Zeros of a Parabola

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Abstract: This activity involves some of the prerequisites of calculus relating to functions and equations. It introduces students to an interesting property of parabolas and a method of proving that property using the TI-89 scripts. They then use the symbolic capacity of their calculator to generalize upon specific results.

NCTM Principles and Standards:
Algebra standards
a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

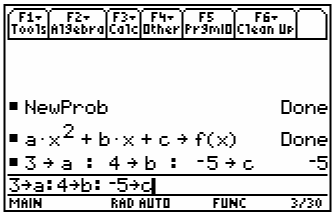
Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

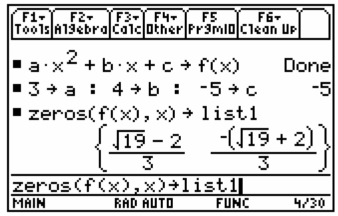
Reasoning and Proof Standard
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Key topic: Prerequisites of calculus relating to functions and equations. Scripts, formal proofs.

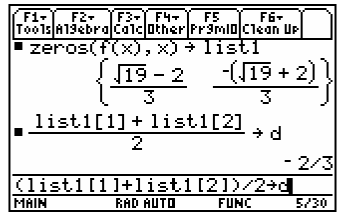
Degree of Difficulty: Elementary
Needed Materials: TI-89 calculator
Situation: Parabolas have many interesting properties. In this activity we'll investigate one of them with the aid of the TI-89 calculator. What is the relationship between the x-coordinates of the zeros of a parabola and the x-coordinate of its vertex?

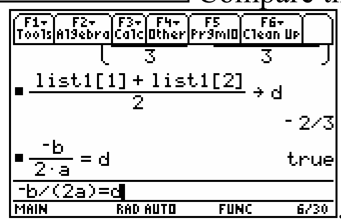
Choose arbitrary values for the coefficients of a parabola in the form of $ax^2 + bx + c$ such that its discriminant is positive. One way to do this is to make sure that a and b have different signs. Store this as f(x) and then find its zeros.





The zeros are given as elements in a list. Store them in a list with the name list1

and then average them and store that result as d: 

Compare this result to the x-coordinate of the vertex of the parabola: $\frac{-b}{2a}$ 

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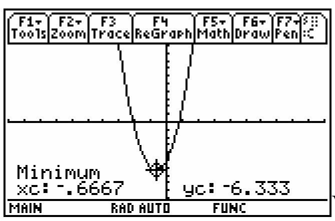
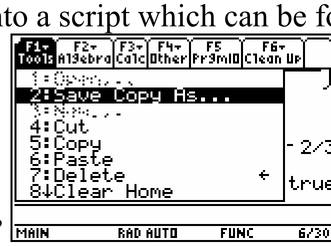
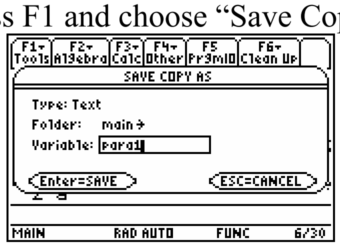
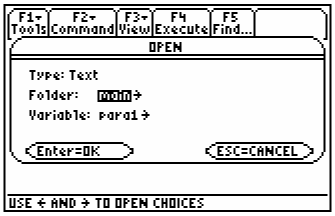


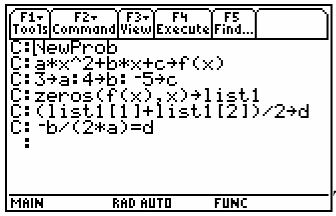
What is our conclusion? The average of the zeros for this parabola is the x-coordinate of its vertex. Is this always true? You could scroll back and change the coefficients of the parabola and execute the steps again, but we'll take a different tack. We'll turn what we've written into a script which can be followed for any parabola: Press F1 and choose "Save Copy As" 

I named this script "para1" 

From the application menu $\boxed{\text{APPS}}$ choose 8:Text Editor: 

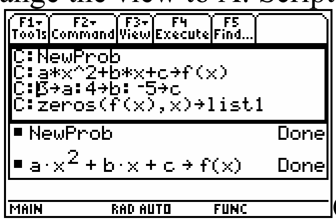
and open the script named para1:



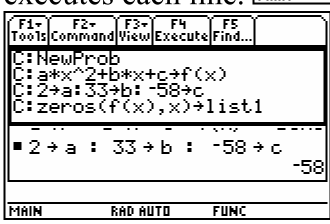


This script contains the commands that were originally typed in. We can play this script for other choices of a, b, and c:

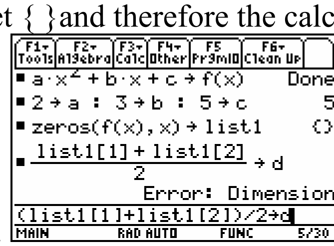
First press F3 to change the view to A: Script View. Pressing F4 executes each line.



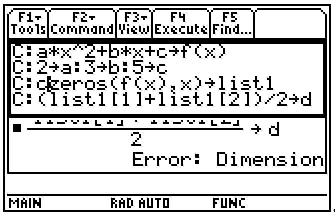
Change the values of a, b, and c:

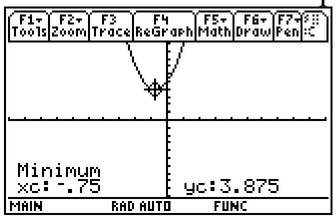


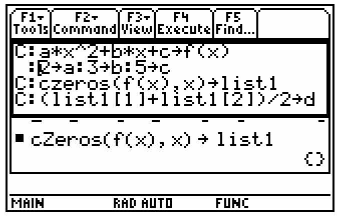
and continue pressing F4 to see that the property is true with these choices. What if the discriminant is negative and the parabola doesn't have real zeros? Scroll up to the top of the script and change the third line of the script to C:2→a: 3→b: 5→c. Run the script from the beginning and observe that the zeros command returns the empty set { }and therefore the calculator can't average the, non existent, elements of this list.



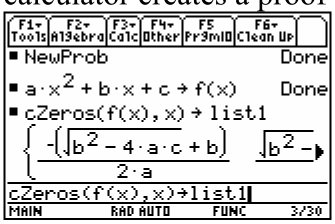
. To fix this problem, we only need to change our statement from "zeros(f(x),x) → list 1" to "czeros(f(x),x) → list 1"

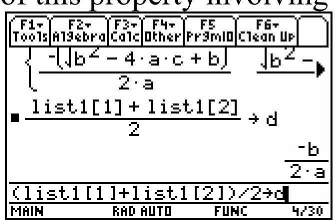


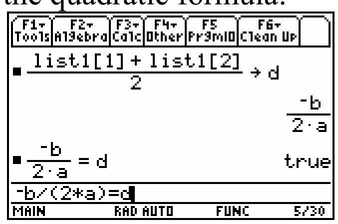
. Now rerun the script with this change and observe that the property is still true. 

Is it always true? Go back to the line "C: 2→a: 3→b: 5→c" and choose 4:Clear Command from the [F2] Command menu: 

which will erase the C: in the line "C: 2→a: 3→b: 5→c". Now run the script again and observe that the calculator creates a proof of this property involving the quadratic formula:







Scripts can be very useful in proving properties. Here we showed that the average of the zeros for this parabola is the x-coordinate of its vertex. We've shown that the property is true even if the zeros of the parabola are complex! Notice that the tangent line to a parabola at its vertex is horizontal - which is also true about the line between its zeros - if they are real. In another activity, we'll generalize our result to other points on parabolas.