Bridge on the River Quad

ID: 9531

Time required
60 minutes

Activity Overview

First, students graph a quadratic function that models the shape of a bridge trestle. They then solve the related quadratic equation by completing the square, recording each step as they complete it. This list of steps is then generalized to deduce the quadratic formula. Finally, students store the formula in their handhelds, compare its results with those of the ***nSolve*** *command, and use it to solve several other quadratic equations.*

Topic: Quadratic Functions & Equations

- *Graph a quadratic function $y = ax^2 + bx + c$ and display a table of integral values of the variable.*
- *Convert a quadratic function $y = ax^2 + bx + c$ to the form $y=a(x-h)^2 + k$ by completing the square and deduce the formula for the roots of a general quadratic equation.*
- *Use the command nSolve to verify the roots of a quadratic equation obtained by the quadratic formula.*

Teacher Preparation and Notes

- *This activity is best used as a transition from solving quadratic equations via factoring and completing the square to using the quadratic formula. It is appropriate for students in Algebra 1. Prior to beginning this activity, students should be familiar with solving quadratic equations by completing the square.*
- *This activity requires students to graph and trace functions, define variables, and use the symbol and template palettes. If students are not familiar with these functions of the handheld, extra time should be taken to explain them.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9531" in the keyword search box.***

Associated Materials

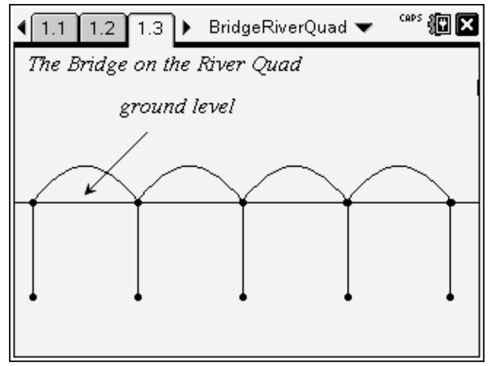
- *BridgeRiverQuad_Student.doc*
- *BridgeRiverQuad.tns*
- *BridgeRiverQuad_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *The Discriminant (TI-Nspire technology) — 9505*
- *Forms of Quadratic Equations (TI-Nspire technology) — 11712*
- *Connecting Factors and Zeros (TI-84 Plus) — 1250*

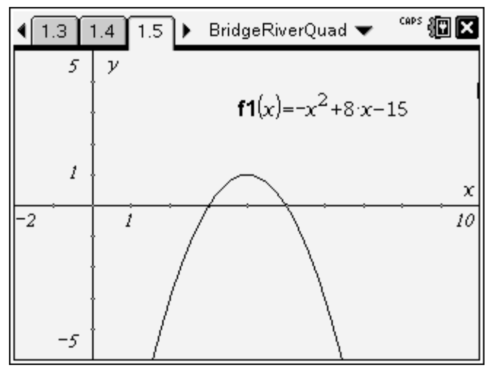
Problem 1 – Solving a quadratic equation by completing the square



Pages 1.2 and 1.3 present the scenario of the problem: a bridge with four parabolic trestles. Students are given the equation $y = -x^2 + 8x - 15$ as a model of the shape of the trestle and prompted to graph it on page 1.5.

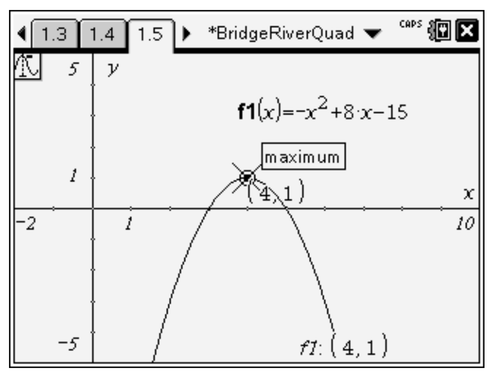
Ask: *How can you find the x-coordinates of the points where this parabola crosses the x-axis?*

Students should realize that they can set the equation equal to 0 and solve for x. Discuss the connection between the function $f(x) = -x^2 + 8x - 15$ and the equation $-x^2 + 8x - 15 = 0$. The function describes the entire graph shown on page 1.5, while the equation is true for only the two points where the graph interests the x-axis.



Students can work independently to complete the square for the equation $-x^2 + 8x - 15 = 0$. Remind them that their first step must be to divide both sides of the equation by –1, in order to make the coefficient of x^2 equal to 1.

The student worksheet provides a place for students to record their work as well as a description of each step. The steps are shown in the solutions at the end of this document.



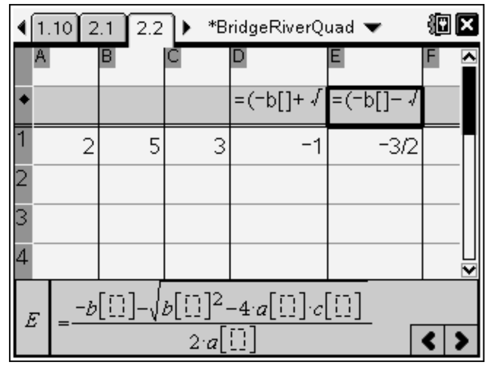
Students check their algebra by comparing the equation that results from completing the square with the coordinates of the vertex of the parabola. In a whole class setting, demonstrate the **Graph Trace** tool (**MENU > Trace > Graph Trace**). Use the right and left arrows to move the cursor along the graph. When you reach the vertex of the parabola, a capital **M** (or **maximum**) will appear on the screen. Press enter. Have students record the coordinates of this point. Remind them that an equation of the form $y = a(x - h)^2 + k$ has a vertex at (h, k).

