Taxicab Geometry – ID: 8643

Time required
40 minutes

Topic: Points, Lines & Planes

- *Construct a line segment joining two points with integral coordinates on a grid and measure its length.*

- *Given the coordinates of the ends of a line segment, calculate its length.*

Activity Overview

In this activity, students begin a study of taxicab geometry by discovering the taxicab distance formula. They then use the definition of radius to draw a taxicab circle and make comparisons between a circle in Euclidean geometry and a circle in taxicab geometry. Lastly, they construct taxicab perpendicular bisectors and discover that only certain pairs of points have taxicab perpendicular bisectors.

Teacher Preparation

- *This activity is designed to be used in a high-school geometry classroom.*

- *Taxicab geometry is a non-Euclidean geometry. For the sake of comparisons, students are best off if they know, before starting this activity, how to find the distance between two points, as well as the definitions and characteristics of circles and perpendicular bisectors in Euclidean geometry.*

- *The screenshots on pages 1–5 demonstrate expected student results.*

- ***To download the .8xv files and student worksheet, go to http://education.ti.com/exchange and enter "8643" in the search box.***

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*

- *The student worksheet helps guide students through the activity and provides a place for students to record their answers.*

- *The .8xv file titled Taxi1 has the initial points for Problem 1 and the .8xv file titled Taxi3 has the initial points for Problem 3. If these files are downloaded to student handhelds, skip **Step 1** for each problem.*

## TI-84 Plus Applications
Cabri Jr.

Taxicab Geometry
ID: 8643

Begin the activity with a brief introduction to taxicab geometry. Taxicab geometry is a form of non-Euclidean geometry—meaning it has different rules and characteristics than the geometry your students are probably used to studying. There are several types of non-Euclidean geometries. Much as it sounds, taxicab geometry considers what it would be like to drive a taxi through a city in which the streets are set up on a grid of parallel and perpendicular lines. Taxicab drivers can not drive through parks and buildings; therefore, they cannot travel from one intersection to another diagonally. Likewise, no diagonal movements are allowed in taxicab geometry.

Problem 1 – Taxicab distances

**Step 1:** *Note: Skip this step if the file Taxi1 is distributed to calculators.*

Open a new Cabri Jr. file. Select **Hide/Show> Axes** to show the coordinate axes. Use the **Hand** tool to drag the origin to the lower left corner of the screen.
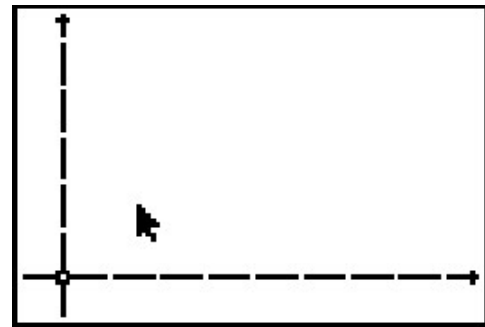
Use the **Point** tool to place two points in the first quadrant. Select **Alph-Num** to label the points *A* and *B*. Then, select **Coord. & Eq.** and show the coordinates for points *A* and *B*.

Use the **Hand** tool to drag point *A* so its coordinates are (2, 1) and drag point *B* so its coordinates are (7, 4).

Save this file as Taxi1.
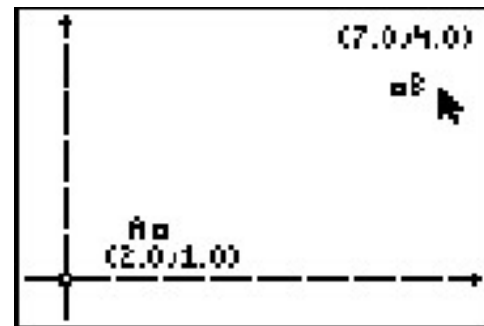
Step 2: Tell students to pretend they are taxicab drivers and need to "drive" from point *A* to point *B* by drawing horizontal and vertical segments between the points. Remind students that no diagonal movements are allowed. Tell them they can make as many, or as few, turns as they wish.

Use the **Segment** tool to construct a set of horizontal and vertical segments that connect point *A* to point *B*.

