

Name _	
Class _	

Problem 1 – The Derivative of $y = \ln(x)$

If (x, y) is a point on y = f(x) and y = g(x) is the inverse of f(x), then (y, x) is a point on g(x). We know that $e^0 = 1$ and $\ln(1) = 0$, so (0, 1) is a point on $y = e^x$ and (1, 0) is a point on $y = \ln(x)$. We could do this for several points and keep getting the same inverse results.

Thus, if $y = e^x$, then $x = e^y$ will be equivalent to $y = \ln(x)$ because they are inverses of one another. Now we can take the implicit derivative with respect to x of $x = e^y$.

$$x = e^{y} \to 1 = \frac{dy}{dx} \cdot e^{y} \to 1 = \frac{dy}{dx} \cdot x \to \frac{1}{x} = \frac{dy}{dx}$$

Use the **limit** command to test this formula. Be careful with your parentheses.

• Find
$$\lim_{h\to 0} \frac{\ln(2+h) - \ln(2)}{h}.$$

• Do the same with
$$\lim_{h\to 0} \frac{\ln(3+h) - \ln(3)}{h}$$
.

• What is
$$\lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h}?$$

Use the derivative command to find the derivative of the logarithmic function
f(x) = ln(x).

F1+ F2+ ToolsA19ebi	raCalcOtherP	FS F6 r9ml0Clea	n Up
t((ln(MAIN	2+h)-1n() Rad auto	2))/h,h FUNC	1, ₫) 0/30

F1+ F2 Too1s A19e	2+ F3+ F4+ braCalcOtherP	F5 F6 r9ml0C1ear	, UP
1/1 /	<u></u>		
<u>d(ln(×</u> Main	:),X) Rad Auto	FUNC	0/30

Problem 2 – The Derivative of $y = \log_a(x)$

What happens if our logarithm has a base other than *e*? We need to know how to take the derivative of the function $y = \log_a(x)$.

First we want to compare y1 = ln(x) and $y2 = log_2(x)$.

To enter $log_2(x)$, use the alpha keys to spell out **log**.

Within the parentheses, enter the expression, then the base.

F1+ F2 Too1s Zoo		v (;2) 34 of \$10	
+PLOTS			
∕y1=1ı	n(x)		
ų́2=`			
ū3=			
- 44 ×			
1 ū5=			
- 46=			
ŭ7=			
<u>`</u> è			
$ \underline{y}2(\mathbf{x})\rangle$	=log(x,2)		
MAIN	RAD AUTO	FUNC	



Do you notice a pattern?

The Logarithmic Derivative

Graph both functions (y1 = ln(x) and y2 = log₂(x)) on the same set of axes. Sketch your graph to the right. What do you notice?

Do the same steps with y1 = ln(x) and $y3 = log_4(x)$. What do you notice?

Too1s	Zoom	Trace	Reġr	aph	Māth	Drau	vPén	: · :
<u> </u>								
MAIN		RAC) AUTI		FU	NC		
(71-) F2+	F3 Trace	(F ¹	1	F5-	F6-	167-	<u>re</u>
Tools	Zoom	Trace	Re9r	aph r	Math	Drau	vPen	÷.
				Ē				
				E				
				ŧ.				
				ŧ				
				ŧ				
MAIN		DOF) AUTI	<u> </u>	FU	ur.		

F1+ F2+ F3 F4 F5+ F6+ F7+8:

Simplify the following ratios.

ln(<i>x</i>)	ln(<i>x</i>)	ln(<i>x</i>)	
$\overline{\log_2(x)}$	$\overline{\log_4(x)}$	$\overline{\log_a(x)}$	

Sometimes the ratio $\frac{\ln(x)}{\log_a(x)}$ is written as $\ln(x) = \ln(a) \cdot \log_a(x)$. We can rewrite this ratio as

 $\log_a(x) = \frac{\ln(x)}{\ln(a)}$ and call it an identity.

Graph the following functions on the same set of axes: y1 = ln(x), y2 = ln(2) ⋅ log₂(x), y3 = ln(3) ⋅ log₃(x). What was the result?

F1+ F2+ Tools Zoom	F3 F4 TraceRe9raph	F5+ F6+ F7+S MathDrawPen⊳	<u> </u>
MAIN	RAD AUTO	FUNC	

What happens when we take the derivative of $y = \log_a(x)$. Use the **derivative** command to find the derivatives of the functions below.

$$f(x) = \log_2(x)$$
 $g(x) = \log_3(x)$ $h(x) = \log_a(x)$

The Logarithmic Derivative

What does $\log_2(e)$ equal? If we use the formula from earlier in this class, we get $\log_2(e) = \frac{\ln(e)}{\ln(2)} = \frac{1}{\ln(2)}$.

Therefore, the general result is $y = \log_a(x) \rightarrow \frac{dy}{dx} = \frac{1}{(x \ln(a))}$.

Problem 3 – Derivative of Exponential and Logarithmic Functions Using the Chain Rule

Now we want to take the derivative of more complicated functions:

Recall: $y = a^u \rightarrow \frac{dy}{dx} = a^u \frac{du}{dx}$ where *u* depends on *x*.

• Suppose that *y* = log_{*a*}(*u*), where *u* depends on *x*. Using the chain rule, take the derivative of this function.

Find the derivative of the following functions with the chain rule.

Identify u(x) and a for each function before you find the derivative.

