Extrema Using Derivatives

ID: 9413

Time required
45 minutes

Activity Overview

*In this activity, students will learn how to find and label extrema using first and second derivatives, be able to inspect a graph and determine which extrema the function has, be able to use **Trace**, **fMin**, and **fMax** to verify the computed answers and find critical values for parametric functions.*

Topic: Derivatives of Higher Order

- *Identify the conditions on $f'(x)$ and $f''(x)$ for a local maximum, minimum and point of inflection.*
- *Solve the equation $f'(x) = 0$ to find the critical points of $f(x)$.*
- *Compute $f''(x)$ at the critical points of f(x) to classify them as extrema or points of inflection.*
- *Graph a function and use **Trace** to approximate its local extrema.*
- *Use **fMin** and **fMax** to verify the manual computation of local extrema.*
- *Find critical points of a parametric function $x = f(t), y = g(t)$ by solving $\frac{g'(t)}{f'(t)} = 0$.*

Teacher Preparation

- *This investigation uses **fMin** and **fMax** verify answers and students will have to restrict the domain when using those commands. Students should be familiar with keystrokes for the **Derivative** command and also be able to graph functions and use the **Trace** command.*
- *The calculator should be set in radian mode before the trigonometric derivatives are taken.*
- *Be sure that the students read the sign rules for minima and maxima in this problem.*
- *Before starting this activity, students should go to the home screen and select **F6:Clean Up > 2:NewProb**, then press ENTER. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9413" in the keyword search box.***

Associated Materials

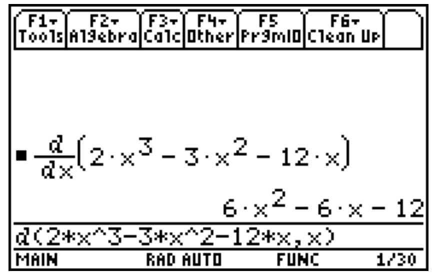
- *ExtremaUsingDerivatives_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Functions and Their Extrema (TI-89 Titanium) — 6436*
- *First Derivative Test (TI-Nspire technology) — 16073*
- *Application of the Derivative (TI-89 Titanium) — 4275*

Problem 1 – The extrema of $y = 2x^3 - 3x^2 - 12x$



Students are to find the first derivative (**F3:Calc>1:derivative**). They should see that there is a common factor of 6 and can be pulled out.

$6x^2 - 6x - 12 = 0 \rightarrow 6(x-2)(x+1) = 0$

Students can use the **solve** command **(F2:Algebra>1:solve)** to find the two critical points (x = 2 and x = –1).

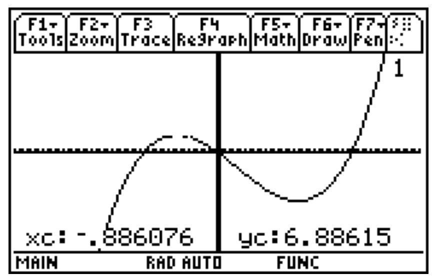
The second derivative can be found by taking the derivative of $6x^2 - 6x - 12$ or using nested derivatives d(d(original function,x),x). The result is $12x - 6$. When students evaluate the critical points, the will should get the following:

$12(2) - 6 = 18 > 0$ so x = 2 is a minimum

$12(-1) - 6 = -18 < 0$ so x = –1 is a maximum

Solving $12x - 6 = 0$, $x = 0.5$. So $x = 0.5$ is a point of inflection.

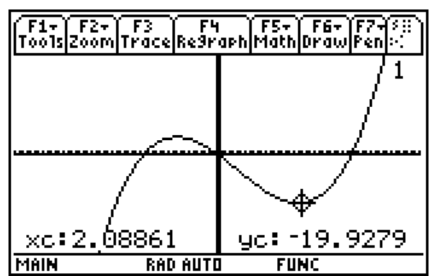
Students are to graph $y = 2x^3 - 3x^2 - 12x$.

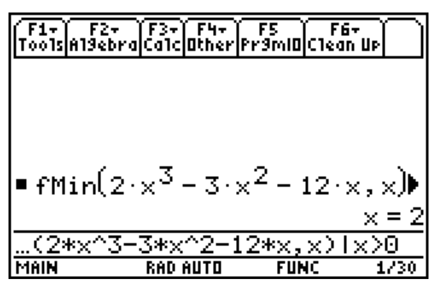


Use **Trace (Graph>F3:Trace)** and enter –1 to move the cursor to the point at x = –1.

Students should see that the two function values on both sides of x = –1 are less than the function value at x = –1.

They will use the same procedure around x = 2 to show that the x-values around x = 2 yields a greater amount than the value at x = 2.

