Exponential Growth

Student Worksheet

Name ______________________

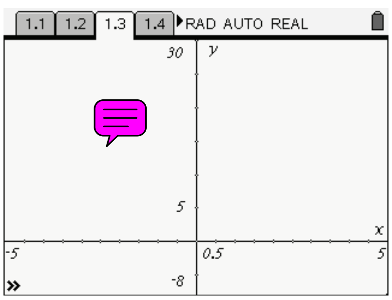
Class ______________________

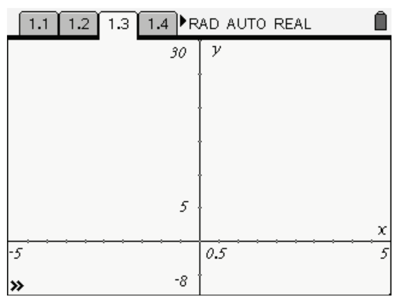
Problem Statement

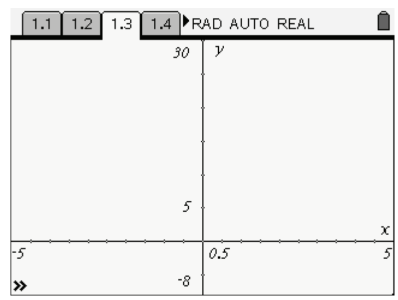
As you have probably noticed by now, graphs of functions can often be categorized into basic families based on their geometric behavior. In addition, the algebraic form of the function can give us insight into how the graph will behave. In this investigation, we will explore one family of functions and make connections between the algebraic and graphical representations of them.

1. On page 1.3 of the *CollegeAlg_ExpGrowth.tns* file, the graph of the function $\mathbf{y}(x) = b^x$ (for $b > 0$) is displayed. The slider bar allows the value of b to be changed. Change the value of b using the slider. Observe how the value of b affects the shape of the graph. Specifically, investigate the effects of b on the graph under the following conditions: $0 < b < 1$, $b = 1$, and $b > 1$ and respond accordingly below.

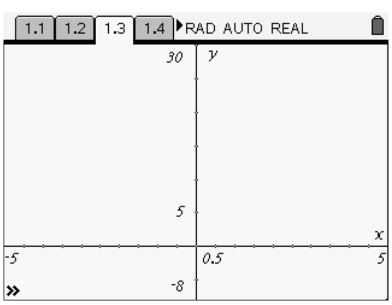
   a. Describe the behavior of the graph when $0 < b < 1$, sketching some of your graphs to illustrate. Explain why you think the graph behaves as it does.

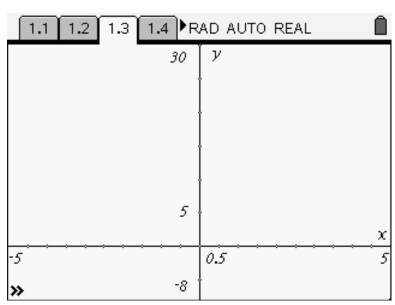


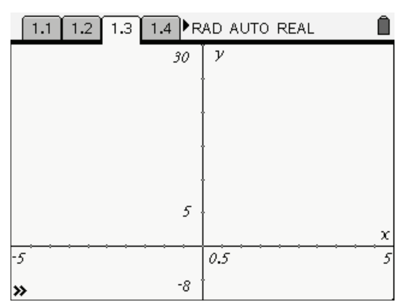




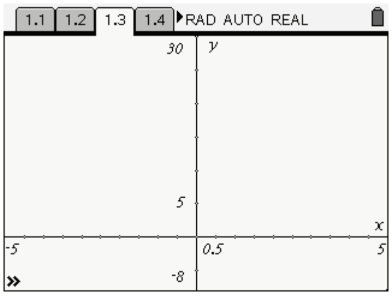
   b. Describe the behavior of the graph when $b = 1$, sketching some of your graphs to illustrate. Explain why you think the graph behaves as it does.

