



Objective: Compare the slopes of a secant line and the tangent line at point P .

In this activity, you will graph $y_1(x)=x^2$ and find the slopes. If point $P(1, 1)$ and point Q are two points on $y_1(x)$ that define a secant line, then the slope of the secant line is $\frac{y_1(x)-y_1(1)}{x-1}$, where x is the x -coordinate of point Q .

Problem 1 – Graphing secant and tangent Lines

Step 1: Press \blacklozenge [F1] to go to the **Y=** screen. At y_1 , press [X] \wedge [2].

Step 2: Press \blacklozenge [F2]. Set the window to -3.5 to 3.5 for the x -axis and -10 to 10 for the y -axis. Use **xscl** and **yscl** of 1 to have tick marks show every unit.

Step 3: Press \blacklozenge [F3] to graph.

Step 4: To draw the tangent line to $y=x^2$ at $(1, 1)$, press [F5] and arrow down to select **Tangent**. When prompted with the question **Tangent at?** Press [1] [ENTER].

- Record the equation of the tangent line at $(1, 1)$. _____
- What is the slope of the line? _____

Step 5: Return to the **Y=** screen. In $y_2(x)$ graph a secant line that passes through point $P(1, 1)$ and point $Q(3, 9)$.

- What is the slope of this line? _____
 - Begin with the point-slope form of an equation. Show the work to explain why you should graph $y_2(x) = 4(x - 1) + 1$.
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- What could be done to bring point Q closer to point $P(1, 1)$? What value does the slope need to approach?



Secant and Tangent Lines

Step 5: On the **Y=** screen, change the slope of the secant line to 3.5. Observe the graph.

Then change the slope to 3 and observe the graph. Note the intersection points P and Q . Repeat one more time, making the slope of the secant line 2.5.

- As point Q approaches point P , what do you notice about the secant and tangent lines?

Problem 2 – Finding average rate of change

Step 6: Turn $y2(x)$ off by pressing $\boxed{F4}$ when the cursor is next to $y2(x)$ on the **Y=** screen.

Step 7: Press $\boxed{\blacktriangledown} \boxed{F4}$ to set up the table to analyze the function numerically. Change **tblStart** to 1 and Δ **tbl** to 0.1.

Step 8: Press $\boxed{\blacktriangledown} \boxed{F5}$ to see the x - and y -coordinates of $y1(x)=x^2$.

The **average rate of change** of function on an interval $[a, b]$ is the slope of the secant line containing $(a, y1(a))$ and $(b, y1(b))$

- Use the table of values of to find the average rate of change of $y1(x)$ on the interval $[1, 1.1]$. Show what you inputted into the calculator (\boxed{HOME}).

Problem 3 – Finding instantaneous rate of change

The **instantaneous rate of a change** of a function at a given point is the slope of the tangent line to the function at that point.

If the tangent line is not known, then the secant line slope can be used as an approximation. The calculation from Problem 2 is an example of such an approximation.

- How can you find an even better estimate of the instantaneous rate of change of $y1(x)$ at $x = 1$?
- Use the calculator to find a better estimate of the instantaneous rate of change of $y1(x)$ at $x = 1$. Show what you inputted into the calculator.



Extension – Rectilinear Motion Application

Use your calculator to answer the following questions. For each answer, be sure to include appropriate units.

Suppose a particle is traveling along a horizontal line such that its position is given by the function $s(t) = t^3$, $t \geq 0$ where s is measured in meters and t is measured in seconds.

1. What is the *average rate of change* of the position function from $t = 2$ to $t = 4$?

2. *Estimate* the *instantaneous rate of change* of the position function at $t = 2$?

The velocity of the particle is given by the function $v(t) = 3t^2$ where v is measured in meters per second and t is measured in seconds.

3. What is the *average rate of change* of the velocity function from $t = 2$ to $t = 4$?

4. *Estimate* the *instantaneous rate of change* of the velocity function at $t = 2$?