EXPLORATIONS

Chapter 15

Features Used

seq( ), SEQUENCE, solve( ), [|], Σ(sum, [ANS], NewProb

Setup

[♦] 1
NewFold econ

Financial Calculations

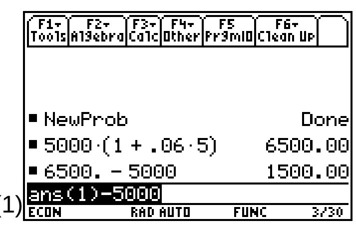
This chapter describes how to use the TI-89 to calculate interest, present worth, loan repayments, and so forth. These methods utilize the time-value-of-money.

Topic 67: Simple Interest

Money that is invested earns interest. The most basic form of interest is known as simple interest. An amount of money with present value P that is invested for N years at an annual interest rate of i has a future value F. For simple interest, the future value is calculated as F=P+NPi=P(1+iN). The future values can be converted back to present value as P=F/(1+iN).

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].
2. Find the payment received after 5 years on a $5000 investment at 6% simple interest (screen 1). The future value is given by F=5000(1+.06∗5)=$6500.

   The total interest paid is 6500-5000=$1500.



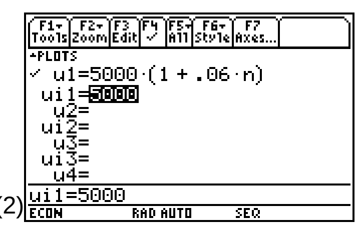
(1)

3. The TI-89 displays this type of sequential calculations in the **SEQUENCE** graphing mode.

   Press [MODE] and set **Graph** mode to **SEQUENCE**.
4. Press [♦] [Y=] and enter the equation for **u1** as a function of the payment period as shown in screen 2.

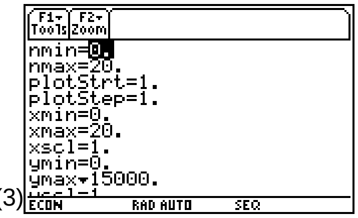
   5000 [(] **1** [+] **.06 n** [)]

   Also enter an initial value of **ui1=5000**.

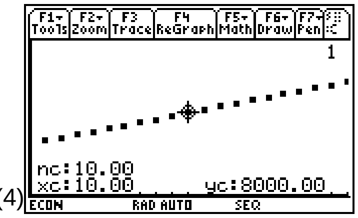


(2)

5. Set the Window variable values in the Window Editor as shown in screen 3.

(3)


6. Press ◆ [GRAPH] to display the sequence for a 20-year period (screen 4). The future value at the 10th year is observed by pressing [F3] **Trace** and moving the cursor to the 10th year (**nc**) where the value is $8,000.

(4)


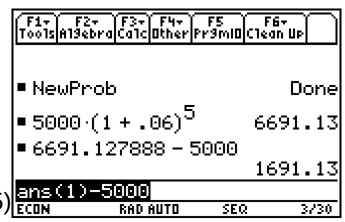
Topic 68: Compound Interest

Compound interest is more common than simple interest and much better for the investor. The interest is calculated on the initial investment plus the interest earned to date. At the starting date, the value of the investment is F(0)=P. At the end of the first interest period, the value of the investment is F(1)=P(1+i); at the end of the second period the value is $F(2)=P(1+i)^2$. The pattern is clear—the value after the nth period is $F(n) = P(1+i)^n$.

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

2. Screen 5 shows how to find the interest on the same $5000 principle at 6% compound interest paid on a yearly basis for 5 years (screen 5). The future value is calculated by F=5000(1+.06)^5= $6691.13.

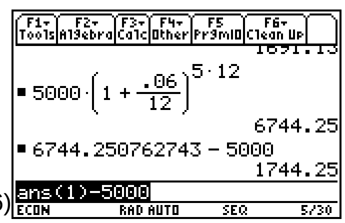
The total interest earned is 6691.13 - 5000=$1691.13, more profitable for the investor than simple interest.

(5)


3. The most common method of interest payment is with monthly compounding. The monthly interest rate is $i_{Month} = i/12$.

Find the future value after 5 years for the $5000 investment at 6% annual interest compounded monthly (screen 6): F=5000(1+.06/12)^(5*12)= $6744.25.

The interest earned is $1744.25, an even more attractive investment.