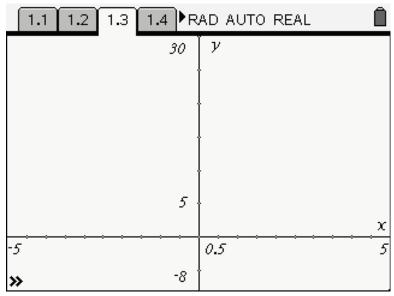


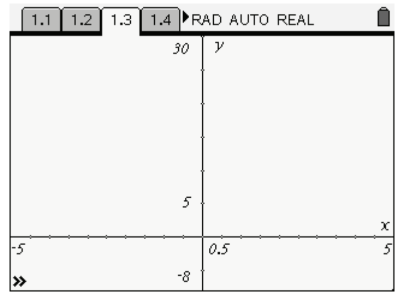




c. Describe the behavior of the graph when $b > 1$, sketching some of your graphs to illustrate. Explain why you think the graph behaves as it does.







d. Explain what would happen if b were negative.

2. On page 2.2 of the *CollegeAlg_ExpGrowth.tns* file, you will see the graph of **f1**$(x) = 2^x$. A tangent line to the curve has been constructed at point T. The slope of the tangent line (which can also be thought of as the instantaneous slope of the function) is displayed. Grab and drag point T along the curve and observe the changing slope.

a. As you drag the point from left to right, what happens to the values of the slope? Explain what the values of the slope tell you about how the function is changing.

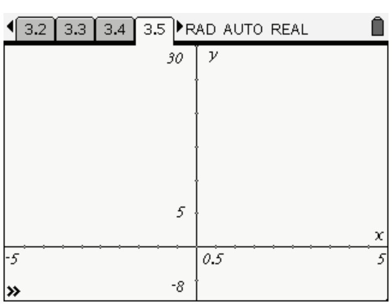
b. In the case where $b = 2$, are there any values that the slope can never equal? Explain why your observation would be so.

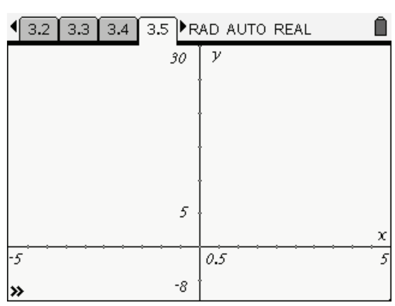
3. On page 3.2 of the *CollegeAlg_ExpGrowth.tns* file, you will again see the graph of **f1**$(x) = b^x$. This time, you will explore the relationship between the slope of the tangent line and the value of the function. Pick a value for b and then move point T. Observe the relationship between the slope and the y-value. Slope is a measure of rate of change in a function. In this example, sometimes the slope is **less than** y, and sometimes it is **greater than** y. There is only one value of b for which the rate of change of the function $y = b^x$ at any point is equal to the value of the function itself.

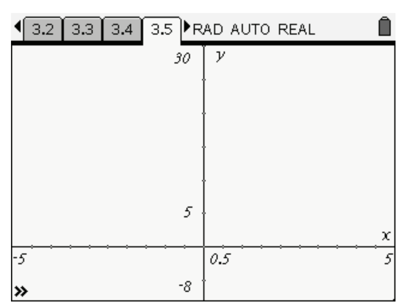
a. To help explore this phenomenon, on page 3.5 of the *CollegeAlg_ExpGrowth.tns* file, a point on the "slope function" corresponding to the point, T, has been plotted (**the dark point**). The coordinates of the point are (x, slope at x). Drag the original point on the tangent line and observe the movement of the "slope point." Describe the movement relative to the shape of the original function below.

In order to better analyze the relationship between the original function and the "slope function," it would be nice to see the path the slope point follows. To do this, select the **Locus** tool from under the Construction menu and then click on the slope point followed by the original point on the function. You will see the path of the slope point appear.

b. Find an approximate value of this number, b, whose slope function is the same as the function itself. (The ratio $\frac{slope}{y}$ displayed on the graph as well as the locus of the slope point should help.) If you need to obtain more decimal accuracy, move your cursor over the number until the number appears grey. Then press to get more digits. Likewise, pressing will decrease the number of digits displayed. Sketch several of your graphs below, showing some of your attempts.







c. This approximation may look familiar. Have you encountered this number before in your study of mathematics? Explain.

Notes