After reading page 1.10, students should put their handhelds away and follow along as you derive the quadratic formula. Stress that you are not doing anything new: you are simply applying the process that the students just used to solve a specific equation to solve a general equation.

Deriving the Quadratic Formula

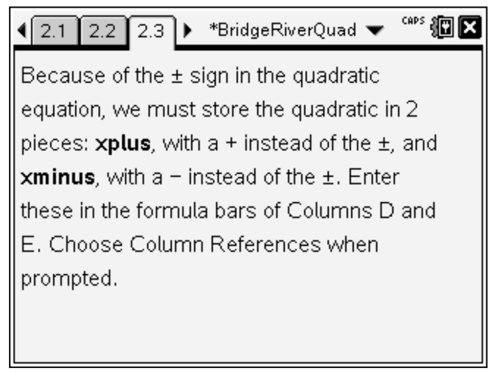
Algebra	Step
$ax^2 + bx + c = 0$	original problem
$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$	divide both sides by *a*
$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$	simplify
$x^2 + \frac{bx}{a} + \frac{c}{a} - \frac{c}{a} = 0 - \frac{c}{a}$	subtract $\frac{c}{a}$ from both sides
$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	add $\left(\frac{b}{2a}\right)^2$ to both sides
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	write the trinomial as a perfect square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{(2a)^2}$	multiply by $-\frac{c}{a}$ to get like denominators
$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	simplify $\left(\frac{b}{2a}\right)^2$
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	combine fractions with like denominators
$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$	take the square root of both sides
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	simplify ($\sqrt{4a^2} = 2a$)
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	subtract $\frac{b}{2a}$ from both sides
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	combine fractions with like denominators

Problem 2 – Using the quadratic formula



Return to the activity at page 2.1. Explain that by storing the quadratic formula in the handheld's memory, students can solve quadratic equations more quickly because they will not need to re-enter the radical and fractional expressions.

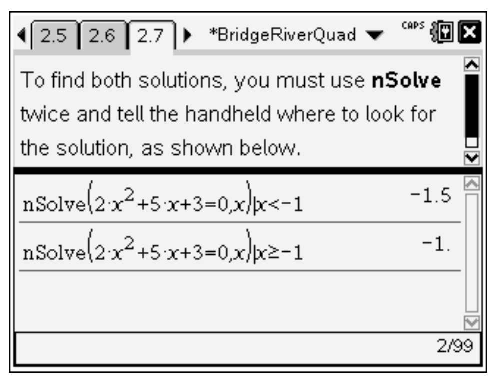
Page 2.2 shows how the coefficients *a*, *b*, and *c* should be entered in Columns A, B, and C, respectively. Page 2.3 gives a strategy for entering the formula in two parts: one for the "plus" portion of the ± sign and one for the "minus" ± portion. If necessary, remind students that the quadratic formula is really two formulas (one for each solution of the quadratic equation) written as one with a ± sign.



Students are to return to page 2.2 and enter the formulas as shown. They may need assistance using the radical and fraction templates (found in the Math Templates, accessed by pressing the templates key). Each time they are prompted, they should choose Column Reference (i.e., **a** means the value in Column A, not the variable **a**).

Students should use parentheses to make sure the correct expressions appear under the radical bar and in the denominator. The handheld provides feedback on this step by laying out the expression in algebraic notation. ***Caution:*** The formula must be *retyped* in Column E, not *copied* from Column D.

Once the formula is stored, it is a simple matter to use. Students can enter more values of *a*, *b*, and *c* in subsequent rows. Remind students that one side of the quadratic function must be equal to 0 before they can apply the quadratic formula.



Use pages 2.5–2.9 of the student TI-Nspire document to lead a discussion of the strengths and weaknesses of the **nSolve** command. For example, it is easy to use if the equation has only one solution, but difficult to use correctly if the equation has more than one solution. Demonstrate how to set upper and lower bounds and/or guesses with the **nSolve** command. Then explain that this command is best used to confirm solutions found with the quadratic formula.

Student Solutions

Problem 1

	Algebra	Step
1.	$-x^2 + 8x - 15 = 0.$	original problem
2.	$\frac{-x^2}{-1} + \frac{8x}{-1} - \frac{15}{-1} = \frac{0}{-1}$	divide both sides by $a = -1$
3.	$x^2 - 8x + 15 = 0$	simplify
4.	$x^2 - 8x = -15$	add 15 to both sides
5.	$x^2 - 8x + 16 = -15 + 16$	add $\left(\frac{-8}{2}\right)^2 = 4^2 = 16$ to both sides
6.	$x^2 - 8x + 16 = 1$	simplify
7.	$(x - 4)^2 = 1$	write the trinomial as a perfect square
8.	$(x - 4)^2 - 1 = 0$	set one side equal to 0
9.	$(x - 4)^2 - 1 = 0$	starting equation
10.	$(x - 4)^2 = 1$	add 1 to both sides
11.	$\sqrt{(x-4)^2} = \pm\sqrt{1}$	take the square root of both sides
12.	$(x - 4) = \pm 1$	simplify
13.	$x - 4 = 1$ or $x - 4 = -1$	break into two equations
14.	$x = 5$ or $x = 3$	solve each

Problem 2

1. $x = -\frac{3}{2};\ x = \frac{2}{5}$

2. $x = -1$

3. $x = \frac{4}{3};\ x = -\frac{3}{2}$

4. $x = \frac{5}{3};\ x = -1$

5. $x = -\frac{7}{11};\ x = 1$

6. $x \approx -1.317;\ x \approx 5.317$

7. $x = -\frac{1}{2};\ x = -4$

8. $x = 1;\ x = -\frac{11}{3}$

9. $x \approx -3.712;\ x \approx 1.212$