(6)


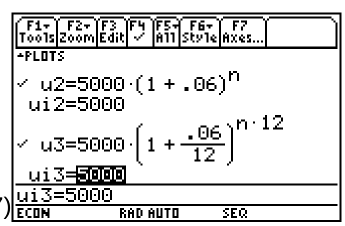
4. The two compound interest examples are compared graphically with the simple interest case by entering them in the Y= Editor.

u2(**n**)= **5000** [(] **1** [+] **.06** [)] [^] **n**

with **ui2**=**5000**

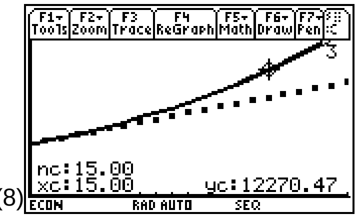
u3(**n**)= **5000** [(] **1** [+] **.06** [÷] **12** [)] [^] [(] **n** [×] **12** [)]

with **ui3**=**5000**

(7)


5. Make the three graphs look different. Highlight the equation for **u2** and press [2nd] [F6] **1:Line**. Highlight **u3** and press [2nd] [F6] **4:Thick**. Then press [◆] [GRAPH].

   Although the two compounding curves look the same, pressing [F3] **Trace** shows that at **nc** = 15 years the three graphs have the future worth of \$9500.00, \$11982.79, and \$12,270.47, for simple interest, yearly compounding, and monthly compounding, respectively.

(8) 

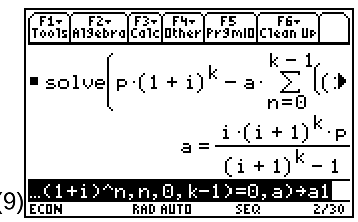
Note: *Use ⊙ and ⊙ to change from one graph to another.*

Topic 69: Loans

The calculation of loan repayment schedules is of great interest in professional as well personal life. Typical loans require an equal periodic payment A made for k payment periods to repay an amount P borrowed at interest rate i per period. At the end of the first payment period, the amount owed is P(1+i) (the principle plus interest for one period) minus one payment A, that is, P(1+i)-A. After the second payment, the remaining amount owed is $(P(1+i)-A)(1+i)-A=P(1+i)^2-A(1+i)-A$. After the *k*th payment, the entire loan and interest is paid, $P(1+i)^k-A(1+i)^{k-1}-A(1+i)^{k-2}-...-A(1+i)-A=0$. Use **solve( )** to find the form of A.

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].
2. Enter the **solve( )** command as shown in screen 9.

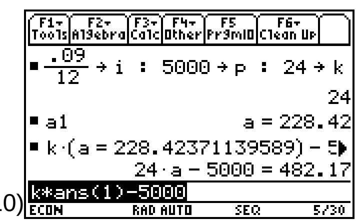
   [CATALOG] **solve (p** [×] [(] **1** [+] **i** [)] [^] **k** [−] **a** [×] [CATALOG] Σ([(] **1** [+] **i** [)] [^] **n** [,] **n** [,] **0** [,] **k** [−] **1** [)] [=] **0** [,] **a** [)] [STO▸] **a1**

(9) 

3. Calculate the total interest paid on a two-year, \$5000 auto loan at an annual interest rate of 9% repaid with monthly payments.

   Enter the interest rate, loan amount, and number of payments (screen 10).

   .09 [÷] **12** [STO▸] **i** [2nd] [:] **5000** [STO▸] **p** [2nd] [:] **24** [STO▸] **k**

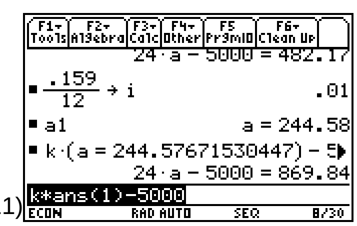
(10) 

4. Display the payment amount (\$228.42) by entering **a1**.
5. Calculate the total interest paid.

   k [×] [2nd] [ANS] [−] **5000**

   The total interest paid is \$482.17.

6. Find the payment for the same debt but with a typical credit card interest rate of 15.9% (screen 11). The monthly payment **a(1)** is \$244.58; and the total interest paid is **k** ∗ **a** − 5000 = \$869.84, nearly twice the total interest for the smaller rate.

