The Logarithmic Derivative

Name ______________________

Class ______________________

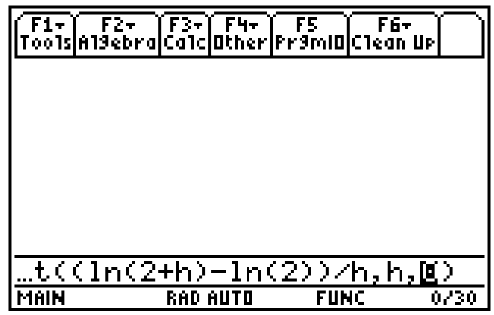
Problem 1 – The Derivative of *y* = ln(*x*)

If (*x*, *y*) is a point on *y* = *f*(*x*) and *y* = *g*(*x*) is the inverse of *f*(*x*), then (*y*, *x*) is a point on *g*(*x*). We know that $e^0 = 1$ and ln(1) = 0, so (0, 1) is a point on $y = e^x$ and (1, 0) is a point on *y* = ln(*x*). We could do this for several points and keep getting the same inverse results.

Thus, if $y = e^x$, then $x = e^y$ will be equivalent to *y* = ln(*x*) because they are inverses of one another. Now we can take the implicit derivative with respect to *x* of $x = e^y$.

$$x = e^y \rightarrow 1 = \frac{dy}{dx} \cdot e^y \rightarrow 1 = \frac{dy}{dx} \cdot x \rightarrow \frac{1}{x} = \frac{dy}{dx}$$

Use the **limit** command to test this formula. Be careful with your parentheses.

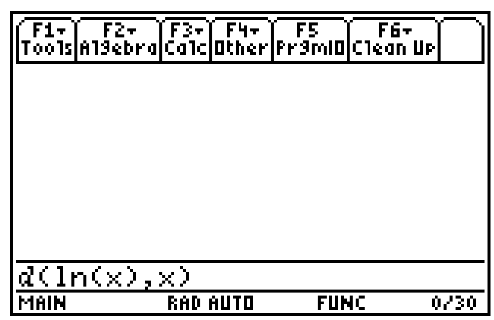


- Find $\lim_{h \to 0} \frac{\ln(2+h) - \ln(2)}{h}$.

- Do the same with $\lim_{h \to 0} \frac{\ln(3+h) - \ln(3)}{h}$.

- What is $\lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$?

- Use the **derivative** command to find the derivative of the logarithmic function *f*(*x*) = ln(*x*).



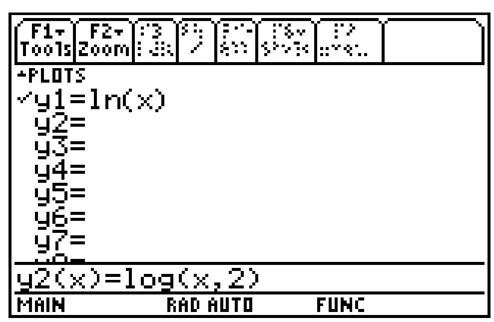
Problem 2 – The Derivative of $y = \log_a(x)$

What happens if our logarithm has a base other than *e*? We need to know how to take the derivative of the function $y = \log_a(x)$.

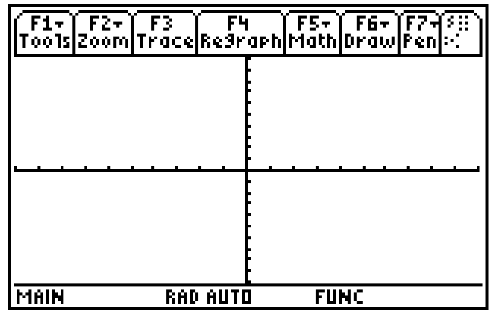
First we want to compare **y1** = ln(*x*) and **y2** = $\log_2(x)$.

To enter $\log_2(x)$, use the alpha keys to spell out **log**.

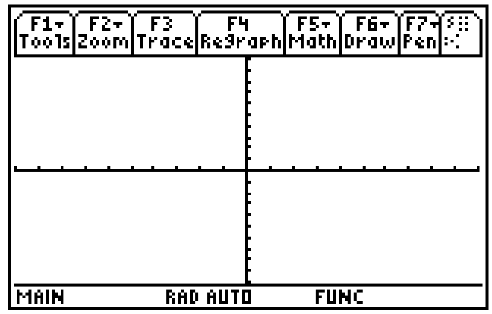
Within the parentheses, enter the expression, then the base.



- Graph both functions (**y1** = $\ln(x)$ and **y2** = $\log_2(x)$) on the same set of axes. Sketch your graph to the right. What do you notice?



- Do the same steps with **y1** = $\ln(x)$ and **y3** = $\log_4(x)$. What do you notice?

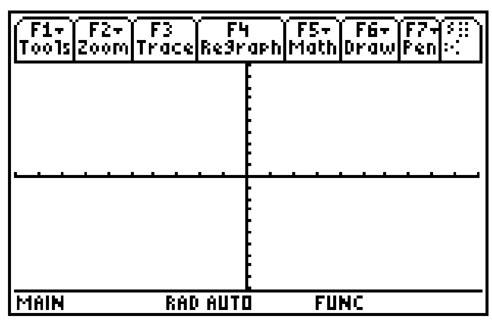


- Simplify the following ratios.

$\dfrac{\ln(x)}{\log_2(x)}$ $\qquad\dfrac{\ln(x)}{\log_4(x)}$ $\qquad\dfrac{\ln(x)}{\log_a(x)}$

Sometimes the ratio $\dfrac{\ln(x)}{\log_a(x)}$ is written as $\ln(x) = \ln(a) \cdot \log_a(x)$. We can rewrite this ratio as $\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$ and call it an identity.

- Graph the following functions on the same set of axes: **y1** = $\ln(x)$, **y2** = $\ln(2) \cdot \log_2(x)$, **y3** = $\ln(3) \cdot \log_3(x)$. What was the result?



What happens when we take the derivative of $y = \log_a(x)$. Use the **derivative** command to find the derivatives of the functions below.

$f(x) = \log_2(x)$ $\qquad g(x) = \log_3(x)$ $\qquad h(x) = \log_a(x)$

- Do you notice a pattern?

What does $\log_2(e)$ equal? If we use the formula from earlier in this class, we get
$\log_2(e) = \frac{\ln(e)}{\ln(2)} = \frac{1}{\ln(2)}$.

Therefore, the general result is $y = \log_a(x) \rightarrow \frac{dy}{dx} = \frac{1}{(x\ln(a))}$.

Problem 3 – Derivative of Exponential and Logarithmic Functions Using the Chain Rule

Now we want to take the derivative of more complicated functions:

Recall: $y = a^u \rightarrow \frac{dy}{dx} = a^u \frac{du}{dx}$ where u depends on x.

- Suppose that $y = \log_a(u)$, where u depends on x. Using the chain rule, take the derivative of this function.

Find the derivative of the following functions with the chain rule.

Identify $u(x)$ and a for each function before you find the derivative.

- $f(x) = 5^{(x^2)}$ $u(x)$=__________ a = ____________

  $f'(x)$ = ________________

- $g(x) = e^{(x^3+2)}$ $u(x)$=__________ a = ____________

  $g'(x)$ = ________________

- $h(x) = \log_3(x^4 + 7)$ $u(x)$=__________ a = ____________

  $h'(x)$ = ________________

- $j(x) = \ln\left(\sqrt{x^6+2}\right)$ $u(x)$=__________ a = ____________

  $j'(x)$ = ________________