## Case File 13

## Life in the Fast Lane: Using skid marks to determine vehicle speed

Estimate the speed of a vehicle from the length of its skid marks.

$$
\left\{\begin{array}{l}
\text { Car Chase Dead Ends } \\
\text { Police left empty handed as thieves escape on pier } \\
\text { HARBERTVILLE, Friday: Downtown was thrown into chaos last night as a dramatic police chase snarled traffic } \\
\text { and sent pedestrians diving for cover. The chase was a result of a robbery at the First United Bank on Maple Blvd. } \\
\text { Three burglars robbed the bank and escaped in an unmarked black luxury sedan. } \\
\text { The pursuing officers could not determine the exact make and model of the car or read the dealer's license plate taped } \\
\text { to the back window. The police lost sight of the vehicle after the thieves reached the waterfront, skidded to a stop } \\
\text { near the end of the pier, and turned into the adjacent loading docks. } \\
\text { The only evidence left at the scene was the car's skid marks. Police have located three cars that match the general } \\
\text { description of the getaway car and that were purchased recently from Luxury Motors, the city's only luxury car } \\
\text { dealer. Police are now working to narrow the list of suspects. }
\end{array}\right.
$$

TO: Chief Detective
FROM: Crime Scene Investigatory Team

Please examine the attached photo of vehicle skid marks, recorded as 738 ft. We estimate that the vehicle was moving at top speed before skidding.

The following persons recently purchased similar sedans that fit the description of the getaway car. Their top speeds are noted.
D. J. Bitterman: Barrington Twister, top speed 105 mph Latoya Sikes: SMC Shade, top speed 115 mph Anwar Al-Dosari: Turner Black Bolt, top speed 140 mph

## Forensics Objective

- determine the speed of a vehicle before its brakes were applied


## Science and Mathematics Objectives

- determine the coefficient of friction between a vehicle and a road surface
- convert between SI units and Imperial units
- rearrange equations to solve for different variables


## Materials (for each group)

- TI-83/TI-84 Plus ${ }^{\text {TM }}$ Family
- Vernier EasyData ${ }^{\text {TM }}$ application
- Vernier EasyLink ${ }^{\text {™ }}$
- Dual-Range Force Sensor
- Hall's carriage or heavy toy car
- thread, string, or yarn
- rubber bands (for a Hall's carriage) or tissue (for a toy car)
- meterstick or metric tape measure
- C-clamp or duct tape
- chalk or tape
- flat, smooth surface (floor or table)


## Procedure

## Part I: Determining the Coefficient of Friction

1. In order to model a vehicle that is skidding, not rolling, you need to prevent its wheels from rotating. If you are using a Hall's carriage, use a tight rubber band between the axles. If you are using a toy car, stuff tissue in the wheel wells. Make sure that the vehicle slides, without the wheels moving, when you push it along.
2. Tie a 15 cm piece of thread, string, or yarn to the front end of the vehicle.
3. Connect the Dual-Range Force Sensor to EasyLink. Set the switch on the force sensor to $\pm 10$ N. Plug EasyLink into the USB port of the calculator. The calculator will turn itself on, and the EasyData screen will appear.
4. Zero the force sensor with nothing attached to it.
a) Place the force sensor on a flat, smooth surface, with its hook parallel to the surface.
b) Select Setupl and choose option 7: Zero....
c) When the force reading is fairly constant, select Zero.
d) Carefully attach the hook on the sensor to the thread connected to the vehicle. Make sure that there is no tension on the thread. The force reading on the Main screen should be $0 \pm 0.01$.
5. Set up the EasyData App to collect one sample every 0.02 seconds for 2 seconds.
a) From the Main screen, select Setup.
b) Select option 2: Time Graph.
c) Select Edit to edit the experiment parameters.
d) Press ©LEAR and then enter $\mathbf{0 . 0 2}$ as the sample interval. Select $\sqrt{N e x t}$ to set the number of samples.
e) Press ©llear and then enter 100 as the number of samples. Select $\sqrt{\text { Next. }}$.
f) When you have confirmed that the experimental parameters are correct ( 0.02 -second sample interval, 100 samples, 2 -second experiment length), select OK to return to the Main screen.
6. When you are collecting data, it is best to have one person pulling the vehicle and a second person pushing the buttons on the calculator.
a) Using a steady force, pull on the force sensor to move the vehicle across a smooth, flat surface at as constant a speed as possible.
b) After the vehicle has started moving, select Start to begin collecting data. If you get a message about overwriting data, select $\sqrt{\mathrm{OK}}$.
c) Continue pulling the vehicle at a constant speed until the calculator screen reads Transferring Data. (Note: Make sure the force sensor remains flat on the surface at all times. If the force sensor is lifted off the surface, your force readings will be inaccurate.)
7. Your graph will look something like the one below.