(11) 

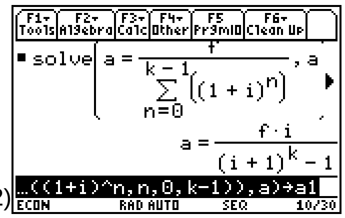
Topic 70: Annuities

An annuity is a financial process in which equal payments, A, are made to an account with an interest rate, i, for a fixed number of periods, k. Usually, the compounding takes place each period. This is often called a uniform series. The first payment earns compounded interest for k -1 periods with a future value of $A(1+i)^{k-1}$; the second payment has a future value of $A(1+i)^{k-2}$; the last payment that is made when the annuity is due has a future value of A. The sum of these terms gives the future value of the annuity, $F= \sum A(1+i)^n$ summed from 0 to k-1. To achieve a future value F, the periodic payment is $A=F/\sum(1+i)^n$.

Example 1: Finding Monthly Payment Amount

1. **solve()** gives a closed form of solution for the computations, but first use **DelVar** so that previous values for **i** and **k** are deleted.

   [CATALOG] **DelVar i** [,] **k**

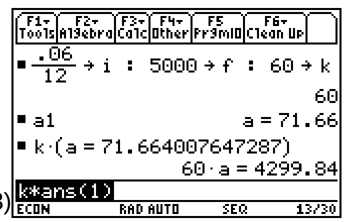


(12)

2. Use the **solve()** command to enter the annuities equation as shown in screen 12.

   [CATALOG] **solve (a** [=] **f** [÷] [CATALOG] **∑(** [(] **1** [+] **i** [)] [^] **n** [,] **n** [,] **0** [,] **k** [−] **1** [)] [,] **a** [)] [STO▸] **a1**

3. To calculate the monthly payments necessary to accumulate \$5000 in 5 years at 6% annual interest rate, enter the variable values as shown in screen 13.



(13)

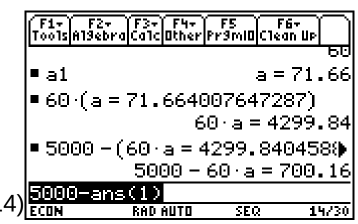
4. Find the monthly payment amount, the total amount paid, and the amount of interest earned (screen 14).

   a1

   x [2nd] [ANS]

   5000 [−] [2nd] [ANS]

   The monthly payment is \$71.66; the total amount paid is \$4299.84; the total interest earned is \$700.16.



(14)

Example 2: Finding Amount to Invest

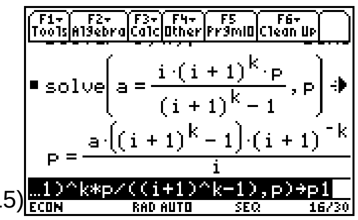
The next example shows how to calculate the present amount to invest, P, required to receive equal periodic payments, A, over a fixed number of periods, k, from an account which earns a compound interest rate i., The equation is the same as the equation in Topic 69 with present value, P = F/(1+ i) ^ k, substituted for future value.

1. Delete the values for **i**, **k**, and **P**.

   [CATALOG] **DelVar i** [,] **k** [,] **p**

2. Enter the equation as shown in screen 15.

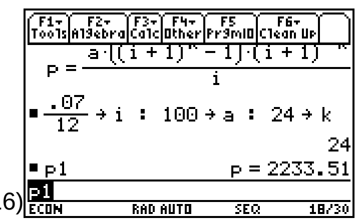
   [CATALOG] **solve (a** [=] **i** [×] [(] **i** [+] **1** [)] [^] **k** [×] **p** [÷] [(] [(] **i** [+] **1** [)] [^] **k** [−] **1** [)] [,] **p** [)] [STO▶] **p1**

(15) 

3. This process is the inverse of loan payments. Instead of receiving an amount of money and paying it back in equal payments, an amount of money is paid to an institution and the equal payments are received.

   Calculate the amount to be paid in order to receive equal monthly payments of a=$100 for k=2 years=24 months from an account that earns i=7% interest per month (screen 16).

   A deposit of $2233.51 (**p1**) returns a total of $2400 over the two-year period.

(16) 

Tips and Generalizations

This chapter has shown how the TI-89 can easily derive and solve time-value-of-money problems. Consider using these examples before you apply for a loan or get a credit card.