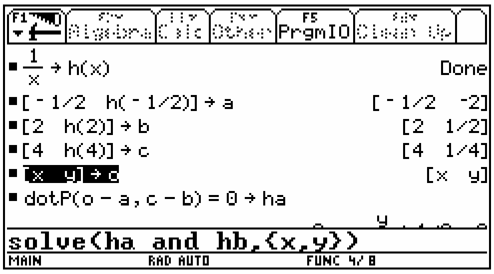
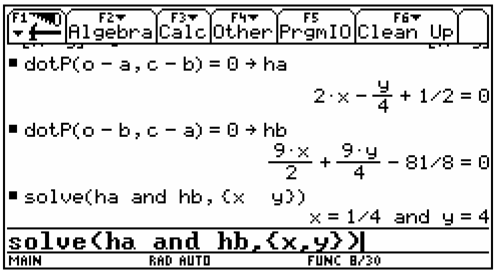
Ages 17-19 – A special attribute of triangles with their vertices on an equilateral hyberbola (C4)

This contribution is part of a paper submitted by M.Gouy, G. Huvent and A.Ladureau "Autour des triangles inscrits sur une hyperbole équilatère". It is a fine example of a meaningful use of CAS in secondary school. All basics are well known from analytic geometry working with or without vectors. The special problems can be solved very easily, even by-hand, but finding a conjecture would require some boring calculations and proving the conjecture would involve large expressions when working with linear equations. The authors present their solution on various platforms (TI, DERIVE and Cabri). Following is the TI-version with some additional comments.

a) Take any three points A, B, C on an equilateral hyperbola and find the orthocenter O of the triangle ΔABC. Do you notice any interesting results?



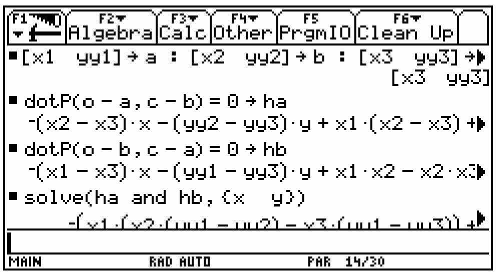


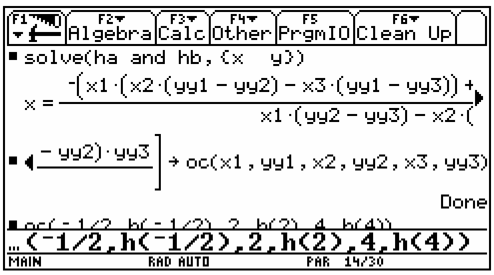
We can see that the orthocenter is a point on the hyperbola. Is this always the case?

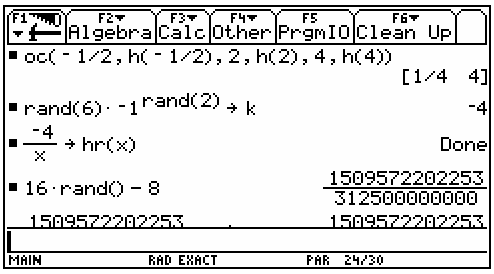
Instead of considering some other triangles we will perform another form of generalization by taking randomly chosen points on the hyperbola and a random equilateral hyperbola. We then find a formula for the co-ordinates of the orthocenter. It could be that this formula is well known by the students from earlier investigations on the triangle.

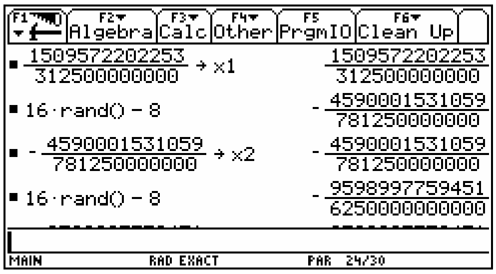
On this occasion the students are encouraged to work with self defined functions and to demonstrate their competence in using the tool by choosing appropriate variable names (eg. y1 cannot be used because it is a reserved system variable).

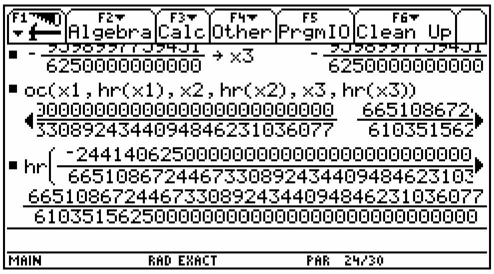
b) Take any randomly chosen points with ($-8 \le x \le 8$) on a random equilateral hyperbola $y(x) = \dfrac{k}{x}$ with k a random integer $-6 \le k \le 6$ and test this feature once more. First find a formula for the co-ordinates of the orthocenter of a triangle.