The force will look very uneven because the range on the $y$-axis is very small. As long as you pull with a constant force, your results should be fine. If your graph has any spikes or dips that are much larger than the rest, select $\sqrt{\text { Main }}$ to return to the Main screen and repeat steps 5 and 6.
8. Compute the average force with which you pulled the vehicle. Since the vehicle was moving at a constant speed (there was no acceleration), the force with which you pulled it was equal to the force exerted by friction (the forces were balanced).
a) Select (Anlyz), then choose option 1: Statistics.... The calculator will ask you to set the bounds for the analysis (the region of the graph that should be included in the calculations). You will be using all of the data you collected.
b) The cursor will be at the far left side of the graph. Select OK to set this point as the left boundary.
c) The cursor will now be at the far right side of the graph. Select $[\mathrm{OK}$ to set this point as the right boundary.
d) The calculator will take a second or two to calculate the mean and standard deviation. When the statistics screen appears, record the mean value in your Evidence Record. This is the average force with which you pulled the vehicle. Select $\sqrt{O K}$ then Main to return to the Main screen.
9. Weigh the vehicle.
a) Disconnect the string from both the vehicle and the force sensor. Hold the force sensor vertically at least 30 cm above your surface so that the vehicle will not touch it when suspended from the string. Attach the string to the hook on the force sensor but not to the car.
b) Select Setup and then select option 7: Zero....
c) When the force reading is fairly constant, select Zero.
d) Carefully attach the vehicle to the thread connected to the sensor. Try not to let the vehicle swing or twist on the thread.
e) When the vehicle is hanging from the force sensor and is relatively stationary, select Start to collect data on the weight of the car. If you get a message about overwriting data, select OK .
f) Let the vehicle hang from the sensor until the screen reads Transferring Data. Then carefully remove the vehicle from the hook on the force sensor.
g) Your screen will display a graph of force vs. time.
10. Repeat step 8, this time with the weight data, to have the calculator find the mean value, which is the average weight of the vehicle. Record this in the Evidence Record.
11. Divide the average force by which you pulled the vehicle, which equals the friction force, by the average weight of the vehicle. The result is $\mu$, the coefficient of friction for the vehicle and the surface it skidded on. (Note: The force and weight are given in newtons, but the resulting coefficient of friction has no units because you divide newtons by newtons.) Record the coefficient of friction in your Evidence Record.

## Part II: Finding Speed from Skidding Distance ○○ ○

12. Use chalk or tape to make a starting line for your measurements on the flat, smooth surface you are using.
13. Simulate the car chase by pushing the vehicle and letting it skid to a stop.
a) Remove the string from the vehicle. Leave the vehicle's wheels locked.
b) Place the vehicle a short distance behind the starting line, and push it toward the starting line with a constant force.
c) Release the vehicle just as its front wheels cross the starting line.
d) Wait until the vehicle is still, and mark the location of its front wheels.
e) Measure the distance from the starting line to the place where the vehicle's front wheels stopped, in centimeters. Record this as the skidding distance in your Evidence Record.
14. Use the speed equation to calculate the speed that the vehicle was traveling when it crossed the starting line and began to skid. The speed equation is $v=\sqrt{2 g \mu d}$ where $v$ is velocity (speed) at the start of the skid, $g$ is acceleration due to gravity ( $9.8 \mathrm{~m} / \mathrm{s}^{2}$ ), $\mu$ is the coefficient of friction, and $d$ is the distance (in meters) the vehicle skidded before stopping. (Note: Be sure that the units of your measurements are consistent with one another!)
15. Repeat steps 13 and 14 at least two more times to fill in your Evidence Record.

