NUMB3RS Activity: Not So Great Expectations
Episode: "Longshot"

Topic: Probability and expected values **Grade Level**: 10 - 12
Objective: To calculate mathematical expectation as it applies to gambling, insurance, manufacturing, and other areas
Materials: TI-83 Plus/TI-84 Plus graphing calculator
Time: 15 - 20 minutes

Introduction
When Charlie explains the contents of the murder victim's notebook, he points out that the racetrack uses a *pari-mutuel* betting system, in which the odds of winning vary with the amount bet on each horse. This system does two things: it extracts a sum for overhead and profit for the racetrack; and, it splits the winnings proportionally among the winning bettors. This activity concentrates on the first part of this by investigating mathematical expectation (expected outcomes).

Discuss with Students
Review the definition of probability and how it is calculated. Time permitting, lead a discussion about how probabilities are calculated in a game as opposed to the insurance company example in the "Extensions," which is based on statistics. Stress that the word "expected" in this context does not mean that anyone actually "expects" to gain or lose a certain amount, but rather the projected *average* gain or loss over a long period of time.

Student Page Answers:
1. $\left(\frac{1}{6}\right)(\$4)+\left(\frac{5}{6}\right)(-\$1) \approx -\$0.17$ **2a.** $\frac{1}{1000}$ **2b.** *–$0.50* **3.** *about –$0.05* **4.** *Because the person already paid the premium, he or she gains only $993, so* $\left(\frac{1}{200}\right)(\$993)+\left(\frac{199}{200}\right)(-\$7) = -\$2$
5. *$2.85* **6a.** *While answers will vary, it should be clear that the $10 saved by not parking in the lot is seen as a positive gain by the parker no matter what. Thus the $5 fine has an expectation of $7.50 (–$2.50 for the ticket compared to +$10.00 for not parking in the lot), a net plus for the parker.* **6b.** *The $100 fine has an expectation of –$40.00, and is a deterrent.* **6c.** *If 'f' is the fine, and any negative expectation can be considered a deterrent, then 0.5(10) – 0.5(f) <0, or any fine greater than $10.00 can be defined as a deterrent.* **6d.** *Factors could include the cost of the towing, the value of one's time to reclaim a towed car, the inconvenience and cost while one is not using it, etc. How students value these items will affect what they see as a fair deterrent.*

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Name: ________________________________ Date: ______________

NUMB3RS Activity: Not So Great Expectations

When Charlie explains the contents of the murder victim's notebook, he points out that the racetrack uses a *pari-mutuel* betting system, in which the odds of winning vary with the amount bet on each horse. This system not only calculates proportional payoffs for the bettors, but also guarantees a profit for the racetrack owners by initially removing a certain percentage of the money bet before the remainder is split among the winners. It is this second part that makes betting a bad investment, since the bettor can always expect to lose in the long run.

This activity examines the *mathematical expectation* of the player. This is the amount a person can expect to win or (more likely) lose in the long run on average.

Consider the case of a simple fair game. Suppose two people each flip a coin, and each person bets \$1. If the coin lands on heads, the first person wins. If it lands on tails, the second person wins. The expectation of each person is found by multiplying the probability of each outcome by the amount won or lost by that outcome, then adding the products. In this example, a person who chooses heads will win $\frac{1}{2}$ of the time and lose $\frac{1}{2}$ of the time. If $P(E)$ denotes the probability of event E, then the expectation is $P(\text{heads})(\$1) + P(\text{tails})(-\$1) = \frac{1}{2} - \frac{1}{2} = 0$. In the long run, each player should win and lose about the same number of times, with an expected "profit" of \$0.

In many cases, not all of the money is returned to the participants, so the expectation can be negative – that is, each player can expect to lose in the long run.

1. Suppose that in a certain game, a player rolls an ordinary six-sided number cube. If the cube lands on 6, the player is paid \$4. If the cube lands on any other number, the player loses \$1. What is the player's mathematical expectation?

2. Suppose that in a certain state lottery, players pick a three-digit number in the range 000 to 999, and win if that number is selected.

- **a.** What is the probability of winning?
- **b.** Suppose the prize is \$499 (the player gets \$500, minus the \$1 cost of the ticket). The player loses \$1 for all other numbers. What is the player's mathematical expectation?

3. An American roulette wheel has 38 numbers on it. Suppose a person bets \$1 on a specific number. The wheel is spun, and if the number picked comes up, the person wins \$35; otherwise, the person loses his or her \$1 bet. What is the mathematical expectation?

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4. The use of probability and mathematical expectation has many applications outside of gambling. In the insurance industry, actuaries (see the “Extensions”) calculate how much to charge so that the company is competitive and can still make a profit. Suppose that for people of a certain age, the probability of dying during the coming year is $\frac{1}{200}$ and a customer of that age buys a $1,000 insurance policy for $7. What is the mathematical expectation of the customer?

5. Sometimes mathematical expectation is positive. Suppose a manufacturer makes a profit of $3 on every item sold but loses $12 on each defective one. If 1% of the items are defective, what is the mathematical expectation (profit)?

6. Mathematical expectation can also be applied to crime deterrence. For a punishment (like a fine) to be a deterrent, the probability of being caught times the fine must be greater than the expected “gain” to the criminal. For example, suppose a sports stadium charges $10 to park in its lot, but there is a street in front that does not permit parking.

- **a.** If half of the people who park illegally get tickets and the fine for a ticket is $5, is this fine a deterrent? Why?
- **b.** What if the fine is $100?
- **c.** What minimum fine do you think would be fair and still be a deterrent? Why?
- **d.** What if the punishment included more than a fine, like the car being towed? What factors could you use to determine the expectation?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

From the point of view of the "player," most of the applications in this activity have a negative expectation. From the point of view of the casino operator or the insurance company, however, these would be positive expectations, since the player's loss is their profit. Farmers and resort operators, for instance, make or lose money depending on the weather, so for them the probability of good or bad weather replaces the probability of a winning number in calculating their expected gain.

Additional Resources

This activity focuses on one part of pari-mutuel betting. The other aspect – dividing the payoff among the winners, is included an applied to jai-alai at:
http://www.cs.sunysb.edu/~skiena/jaialai/excerpts/node15.html

For an academic paper that relates pari-mutuel betting to financial markets, see:
http://dimacs.rutgers.edu/Workshops/Markets/pennock.pdf#search=%22pari-mutuel%22

For the Student

Fairly splitting wagered money plays a significant role in the history of mathematics. Two French mathematicians, Blaise Pascal and Pierre de Fermat, discussed the following problem in a series of letters in the seventeenth century. Suppose two people are playing a game in which each person has an equal chance of winning a point, and the first person to win three out of five possible points is the winner. The game is interrupted with one player leading 2 – 1. What would be a fair way to divide the money in the pot?

Make a conjecture about the answer to this. Write an argument giving reasons why this conjecture is correct using just your own thoughts. Then research Pascal and Fermat's arguments.

Related Topic

An *actuary* is an applied mathematician who uses probability to set rates for insurance companies. This occupation is frequently listed among the most lucrative and desirable jobs (see any edition of *Jobs Rated Almanac*). Students who enjoy mathematics and like to apply it may find the field very interesting. For more information, including specific information for high school students and sample exams to become an actuary, see:
http://www.beanactuary.org

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