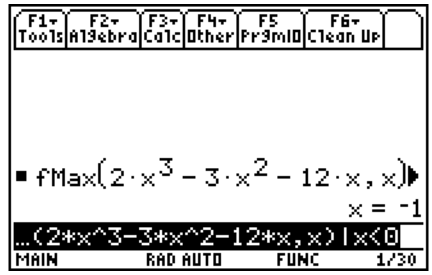




Students are to use the **fMin** and **fMax** commands to verify the maximum and minimum.

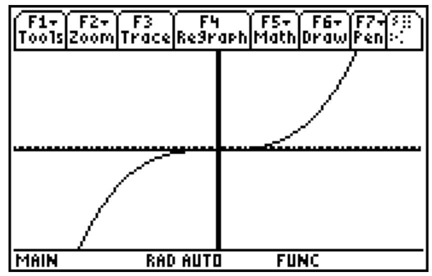
If students use **fMin** (**F3:Calc>6:fMin)** on the function without any qualifiers on x, they will get $-\infty$. So since we know that the minimum is greater than 0, they need to add **|x>0** to the end of the command.

If students use **fMax** (**F3:Calc>7:fMax)** on the function without any qualifiers on x, they will get ∞. So since we know that the maximum is less than 0, they need to add **|*x*<0** to the end of the command.



Problem 2 – The extrema of $y = x^3$

The student screen should look like the one at the right.



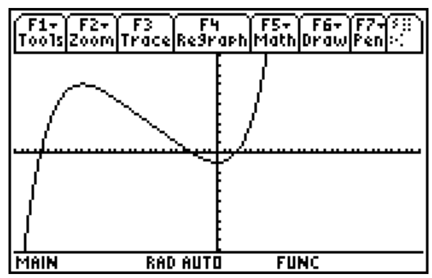
There are no extrema to be seen on the graph. However, the function changes concavity at $x = 0$. Thus we have a point of inflection but no extrema.

The first derivative is $3x^2$.

$3x^2=0$ means that $x=0$ which is a point of inflection.

Problem 3 – Extrema for other functions

Students will now graph $g(x) = (x+1)^5 - 5x - 2$

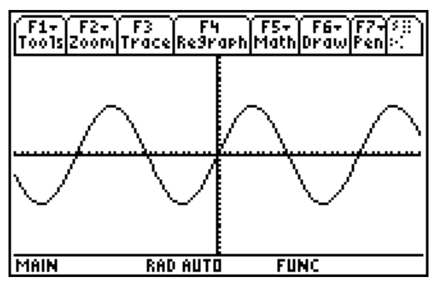


They will need to adjust the window settings. Students will use the graph and the **fMin** and **fMax** command to find the extrema.

Minimum: $x = 0$; Maximum: $x = –2$

Inflection Point: $x = –1$.

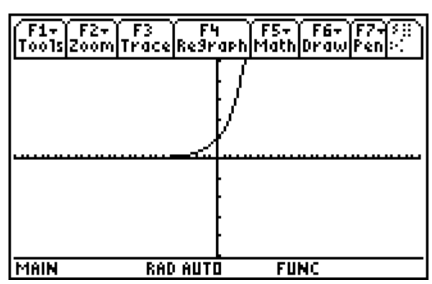
Graph h(x) = sin(3x). There will be multiple minima, multiple maxima, and multiple points of inflection.



Minima: $x = \frac{\pi}{2} \pm \frac{2n\pi}{3}$; Maxima: $x = \frac{\pi}{6} \pm \frac{2n\pi}{3}$

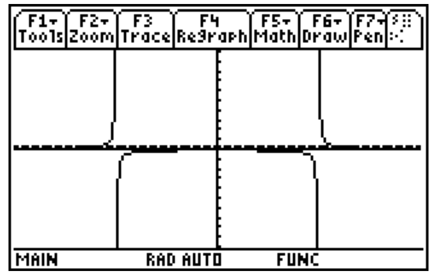
Inflection Point: $x =$ $x = \frac{n\pi}{3}$–1.

Graph j(x) = e^{4x}. There are no extrema.



$j'(x) = 4e^{4x} > 0$ so there are no critical points. Also, the function is always concave up so there are no points of inflection.

Graph $k(x) = \dfrac{1}{x^2 - 9}$.



Note that there is a maximum at $x = 0$ but no minimum. There are two vertical asymptotes ($x = 3$, $x = –3$) where the function is not defined. There are no points of inflection because the function changes concavity at the asymptotes.

Remind students that local minimum and local maximum mean just that. The values around $x = 0$ are smaller than at $x = 0$ but all of them are smaller than the values of the function when $x > 3$ or $x < –3$.