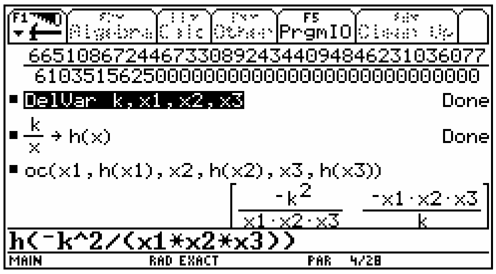


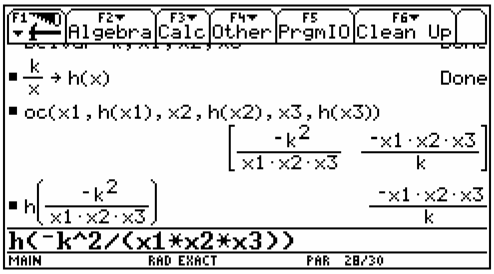








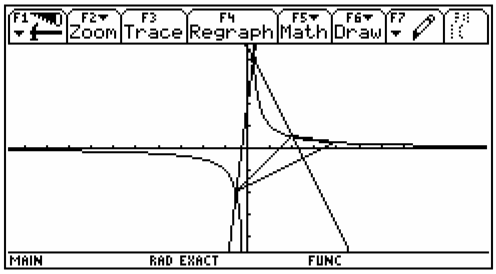




We can see that the hyperbola passes through the orthocenter. To answer the question from above; "This is always the case".

c) Give a graphic representation of the triangle, the hyperbola and the orthocenter using the data from task (1).

Producing the graphic representation needs competence in applying appropriate methods.

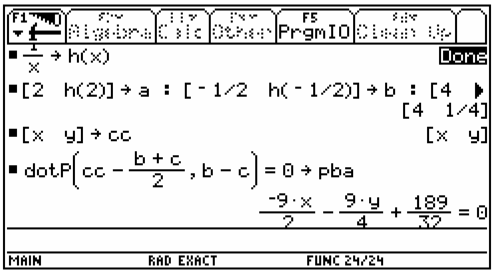


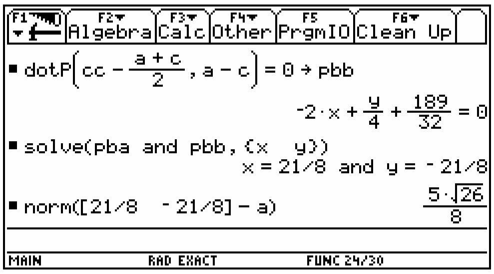
d) Find the circumcircle of the given triangle. Add the graph of this circle and point *O'* which lies symmetric to orthocenter *O* with respect to the origin. Describe your results.

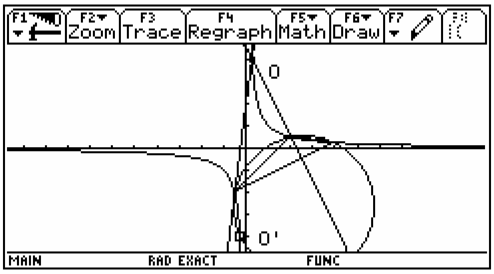
Prove your conjecture.

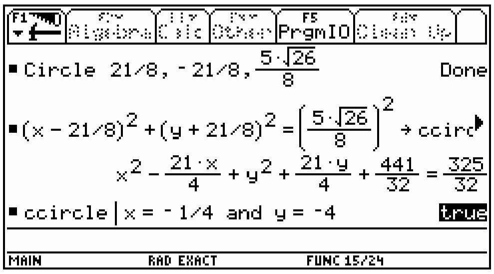
pba and pbb are the perpendicular bisectors of the sides of the triangle. Here again the student proceeds from a special situation to the generalized one. The student must apply the well known method for finding the centre of the circle and then use a strategy for solving the general proof. Access to CAS enables the student to focus on the process and not get lost in manipulations of huge expressions full of variables and indices. We are not suggesting that these skills are unnecessary in the presence of a CAS, but that there may be other occasions to practice and assess them. At this moment we want students to focus on the problem solving process rather than routine manipulations.

