

Optimization

ID: 9609

Activity Overview

Students will learn how to use the second derivative test to find maxima and minima in word problems and solve optimization problems in parametric functions.

Topic: Application of Derivatives

- Find the maximum or minimum value of a function in an optimization problem by finding its critical points and applying the second derivative test. Use **Solve** (in the Algebra menu) to check the solution to f'(x) = 0.
- Use the command fMin or fMax to verify a manually computed extremum.
- Solve optimization problems involving parametric functions.

Teacher Preparation and Notes

- This investigation uses **FMax** and **fMin** to answer questions. Students will have to take derivatives and solve on their own.
- Before starting this activity, students should go to the home screen and select F6:Clean Up > 2:NewProb, then press ß
 This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.
- This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.
- To download the student worksheet, go to education.ti.com/exchange and enter "9609" in the quick search box.

Associated Materials

• CalcWeek33_Optimization_Worksheet_Tl89.doc

Suggested Related Activities

To download any activity listed, go to <u>education.ti.com/exchange</u> and enter the number in the quick search box.

- Application of the Derivative (TI-89 Titanium) 4275
- Find Optimization Points with Derivatives (TI-89 Titanium) 3239
- Where Should it Go? Optimization Exercise (TI-Nspire CAS technology) 10204
- Maximizing Area (TI-Nspire technology) —10043
- Design a Better Drink Can (TI-Nspire technology) 8272

🐌 TImath.com

Problem 1 – Optimization of distance and area

Students will graph the equation y = 4x + 7. They need to minimize the function $s = \sqrt{x^2 + y^2}$ where *x* and *y* are the coordinates of a point on the line. The constraint is the equation of the line.

They are to rewrite the function using one variable: $s = \sqrt{x^2 + (4x + 7)^2} = \sqrt{17x^2 + 56x + 49}$.

To find the exact coordinates of the point, students will take the first derivative (Menu > Calculus > Derivative), and solve for the critical value (Menu > Algebra > Solve), and take the second derivative. Since the second derivative is always positive, there a minimum at the critical value of $x = -\frac{28}{17}$.

To find the *y*-coordinate, students should substitute the value of *x* into the original equation y = 4x + 7. To find the distance, they should substitute the *x*- and *y*-values into the function $s = \sqrt{x^2 + y^2}$. The point is (-1.647, 0.412) and the distance is 1.698 units.

Students are to maximize the function $A = I \cdot w$. The constraint is 2I + 2w = 200. Since I = 100 - w students can rewrite the function as $A = (100 - w)w = 100w - w^2$.

Students will take the first derivative and solve to find the critical value is w = 50. The second derivative is always negative so we have a maximum. When w = 50 m, then l = 50 m.

The maximum area is 2500 m².

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$\frac{d}{d\omega} (100 \cdot \omega - \omega^{2}) \qquad 100 - 2 \cdot \omega$ = solve(100 - 2 \cdot \overline 0, \overline 0) = solve(ans(1)=0, \overline 0) MAIN RAD AUTO FUNC 2/30 $\frac{F1 \cdot F2 \cdot F3 \cdot F4 \cdot F5}{Tools Al3ebra Calc 0 ther Pr3mid Clean UP}$ = $\frac{d}{d\omega} (100 - 2 \cdot \omega) \qquad -2$ = $fMax (100 \cdot \omega - \omega^{2}, \overline 0) \qquad \omega = 50$	
$= \frac{d}{d\omega} (100 \cdot \omega - \omega^2) \qquad 100 - 2 \cdot \omega$ = solve(100 - 2 \cdot w = 0, \omega) = solve(ans(1)=0, \omega) MAIN BAD AUTO FUNC 2/30	
• solve(100 - 2 · ω = 0, ω) ω = 50 solve(ans(1)=0, ω) MAIN RAD AUTO FUNC 2/30 F1- F2- F3- F4- F5 F6- Tools/A19ebra/Calc Other Pr3miD(Clean UP) • $\frac{d}{d\omega}(100 - 2 \cdot \omega)$ -2 • $fMax(100 \cdot \omega - \omega^2, \omega) = 50$	$= \frac{d}{d\omega} \left(100 \cdot \omega - \omega^2 \right) \qquad 100 - 2 \cdot \omega$
$w = 50$ $solve(ans(1)=0,w)$ MAIN RAD AUTO FUNC 2/30 $F1* F2* F3* F4* F5 F6*$ Tools[A19ebra[Calc]Other]Pr3mil[Clean UP] $= \frac{d}{dw}(100 - 2 \cdot w) -2$ $= fMax(100 \cdot w - w^{2}, w) w = 50$ $FMax(100 \cdot w - w^{2}, w) w = 50$	■ solve(100 - 2·w = 0,w)
$\frac{\text{solve}(\text{ans}(1)=0, \omega)}{\text{MAIN}}$ $\frac{\text{RAD AUTO} \text{FUNC} 2/30}{\text{F1} + F2 + F2 + F3 + F4 + F5} F6 + F7}$ $\frac{\text{F1} + F2 + F2 + F3 + F4 + F5}{\text{Tools} \text{Algebra}(\text{Calc}) \text{Other} \text{Prgmin}(\text{Clean UP})}$ $= \frac{d}{d\omega} (100 - 2 \cdot \omega) -2$ $= \text{fMax}(100 \cdot \omega - \omega^2, \omega) \omega = 50$ $\frac{\text{FMax}(100 \cdot \omega - \omega^2, \omega)}{(100 \cdot \omega - \omega^2, \omega)} \omega = 50$	ω = 50
MAINRAD AUTOFUNC2/30F1+F2+F3+F4+F5F6+ToolsA13ebraCalcutherPr3miDClean UP $\frac{d}{dw}(100 - 2 \cdot w)$ -2 $\frac{d}{dw}(100 \cdot w - w^2, w)$ $w = 50$ FMax(100 \cdot w - w^2, w) $w = 50$	solve(ans(1)=0,ω)
$\frac{d}{d\omega}(100 - 2 \cdot \omega) = 50$	MAIN RAD AUTO FUNC 2/30
$= \frac{d}{d\omega} (100 - 2 \cdot \omega) -2$ $= \frac{f^2}{d\omega} (100 - 2 \cdot \omega) -2$	
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$= \frac{1}{d\omega} (100 - 2.0) = -2.0 = -2.$	
$fMax(100 \cdot \omega - \omega^2, \omega) \qquad \omega = 50$	$= \frac{1}{dw} (100 - 2 \cdot w) = -2$
$\frac{-1}{2} \max\{100 \ \text{w} \ \text{w}, \text{w}\} \ \text{w} = 00$	■ £Max(100=
	$\frac{-1}{2} \max(100 \pm 0.02 \pm 0.02)$
MAIN RAD AUTO FUNC 2/30	<u> </u>

