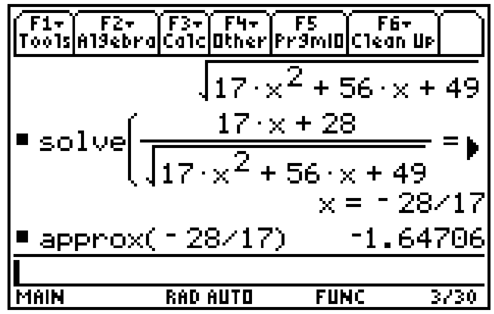
- *Application of the Derivative (TI-89 Titanium) — 4275*
- *Find Optimization Points with Derivatives (TI-89 Titanium) — 3239*
- *Where Should it Go? Optimization Exercise (TI-Nspire CAS technology) — 10204*
- *Maximizing Area (TI-Nspire technology) —10043*
- *Design a Better Drink Can (TI-Nspire technology) — 8272*

Problem 1 – Optimization of distance and area

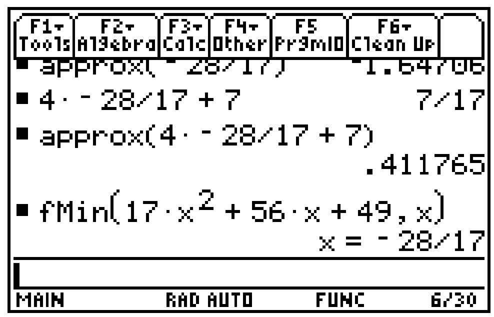


Students will graph the equation $y = 4x + 7$. They need to minimize the function $s = \sqrt{x^2 + y^2}$ where x and y are the coordinates of a point on the line. The constraint is the equation of the line.

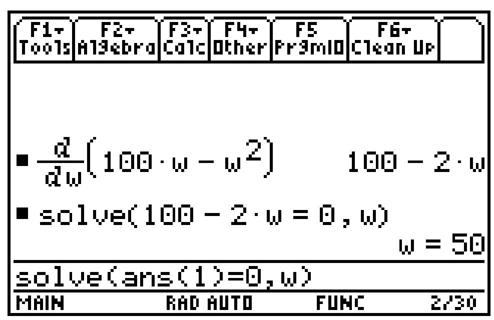
They are to rewrite the function using one variable: $s = \sqrt{x^2 + (4x+7)^2} = \sqrt{17x^2 + 56x + 49}$.



To find the exact coordinates of the point, students will take the first derivative **(Menu > Calculus > Derivative)**, and solve for the critical value **(Menu > Algebra > Solve)**, and take the second derivative. Since the second derivative is always positive, there a minimum at the critical value of $x = -\dfrac{28}{17}$.



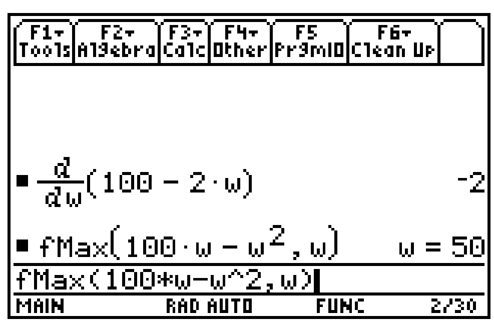
To find the y-coordinate, students should substitute the value of x into the original equation $y = 4x + 7$. To find the distance, they should substitute the x- and y-values into the function $s = \sqrt{x^2 + y^2}$. The point is (–1.647, 0.412) and the distance is 1.698 units.



Students are to maximize the function $A = l \cdot w$. The constraint is $2l + 2w = 200$. Since $l = 100 - w$ students can rewrite the function as $A = (100 - w)w = 100w - w^2$.

Students will take the first derivative and solve to find the critical value is $w = 50$. The second derivative is always negative so we have a maximum. When $w = 50$ m, then $l = 50$ m.

The maximum area is 2500 m^2.

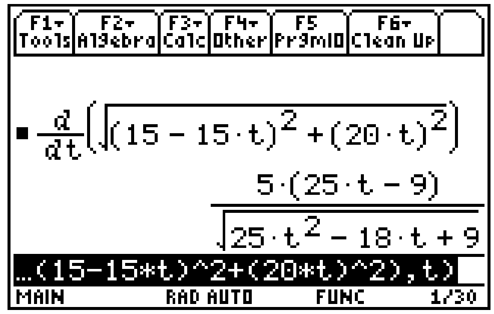


Problem 2 – Optimization of time derivative problems

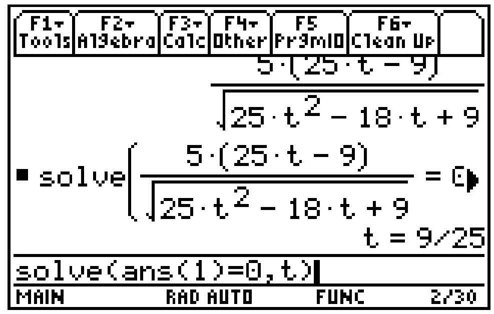
Remind students to use *t*, for time, instead of *x*. The position equations are the constraints.

The boat heading north is going from the right angle to the point northward. Its position equation is $y = 20t$.

The boat heading west is going to the right angle. At 1 pm, it is one hour from the arrival time 2 pm so it is 15 km away. Its position equation is $x = 15 - 15t$.

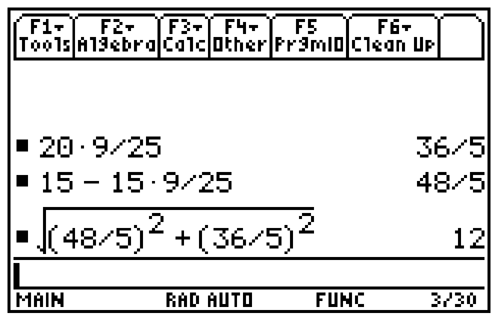


Students are to minimize the distance function
$s = \sqrt{x^2 + y^2} = \sqrt{(15 - 15t)^2 + (20t)^2}$.



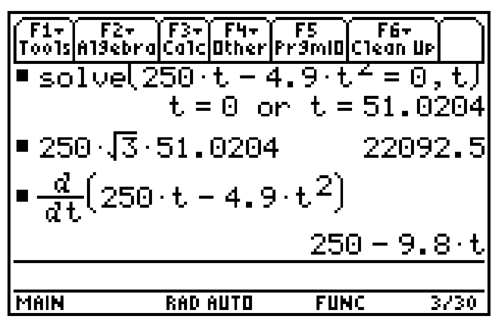
There is a restriction of $0 < t < 1$ because the boats are only moving for 1 hour. Students will solve the first derivate to find the critical time is ***t* = 9/25**. Since the second derivative is always positive, there is a minimum.

The time at which the distance between the boats is minimized is (9/25) · 60 = 21.6 minutes after 1 pm or about 1:22 pm. The distance between the two boats is 12 km.



Extension – Parametric Function

To rewrite the parametric equations, students will need to know that sin(30°) = 0.5 and cos(30°) = $\frac{\sqrt{3}}{2}$.



To find when the projectile hits the ground, students are to set $y = 0$ and solve. ($t = 0$ and $t = 51.02$). Substituting these values into the *x* function gives how far away it lands (22,092.5 units). Students can find the maximum height when $\frac{dy}{dt} = 0$ ($t \approx 25.51$). Substituting this value in the function for *y* students should get 3188.78 units high.