Chi-Square Distributions

ID: 9738

Time required
30 minutes

Activity Overview

In this activity, students first compare the chi-square distribution to the standard normal distribution (for five degrees of freedom). They will also determine how the Chi-Square distribution changes as they increase the degrees of freedom. Students then confirm critical values of χ^2 and finish the activity by constructing confidence intervals for real-life scenarios.

Topic: Continuous Distributions and their Properties

- *Graph the probability density function of the χ^2 distribution*
- *Calculate probabilities*
- *Calculate a confidence interval*

Teacher Preparation and Notes

- *Students should already be familiar with the normal distribution and it characteristics, as well as finding and interpreting confidence intervals for normal distributions (with known population standard deviation or sample size larger than 30).*
- *Using a confidence interval to make a decision, as done in Problem 3, is a precursor to hypothesis testing.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9738" in the keyword search box.***

Associated Materials

- *Chi_Square_Distributions_Student.doc*
- *Chi_Square_Distributions.tns*
- *Chi_Square_Distributions_Soln.tns*

Suggested Related Activities

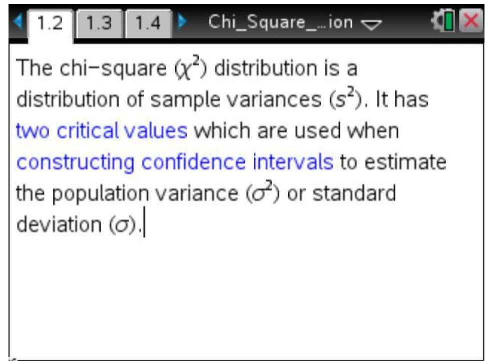
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *F Distribution (TI-Nspire technology) — 9781*
- *Is it Rare? (TI-Nspire technology) — 9095*
- *Candy Pieces (TI-Nspire technology) — 9997*
- *Cancer Clusters (TI-Nspire technology) — 9996*

Problem 1 – Characteristics of the Chi-Square Distribution

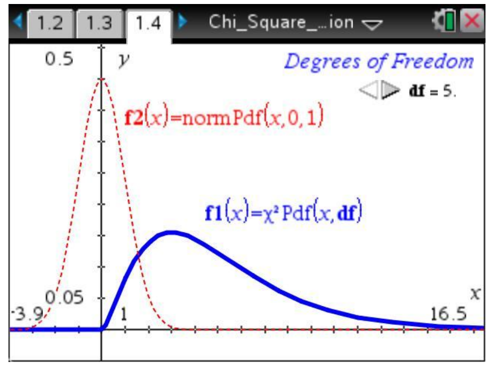
Step 1: Discuss the basics of the chi-square distribution (describes variance) and that the degrees of freedom (d.f.) for this distribution are one less than the sample size.

Point out that the chi-square distribution requires that samples be taken from a normally distributed population.

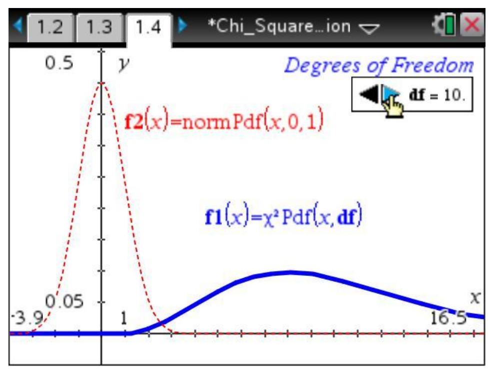


Step 2: Students are to advance to page 1.4 to view the chi-square distribution for *n* = 6 in bold. They can discuss how this distribution is different from the standard normal distribution.

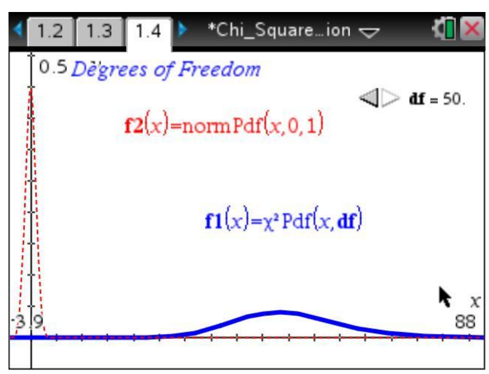
The chi-square distribution is skewed to the right and none of the values are negative.