Name: $\qquad$
Date: $\qquad$

## Evidence Record

|  | Value |
| :---: | :---: |
| Average Sliding Friction (N) <br> (Equal to average force used <br> to pull vehicle) |  |
| Average weight of vehicle (N) |  |
| Coefficient of friction, $\mu$ <br> $\mu=$ Sliding friction (N) <br> Weight of vehicle (N) |  |


| Trial <br> Number | Skidding <br> Distance $(\mathrm{cm})$ | Speed <br> $(\mathrm{cm} / \mathrm{s})$ |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

## Case Analysis

1. Rearrange the speed equation to solve for $d$, the skidding distance. Show your work.
2. In most cases, an accident investigator cannot accurately compute $\mu$, the coefficient of friction. However, tests have been done to establish a range of values for $\mu$ that apply to most situations. In general, the coefficient of friction for a car on an asphalt road is between 0.5 and 0.9. Using this information and the equation from question 1 , fill in the final two columns in the table.

| Coefficient <br> of Friction | Speed of <br> Car <br> $(\mathbf{c m} / \mathbf{s})$ | Speed of <br> Car (mph) | Skidding <br> Distance <br> (cm) | Skidding <br> Distance <br> (ft) |
| :---: | :---: | :---: | :---: | :---: |
| 0.5 | 1341 | 30 |  |  |
| 0.9 | 1341 | 30 |  |  |
| 0.5 | 2012 | 45 |  |  |
| 0.9 | 2012 | 45 |  |  |
| 0.5 | 2682 | 60 |  |  |
| 0.9 | 2682 | 60 |  |  |
| 0.5 | 4024 | 90 |  |  |
| 0.9 | 4024 | 90 |  |  |

3. In general, what happens to the skidding distance when the speed doubles?
4. Give an example of a situation in which the smaller coefficient of friction (0.5) may apply.
5. Give an example of a situation in which the larger coefficient of friction (0.9) may apply.
6. Using the length of the skid marks from the crime scene report, calculate the speed of the getaway car, in miles per hour, assuming the smaller coefficient of frictionis correct.
7. Calculate the speed of the car again, in miles per hour, assuming the larger coefficient of friction is correct.
8. Considering the conditions of the road and the getaway car, which coefficient of friction do you think most likely applied during the car chase? Explain your answer.
9. Based on your answer to question 8 , which suspect's car was most likely involved in the car chase?

## Case File 13

## Life in the Fast Lane: Using skid marks to determine vehicle speed

## Teacher Notes

## Teaching time: one or two class periods

This lab introduces the coefficient of friction and uses it to determine the speed of a vehicle from the distance of its skid.

## Tips

- Preparing the vehicle and computing the coefficient of friction should take about 20 minutes. This depends on how steadily the students pull the vehicle. It may take significantly longer.
- It is important that students zero the force sensor before each force measurement because the sensor is affected by very small movements.
- You may want to review balance of forces and Newton's laws of motion with the students before they begin the lab.


## Lab Preparation

- If the vehicle you use is fairly lightweight, you may need to add some weight to it to make it slide more stably and in a straighter line.
- Use as smooth a surface as possible (not carpet) to slide the vehicles on. This will enable the students to pull the vehicle with more-constant force and reduce the error in their calculation of $\mu$.


## Background Information

If skid marks are present at the scene of an accident, one member of an investigative team is dispatched with a tool, called a trundle wheel, that looks like a walking cane with a small tire on the bottom. This tire is attached to a device similar to a pedometer. It counts the number of times the tire rotates and converts that number into distance traveled, in meters. In this way, the investigator can accurately follow the path of the skid mark and determine its length.

From the length of the skid mark, it is possible to estimate the speed of the vehicle at the start of the skid. This calculation is based on Newton's laws of motion and the formula for the kinetic energy of a moving body. The derivation of the formula is as follows:

$$
\begin{gathered}
F_{f}=\mu m g \\
E_{k}=1 / 2 m v^{2}=F_{f} d \\
1 / 2 m v^{2}=(\mu m g) d \\
v=\sqrt{2 g \mu d}
\end{gathered}
$$

where $v$ is the speed (velocity) at the start of the skid, $m$ is the mass of the vehicle, $g$ is the acceleration due to gravity ( $9.8 \mathrm{~m} / \mathrm{sec}^{2}$ ), $\mu$ is the coefficient of friction, $F_{f}$ is the force of friction, $E_{k}$ is kinetic energy, and $d$ is the length of the skid mark.