Show the coordinates of the endpoints of the segments with **Coord. & Eq**. If the endpoints are not at integer values, use the **Hand** tool to drag the points so the coordinates are all integer values and each segment is either horizontal or vertical.
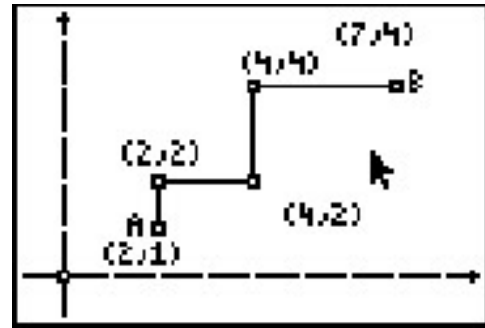
Step 3: Once students have traveled from *A* to *B*, they should measure the distance they traveled. They can do this by using the **Measure > D. & Length** tool.

Determine the greatest and shortest distances traveled in your classroom.
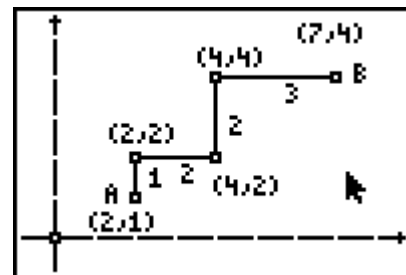
Step 4: Students should repeat the trip from *A* to *B*, but this time make a different path. Once done, have them find the distance of this second trip.

Tip: To avoid a cluttered screen, return to the screen with only points *A* and *B* using one of these methods:

- use **Undo > Explore**

- use **Hide/Show > Object**

- open the saved file *Taxi1*

Determine the shortest distance that any student made from *A* to *B*. The shortest possible distance is 8. Ask students how this can be done with just one turn. (See figure at right.)
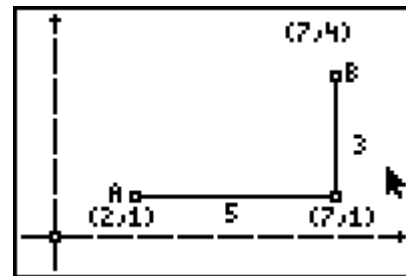
Step 5: Ask students how they would normally determine the distance between two points (outside of taxicab geometry). They might discuss the distance formula or the Pythagorean Theorem. Have students measure the distance from *A* to *B* directly and then answer Question 1 on their worksheets.
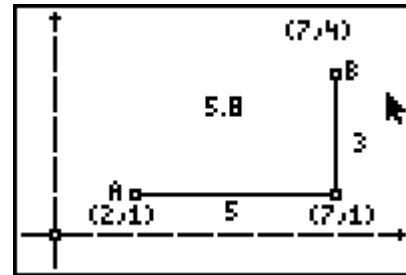
Note: Students can use the **Measure >D. & Length** tool without drawing a segment from *A* to *B*.

Step 6: Students should now clear everything off the screen except points *A* and *B*. Drag the points such that the coordinates of *A* are (1, 4) and the coordinates of *B* are (5, 2).
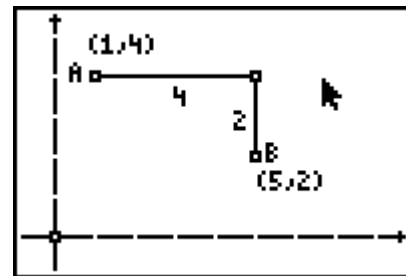
Students should construct a path from *A* to *B* that is the shortest possible path, and that only one turn is made. (There are two possible ways: down and then to the right, or to the right and then down.) Then find the length of the two segments using the **Measure > D. & Length**.

Students should try to think of a way they could find the two lengths of the segments they drew if given only these coordinates. This is Question 2 on the worksheet. (Subtract the *x*-values, take the absolute value, subtract the *y*-values, take the absolute value, and add the absolute values.)