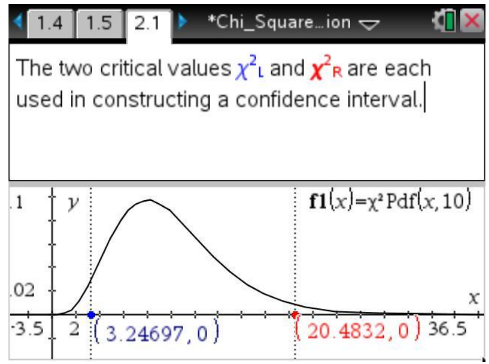
Step 3: Students are to **click** the **df** arrows to change the degrees of freedom to 10, 25, and 50 and observe how the distribution changes.



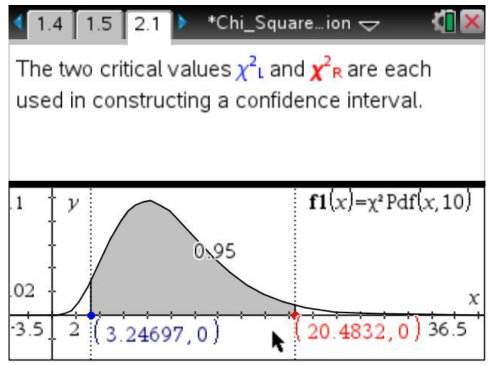
Students will need to adjust their viewing window to see the distributions for the greater degrees of freedom. They will see that as the number of degrees of freedom increases, the distribution becomes less skewed and more symmetric. They may also conjecture that the mean equals the number of degrees of freedom (this is true).



Problem 2 – Critical Values for a Chi-Square Distribution



Step 1: Students are to advance to page 2.1. Explain that the L and R notation in the critical values represent left and right.

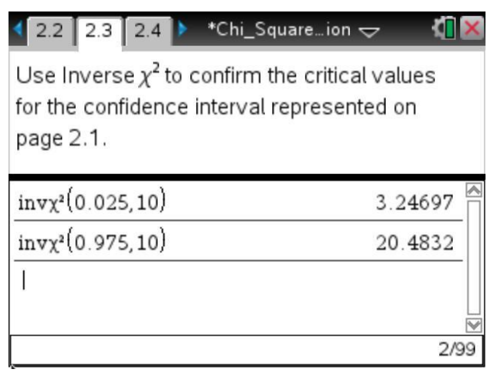


Step 2: Students can use the **Integral** tool (menu **> Analyze Graph > Integral**) to find the area between the critical values.

(**Note:** They should first select the *Graph* application on the screen.)

They need to select the left critical point and then the right critical point. (Going from right to left will show a negative area, although the absolute value of the area will be correct.)

Students will see that 95% of the area lies between the critical values, so the graph represents a 95% confidence interval. If desired, you can have students confirm that the remaining 5% is distributed equally between both tails. (Students may need to change the window to extend far enough out the right.)

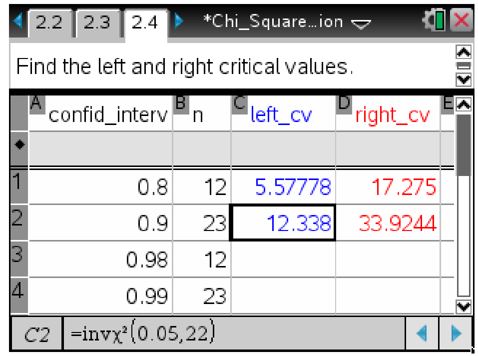


Step 3: Instruct students to use the *Calculator* application on page 2.3 to confirm the critical values for a 95% confidence interval when *n* = 11 (d.f. = 10).

The Inverse χ^2 command can be selected from the menu (menu **> Statistics > Distributions > Inverse** χ^2) or typed as **invchi2**. Either way, first enter the area to the left of the critical value and then the degrees of freedom.