Notice that, in the final equation, the weight of the vehicle does not appear. Only the speed of the car, the acceleration due to gravity, and the coefficient of friction-which is related to the vehicle's tires and the road's surface-determine the stopping distance.

## Modifications

If time permits, you can have the students repeat the experiment on different kinds of surfaces, such as carpet or wood. They could also experiment with different weights in the vehicle. This should help to reinforce the meaning and applications of the coefficient of friction.

## Sample Data (using a Hall's Carriage)

|  | Value |
| :--- | :--- |
| Average Sliding Friction (N) <br> (Equal to average force used <br> to pull vehicle) | 0.5770 |
| Average weight of vehicle (N) | 2.092 |
| Coefficient of friction, $\boldsymbol{\mu}$ <br> $\boldsymbol{\mu}=\underline{\text { Sliding friction (N) }}$ Weight of vehicle (N) | 0.276 |


| Trial <br> Number | Skidding <br> Distance (cm) | Speed <br> $\mathbf{( c m / s )}$ |
| :---: | :---: | :---: |
| $\mathbf{1}$ | 15.8 | 92.5 |
| $\mathbf{2}$ | 19.2 | 102 |
| $\mathbf{3}$ | 22.6 | 111 |

## Case Analysis Answers

1. Rearrange the speed equation to solve for $d$, the skidding distance. Show your work.

$$
\begin{gathered}
v=\sqrt{2 g \mu d} \\
v^{2}=2 g \mu d \\
d=v^{2} \div 2 g \mu
\end{gathered}
$$

2. In most cases, an accident investigator cannot accurately compute $\mu$, the coefficient of friction. However, tests have been done to establish a range of values for $\mu$ that apply to most situations. In general, the coefficient of friction for a car on an asphalt road is between 0.5 and 0.9. Using this information and the equation from question 1 , fill in the final two columns in the table.

| Coefficient <br> of Friction | Speed of <br> Car <br> (cm/s) | Speed of <br> Car (mph) | Skidding <br> Distance <br> (cm) | Skidding <br> Distance <br> (ft) |
| :---: | :---: | :---: | :---: | :---: |
| 0.5 | 1341 | 30 | 1835 | 60.2 |
| 0.9 | 1341 | 30 | 1019 | 33.4 |
| 0.5 | 2012 | 45 | 4131 | 136 |
| 0.9 | 2012 | 45 | 2295 | 75.3 |
| 0.5 | 2682 | 60 | 7340 | 241 |
| 0.9 | 2682 | 60 | 4078 | 134 |
| 0.5 | 4024 | 90 | 16,523 | 533 |
| 0.9 | 4024 | 90 | 9179 | 301 |

3. In general, what happens to the skidding distance when the speed doubles? Stopping distance more than doubles.
4. Give an example of a situation in which the smaller coefficient of friction (0.5) may apply. Situations include wet or icy road, old tires, tires that are incorrectly inflated, and bad brakes.
5. Give an example of a situation in which the larger coefficient of friction (0.9) may apply. Situations include dry road, new and properly inflated tires, and good brakes.
6. Using the length of the skid marks from the crime scene report, calculate the speed of the getaway car, in miles per hour, assuming the smaller coefficient of friction is correct. 105 mph
7. Calculate the speed of the car again, in miles per hour, assuming the larger coefficient of friction is correct.
141 mph
8. Considering the conditions of the road and the getaway car, which coefficient of friction do you think most likely applied during the car chase? Explain your answer.
The car was new, so the tires were probably new, but the road was probably wet due to the rain. Answers may vary, but students should clearly support their arguments.
9. Based on your answer to question 8, which suspect's car was most likely involved in the car chase?
Answers will vary, depending on the answer to question 8. Given the uncertainty in the coefficient of friction, any one of the three may be the culprit.