Step 7: Again, students should return to a view with just points *A* and *B*. Using the coordinates (3, −1) and (6, 3), find the taxicab distance from *A* to *B*.
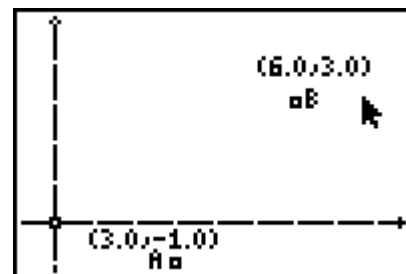
Depending on the ability level of your class, either display the following formula or have students write it themselves, based on what they already know. This is Question 3 on the worksheet.
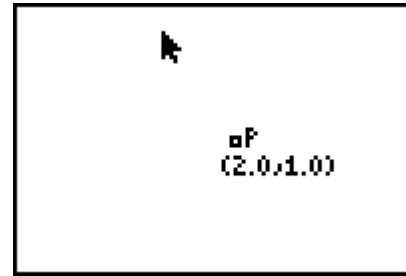
taxicab distance = $|y_2 - y_1| + |x_2 - x_1|$

Problem 2 – Taxicab circles

Step 1: Open a new Cabri Jr. file. If needed, select
Hide/Show > Axes to hide the coordinate
axes.

Use the **Point** tool to place one point in the
middle of the screen.

Label the point *P* and find its coordinates.
Drag it so the *x*-value is an integer and the
y-value is 1.0.

Step 2: Discuss with students the definition of a circle
(set of points equidistant from center point).
Students should try creating a taxicab circle
with point *P* as the center and a radius of 3.
Students will need to plot every point that is
3 units from *P*. Remind them that no diagonal
movements are permitted.

To help students with this task, suggest that
they think about the coordinates of the points
on the circle as *x* units left or right and *y* units
up or down in relation to the center. This way,
x + *y* will always equal the radius of the circle.

Note: This process is time-consuming and
may be better accomplished via teacher
demonstration.
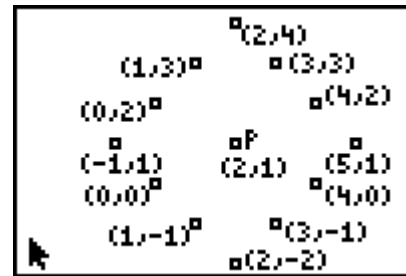
Step 3: Ask students how many points make up the
circle (12). Lead a class discussion about the
differences between a circle in Euclidean
geometry and a circle in taxicab geometry.
This is Question 4 on the worksheet. (shape,
infinite v. finite number of points)

Ask students to make a conjecture about
increasing the radius and the resulting number
of points in the taxicab circle (number of points
increase as radius increases, the number of
points equals 4 times the radius).

Problem 3 – Taxicab perpendicular bisectors

Step 1: *Note: Skip this step if the file Taxi3 is distributed to calculators.*

Open a new Cabri Jr. file. Show the coordinate axes and drag the origin to the lower left of the screen.

Use the **Point** tool to place two points on the screen. Label these points *G* and *H* and find their coordinates. Drag them so *G* is at (2, 3) and *H* is at (5, 2).

Save this file as Taxi3.
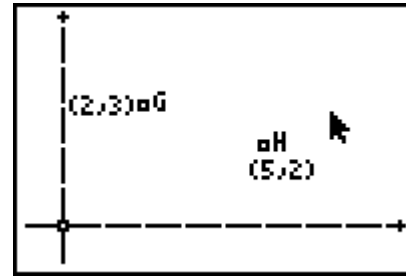
Step 2: Every point that is equidistant from points G and H should be plotted.

Once these points are plotted, explain that students have just created a taxicab perpendicular bisector. Students should discuss similarities and differences between perpendicular bisectors in Euclidean and taxicab geometry. This is Question 5 on the worksheet. (equidistance, infinite v. finite)
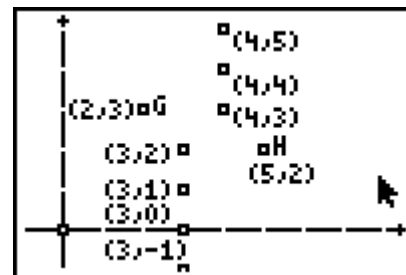
Step 3: Return to a view with just points *G* and *H*. Have students see if perpendicular bisectors can be drawn for each of the following pairs of points:

- (1, 4) and (3, 4)   yes

- (4, 2) and (3, 4)   cannot be done

- (3, 4) and (7, 2)   yes

- (4, 2) and (3, 4)   cannot be done

Note: This is a good opportunity to have students work in groups.

See if students can identify the condition required for two points to have a perpendicular bisector (the minimum distance between them must be an even number of units). This is Question 6 on the worksheet.