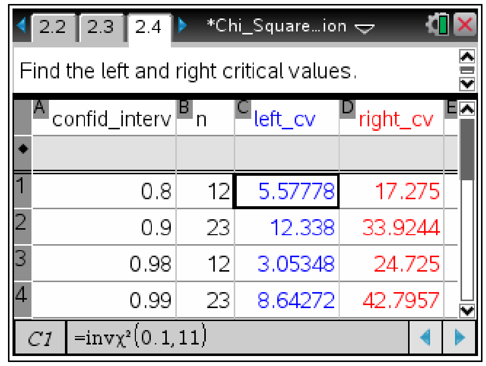
Step 4: Let students work independently to find the critical values for the confidence intervals and sample sizes listed on page 2.4. Students will need to enter an " = " sign before entering the *invX*^{2} command to obtain a result.

If checking answers against a table in a book, be aware that some books give the area to the right of each critical value, rather than to the left.



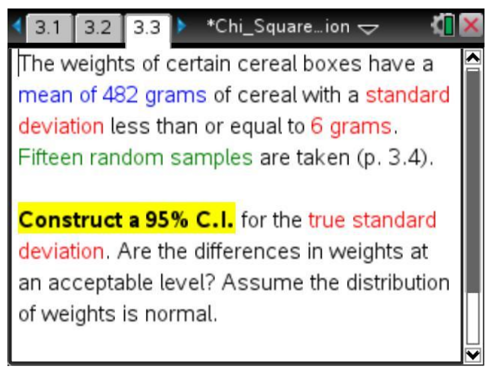
Students are to compare the critical values for *n* = 12 for the 80% and 98% levels and explain why, at the 98% level, the left value is smaller and the right value greater.

(greater confidence = more room for error)



Problem 3 – Constructing a Confidence Interval

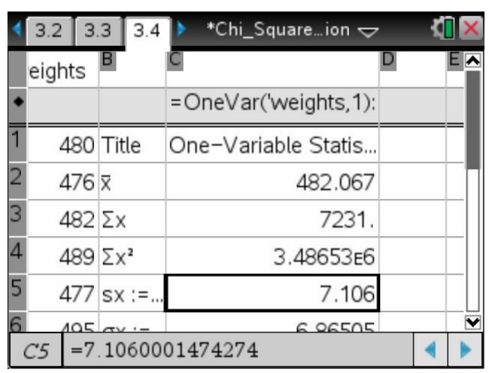
Step 1: Introduce the formulas for the confidence intervals for the population variance and standard deviation (shown on pages 3.1 and 3.2). Then students are to read the problem on page 3.3. Ask them what they need to be able to construct the confidence interval.



Step 2: Students can find the sample standard deviation on page 3.4 by selecting menu **> Statistics > Stat Calculations > One-Variable Statistics**.

The standard deviation of the sample is about 7.106.

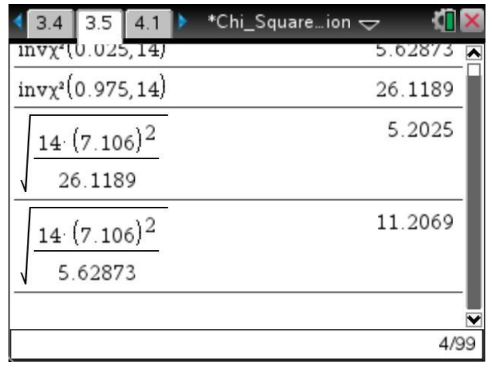
Make sure the students set the xList to "weights."



Step 3: On page 3.5, students should find each of the critical values and then use them to construct the 95% confidence interval.

We are 95% confident that the true standard deviation lies between 5.2025 and 11.2069. Since 6 is in the interval, the weights are considered consistent, that is, the differences are acceptable.

Point out that if the standard deviation had to be less than or equal to 5 grams, then the differences would not be considered acceptable because 5 is outside the interval.



Problem 4 – Practice

Have students work though the questions in Problem 4 on their own. When everyone is finished, review the answers and address any questions.