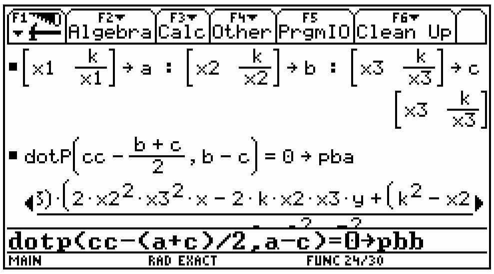
The students will find out that the hyperbola surprisingly passes through Point O. Instead of recalculation with another triangle they can immediately consider the general proof.

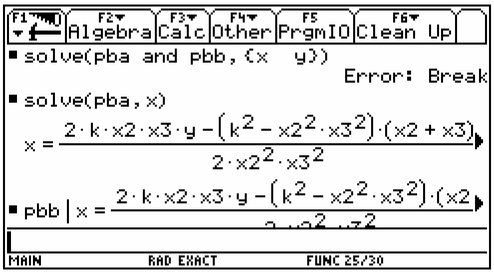


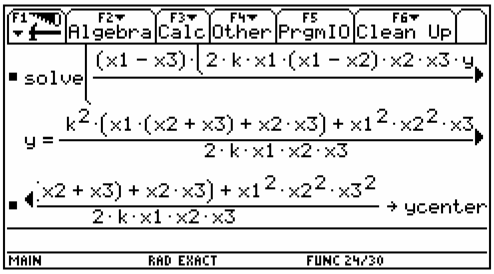


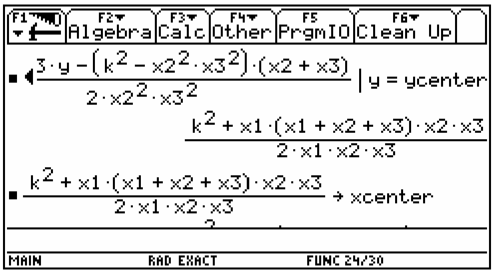


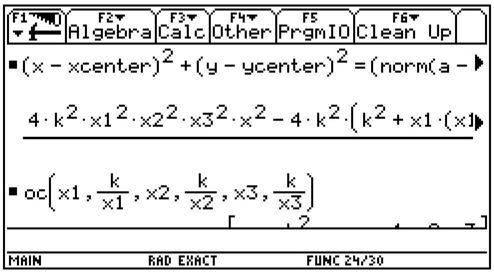


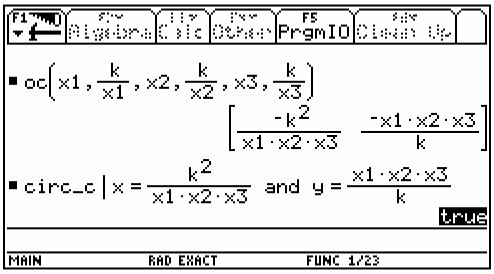




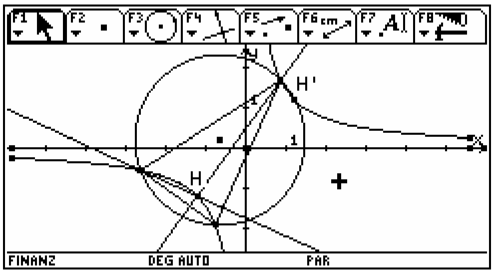


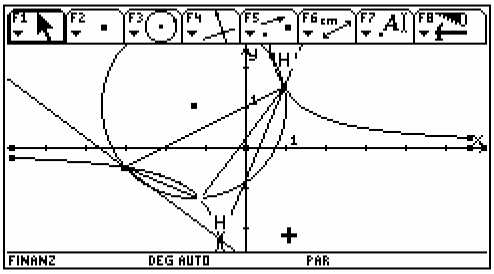






A fine accomplishment of this investigation is the realization that a Dynamic Geometry Tool such as Geometer´s Sketchpad or Cabri can be used to represent this problem. Both of these programs can be used on the Voyage 200. Doing this we demonstrate the multiple representations of one problem – analytical, graphical and dynamic– addressing various conceptual levels of the students.





M.Gouy, G.Huvent & A.Ladureau, Autour des triangles inscrits sur une hyperbole équilatère, 2003

The full article can be found at *perso.wanadoo.fr/gery.huvent* *in the rubric IREM de Lille.*