

Activity 10

Analysis of a Bouncing Ball

Objectives

- Explore the relationship among position, velocity, and acceleration
- Connect mathematical relationships to real-world phenomena

Materials

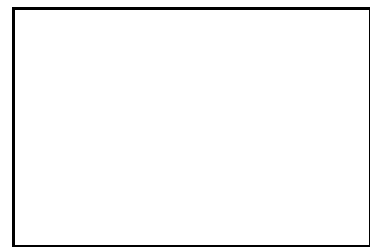
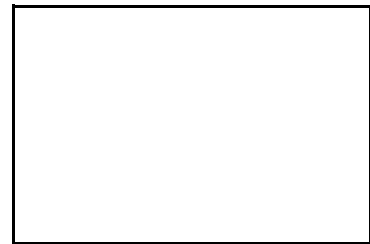
- TI-84 Plus / TI-83 Plus
- Calculator-Based Ranger™ (CBR™)
- Ball (Racquetballs work well, as do basketballs or other balls that will bounce. Tennis balls may not give good results because the ball may absorb the sound wave rather than reflect it.)

Introduction

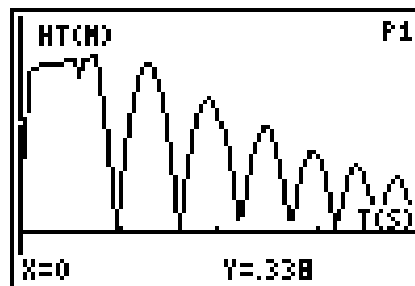
This activity uses the sonic motion detector and CBR™ unit to record and plot the height from the ground versus time for a bouncing ball. The ball is released underneath a motion detector, and its motion is recorded. One bounce is selected and analyzed. During the one bounce, the ball is under the influence of gravity. Because the ball's speed is small at all times, the air resistance is negligible, and therefore the ball can be studied as an object in free fall.

Exploration

1. Predict the height from the ground versus time graph for a ball bouncing underneath the motion detector. Make a sketch to illustrate your prediction.
2. Predict the velocity of the ball as a function of time for the ball during a single bounce.
3. Predict the acceleration of the ball as a function of time for the ball during a single bounce.



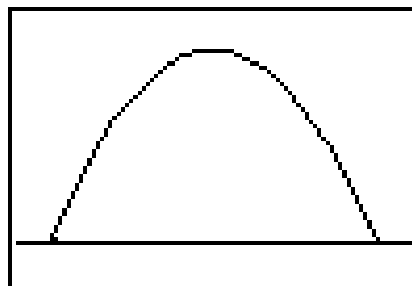
4. Attach the CBR™ to the graphing handheld. From the **Apps Menu**, select **2:CBL/CBR**, and execute the **3:RANGER** program. From the main menu, select **3:APPLICATIONS**, select **1:METERS**, and then select **3:BALL BOUNCE**. Hold the ball at least $\frac{1}{2}$ meter away from the CBR. When you hear the clicking sound indicating that the CBR is activated, drop the ball under the detector. Be sure to hold the CBR steady.



Your graph should be similar to the one shown above. How does this compare with your prediction?

If you are not satisfied with your results, press **ENTER** and select **5:REPEAT SAMPLE** to recollect the data. The time values for the collected data are stored in **L1**, and the heights of the ball as measured from the ground are stored in **L2**. (The program converts the distances from the CBR into heights from the ground.)

5. The next step is to select one of the bounces for analysis. From the **LIST OPS Menu**, select **8>Select** and input **L5, L6**. This will allow you to store a portion of the data to different lists. The graph of the ball will appear with the words "Left Bound?" Use the right arrow key to move the cursor to the beginning of the first bound, and press **ENTER**. The words "Right Bound?" appear. Use the right arrow key to move the cursor to the end of the first bound, and press **ENTER**. The graphing screen shows only the selected data. If you would like to have the data take up the entire window as shown above, press **ZOOM** **9**. If you are unhappy with your graph, return to the **STATPLOT** and adjust **Plot1** by inputting **L1** as the Xlist and **L2** as the Ylist, and then repeat the select procedure.



6. The height data for the selected curve is now stored in **L6**, and the time data is stored in **L5**. The quadratic regression will find the best-fitting equation for this data. The height of the ball from the ground with the acceleration due to gravity acting on it is $d = d_0 + v_0t + \frac{1}{2}gt^2$. In this equation, d_0 is the initial height, v_0 is the initial velocity, and g is the acceleration due to gravity. The quadratic regression fits the data with an equation in the form $y = ax^2 + bx + c$. Place the letters a , b , c , y , or x in the spaces below to show which of these correlate to the constants that are used in physics for the basic equation of motion with constant acceleration.

$$\frac{1}{2}g \quad \underline{\hspace{2cm}} \quad t \quad \underline{\hspace{2cm}} \quad d \quad \underline{\hspace{2cm}}$$

7. Find a quadratic regression for these data. From the **CALC STAT Menu**, select **5:QuadReg**. Input the quadratic regression as shown at the right. When using the **QuadReg** function, you must first indicate the lists where the data are stored and then the equation variable where you want to store the equation. Record the values of a , b , and c .

