NUMB3RS Activity: The Art Gallery Problem
Episode: "Obsession"

Topic: Securing art galleries   **Grade Level:** 8 - 12

Objective: Students will use the TI-Navigator™ system to discover the conclusion of Chvatal's Art Gallery Theorem.

Time: 25 - 30 minutes

Materials: TI-83/84 Plus graphing calculators, TI-Navigator system, and the following activity settings files:
*Art_Gallery1.act, Art_Gallery2.act, Art_Gallery3.act, Floor_Function.act*
To download these files, go to **http://education.ti.com/exchange** and search for "6582."

Introduction

In the *NUMB3RS* episode "Obsession," pop star Skylar Wyatt is being stalked in her home by an intruder. Oddly, none of her home's security cameras pick up an image of the stalker. This leads Charlie to consider the placement of the home's cameras as it relates to Chvatal's Art Gallery Theorem. This theorem proves that a maximum of $\left\lfloor \dfrac{n}{3} \right\rfloor$ cameras are needed to guard a room comprised of *n* vertices. This is based on the assumption that each camera can see in all directions at one time, but cannot see through walls.

## Part I: Introduction to the Art Gallery Problem

1. a. Students should work individually during this part of the activity.

   b. Launch TI-Navigator on the computer and press **Begin Class** to start the session.

   c. Have students log into `NavNet` on their calculators. Instruct students to press ① `Activity Center`.

2. a. Load the ***Art_Gallery1.act*** activity settings file into Activity Center. Activity Center is set up so that students can submit up to 8 points. Press **▶ Start Activity** to display the image of a non-convex polygon on each student's calculator, as shown below.
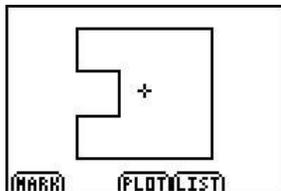
   b. Select the 'Show Student Names' box. This will allow you to display individual student results.

3. a. Explain that the polygon on their calculators represents the floor plan for an art gallery. Their task is to mark the location of cameras (with a point) that will enable the entire art gallery to be viewed.

   b. Students should find an arrangement that requires the fewest number of cameras needed so as to minimize the cost of surveillance.

   c. Press ■ **Stop Activity**. Click the 'List-Graph' tab, and highlight each student name in the Display Name column. Then click 'Hide.' Now highlight the points for one student at a time, to reveal the positioning of their cameras (hold down the shift key as you click on each of the points for an individual student).

   d. Review and discuss the camera locations for several students in the class. It should be apparent that there are a number of ways to use only two cameras.

4. a. Load the *Art_Gallery2.act* activity settings file into Activity Center to display another art gallery. Press ▷ **Start Activity** to begin.

   b. Students will see a convex polygon on their calculators, as shown below. This polygon has the same number of vertices as the previous polygon.
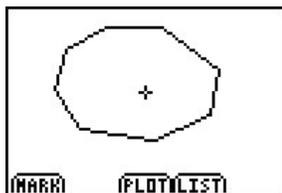
   c. Start the activity and instruct students to submit points using the same methods as Step 3.

   d. Press ■ **Stop Activity**. Review students' results as in Step 3.

   e. Discuss with students that any convex polygon requires only one camera.

   f. Also point out that this polygon has the same number of vertices as the previous polygon and therefore, there is not a one-to-one correspondence between the number of vertices and the number of cameras needed.

5. a. Load the *Art_Gallery3.act* activity settings file into Activity Center to display another art gallery. Press ▷ **Start Activity** to begin.

   b. Students will see a non-convex polygon on their calculators, as shown below.
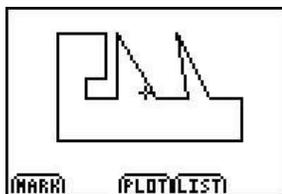
   c. Instruct students to submit the location of cameras as was done in Step 3.

   d. Press ■ **Stop Activity**. Review students' results using **Screen Capture**.

Part II: The Art Gallery Problem Solved

6. a. In Part I, students gained familiarity with the art gallery problem. Now they will work in groups to create polygons that require the <u>maximum</u> number of cameras for a given number of vertices.

  b. Divide the class into groups of 4 students.

  c. Give each group a specific number of vertices to work with (use between 9 and 18 vertices).

  d. Instruct each group to sketch (on paper) a polygon with the given number of vertices such that a <u>maximum</u> number of cameras are needed. (Students should have realized from Part I that the polygon must be non-convex.) Circulate throughout the room to check their progress.

  e. Have one student from each group exit out of `NavNet` and work from the home screen. Have these students press 2nd [QUIT] to return to TI-Navigator Home and to exit NavNet select `4: EXIT APP`.

  f. Have these students press 2nd [FORMAT] and configure their calculator to turn off the axes as seen in the screen below.

  g. Have the selected students press GRAPH and 2nd [DRAW] **2:Line(**. This will set up the calculator screen to draw lines. Tell students that they will recreate their sketches on the calculator by moving the cursor to a location on the screen and pressing ENTER to mark the location of the first vertex. Using the arrow keys, they can move to the location of the next vertex and press ENTER to lock it in place. Press ENTER again and move to the next vertex and lock it in place. Repeat this process until the polygon is completed. *[Note: The polygon should not extend into the bottom of the screen, where the x- and y-coordinates are displayed. Otherwise this portion of the polygon will be cut off in Activity Center.]*

7. a. When each group has completed their sketches, use **Screen Capture** to collect their images.

  b. Save each image to the computer and load them, one at a time, into Activity Center as a background image.

  c. Set 'Contribute' to Points and press ▶ **Start Activity** to begin. Have students use one calculator from each group to locate cameras as they did in Part I. (From Part I, the configuration still allows students to mark up to 8 points.)

  d. Press ■ **Stop Activity**.

  e. Use the *Hide* feature of Navigator to look at each group's responses.

  f. Keep a record of the number of vertices and the number of cameras needed.

  g. Use the results from this step to deduce that $\left\lfloor \dfrac{n}{3} \right\rfloor$ cameras are needed to guard a room comprised of *n* vertices.

*__The goal of this activity is to give your students a short and simple snapshot into a very extensive
math topic. TI and NCTM encourage you and your students to learn more about this topic using the
extensions provided below and through your own independent research.__*

Extensions

- This activity can also be used as an introduction to the floor function $f(x) = \lfloor x \rfloor$.

    a. Load the activity settings file ***Floor_Function.act***. This sets up Activity Center so
    that students can submit one equation.

    b. Press ▶ **Start Activity** to begin. Have one student press MATH and select **2:INT(**.
    Have the student submit the equation $Y_1 = \texttt{int(X/3)}$.

    c. Press ■ **Stop Activity** . Explain to students that this function can be used to
    model the art gallery problem, where the *x*-values represent the number of vertices
    and the *y*-values represent the number of cameras needed. Make sure it is clear to
    students that only integer values of *x* make sense in the context of the problem.

    d. Students may be confused as to the value of this step function at its discontinuities.
    Make sure that students understand that at these points on the graph, the
    corresponding *y*-value is $x \div 3$.

- Two additional *NUMB3RS* activities for "Obsession" explore the Art Gallery Problem.
  These activities can be downloaded for free from the Web sites listed below.

    **http://www.cbs.com/primetime/numb3rs/ti/activities/
    Act2_GuardingtheGoodsI_Obsession_final.pdf**

    **http://www.cbs.com/primetime/numb3rs/ti/activities/
    Act3_GuardingtheGoodsII_Obsession_final.pdf**

- If you would like to learn more about TI-Navigator, visit **http://education.ti.com/navigator**.