Problem 2 – Optimization of time derivative problems

Remind students to use t, for time, instead of x. The position equations are the constraints.

The boat heading north is going from the right angle to the point northward. Its position equation is y = 20t.

The boat heading west is going to the right angle. At 1 pm, it is one hour from the arrival time 2 pm so it is 15 km away. Its position equation is x = 15 - 15t.

Students are to minimize the distance function

$$s = \sqrt{x^2 + y^2} = \sqrt{(15 - 15t)^2 + (20t)^2}$$
.

There is a restriction of 0 < t < 1 because the boats are only moving for 1 hour. Students will solve the first derivate to find the critical time is t = 9/25. Since the second derivative is always positive, there is a minimum.

The time at which the distance between the boats is minimized is $(9/25) \cdot 60 = 21.6$ minutes after 1 pm or about 1:22 pm. The distance between the two boats is 12 km.

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• <u>d</u> [[[]	$(5 - 15 \cdot t)^2$	² +(20·t) ²)
	5.	(25·t - 9	9)
	J25 ··	t ² - 18 · 1	է+9
(15-15 Main	5*t)^2+(20 RAD AUTO	0*t)^2), FUNC	t) 1/30
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		t ² - 18 · ·	t+9
∎solve	$\frac{5 \cdot (25)}{25 \cdot t^2}$	t-9) 18·t+9	= C)
	, 1 20 0	t=	9/25
<u>solve(a</u> Main	ans(1)=0,1 Rep euto	EUNC	2/30
/ <u></u>			~~~
Too1s A19eb	raCalcOtherP	r9mIO Clean	UP
■ 20+9/:	25		36/5
• 15 - 13	5.9/25		48⁄5
■ (48⁄5) ² +(36/5)) ²	12
MAIN	RAD AUTO	FUNC	3/30

Extension – Parametric Function

To rewrite the parametric equations, students will need

to know that $\sin(30^\circ) = 0.5$ and $\cos(30^\circ) = \frac{\sqrt{3}}{2}$.

To find when the projectile hits the ground, students are to set y = 0 and solve. (t = 0 and t = 51.02). Substituting these values into the x function gives how far away it lands (22,092.5 units). Students can find the maximum height when $\frac{dy}{dt} = 0$ ($t \approx 25.51$). Substituting this value

in the function for y students should get 3188.78 units high.

F1+ F2+ ToolsA19eb Solve	F3+ F4+ ra[ca]c0therP .250·t - 4 t = 0 or	F5 F6 r9m10[C1eal •9·t ² = • t = 51	.0204	
■ 250 · \ 3	51.0204	22	092.5	
■ <u>_d</u> (25	0·t-4.9	t ²)		
		250 -	9.8·t	
MAIN	RAD AUTO	FUNC	3/30	
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F1+ F2+ ToolsA19eb	F3+ F4+ raCa1cOtherP	F5 F6 r9ml0C1ea		
F1+ F2+ ToolsA13ebi ut	ra[ca1c 0ther P :250 - 9.8	r9m10Clean 250 - t = 0, t = 25	, 9.8∙t t) .5102	
E 250 · 25	ra[ca1c 0ther P 250 - 9.8 5.5102 - 4	F5 F6 250 - t = 0, t = 25 .9 (25. 31	9.8·t 9.8·t t) .5102 5102 88.78	