QuadReg L5, L6, Y1

8. What is the value for the acceleration of gravity as calculated in the regression equation? Explain your reasoning.
9. How does this compare with the accepted value, which is -9.8 m/s^2 ?
10. Consider the fact that the chosen section of the data did not begin at time $t = 0$. Do the values of b and c in the regression equation have physical significance for this data set? Explain your answer.
11. How well does the regression equation fit the data?

- 12.** The velocity of the ball is the instantaneous rate of change of the height as a function of time. This is also the slope of the line tangent to the height versus time graph. The tangent function in the **DRAW Menu** will draw the line tangent to the graph and give the equation of the line on the graphing handheld. From the graph, select **5:Tangent** from the **DRAW Menu**. This command will return the cursor to the graph. Use the arrow keys to position the cursor at a point on the left side of the parabola. Record the value for the time below, and then press **[ENTER]**. Record the value of the slope of the tangent line as the velocity. Repeat the same procedure to record a time and corresponding value for the slope of the tangent line at the top of the parabola and on the right side of the parabola.

Time and velocity for a point on the left side:

Time _____ Velocity _____

Time and velocity for a point at the top:

Time _____ Velocity _____

Time and velocity for a point on the right side:

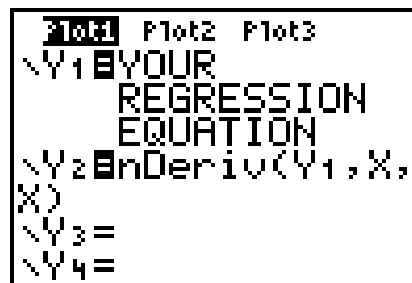
Time _____ Velocity _____

Explore by drawing other tangent lines until you develop an understanding of how the velocity changes when the ball is in the air. To clear the drawn tangent lines, select **1:ClrDraw** from the **DRAW Menu**.

- 13.** Think about the velocity of the ball as it leaves the ground, reaches the maximum height, and returns to the ground. Use this information along with the values of the slopes of the tangent lines drawn above, and refine your prediction of the graph of velocity versus time for the bounce. Sketch and explain your new prediction.



14. The velocity of the ball is the derivative of the position. To examine the velocity of the ball, graph the numerical derivative of the regression equation with your regression equation and data. From the **MATH Menu**, choose the numerical derivative, and set up as shown. The numerical derivative requires three arguments: the equation, the variable, and the value at which it will be calculated. Because you are graphing the derivative, the third argument is x to indicate that the numerical derivative should be calculated at all values of x .



Plot1 Plot2 Plot3
 $\sqrt{Y_1}$ \square YOUR
 REGRESSION
 EQUATION
 $\sqrt{Y_2}$ \square nDeriv($Y_1, X,$
 X)
 $\sqrt{Y_3}$ =
 $\sqrt{Y_4}$ =

15. Adjust the viewing window settings so that you can see the height and velocity graphs together. Make a sketch of the height versus time plot along with the velocity graph. Label the axes, including numerical values on the sketch.



16. How does the velocity versus time graph compare with your prediction? Explain any differences.
17. Analyze the two graphs.
- When is the velocity of the ball positive? When is it negative? When is it zero?
 - Compare the velocity and height graphs. Describe the motion and position of the ball when the velocity is positive, negative, and zero.
18. Define acceleration in terms of velocity without using the word *derivative*. To what does this correspond on the velocity versus time graph?
19. The slope or instantaneous rate of change of the velocity versus time graph is the acceleration. Is the slope of the velocity versus time graph constant? What is the slope of the velocity versus time graph when the velocity is zero? Is the slope of the velocity versus time graph ever zero?

- 20.** Given your graph of velocity, refine your prediction for acceleration versus time.
- 21.** Because the acceleration of the ball is the instantaneous rate of change or derivative of the velocity, graph the acceleration of the ball by inputting **Y3** as **nDeriv(Y2,X,X)**. Turn off the position and velocity graphs by moving the cursor on top of the equal sign and pressing **ENTER**. Turn off the statistical plot by moving the cursor to the top of **Plot1** in the **Y=** editor and pressing **ENTER**. Adjust the y-values of the viewing window to see the graph. Record the viewing window settings. Display the graph of acceleration versus time and use **TRACE** to determine the value of the acceleration at several different times. Sketch the graph, labeling the axes.

The values of the acceleration at several times:

$$t1 = \underline{\quad} a = \underline{\quad}$$

$$t2 = \underline{\quad} a = \underline{\quad}$$

$$t3 = \underline{\quad} a = \underline{\quad}$$



- 22.** What does the graph of the acceleration versus time of the ball bounce show regarding the acceleration of the ball during the entire time?
- 23.** What should the velocity versus time plot look like for all of the bounces that you collected initially?
- 24.** Turn on the plot of **L1** and **L3** to check your prediction. Explain why the velocity versus time graph section for each bounce shows parallel lines.
- 25.** Summarize the relationships among the height, velocity, and acceleration of the bouncing ball.