Triangle Inequalities

ID: 9425

Time required
30 minutes

Topic: Right Triangles & Trigonometric Ratios

- *Derive the Triangle Inequality as a corollary of the Pythagorean Theorem and apply it.*
- *Derive the Triangle Side and Angle Inequality as a consequence of the Pythagorean Theorem and apply it.*

Activity Overview

Students begin this activity measuring the sides and angles of isosceles and scalene triangles to conclude that, for a triangle, congruent angles are opposite of congruent sides, the largest angle is opposite the longest side, and the smallest angle is opposite the shortest side. Students then extend a side of a triangle to discover that the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles. This leads to the fact that the measure of an exterior angle is greater than the measure of either remote interior angle. Last, students use these discovered facts to prove that the perpendicular segment from a point to a line is the shortest segment from the point to the line.

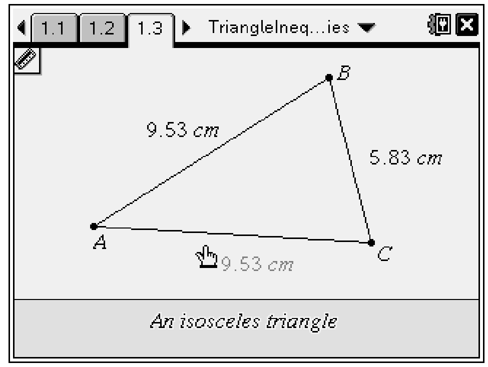
Teacher Preparation

- *This activity is designed to be used in high school geometry classroom. This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *Before beginning this activity, students should know how to classify a triangle by its side and angle measures and know that perpendicular lines intersect to form right angles.*
- *Students should also be aware of the terminology for the parts of an isosceles triangle (e.g., base, base angles, legs, vertex angle).*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9425" in the quick search box.***

Associated Materials

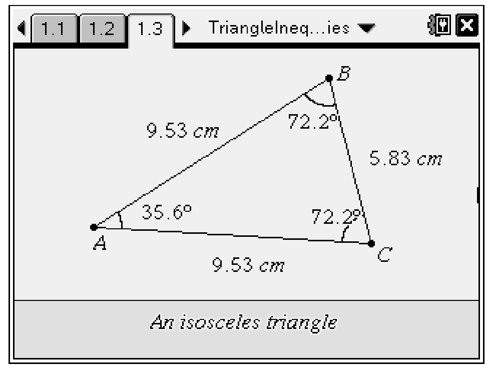
- *TriangleInequalities_Student.doc*
- *TriangleInequalities.tns*

Problem 1 – An isosceles triangle



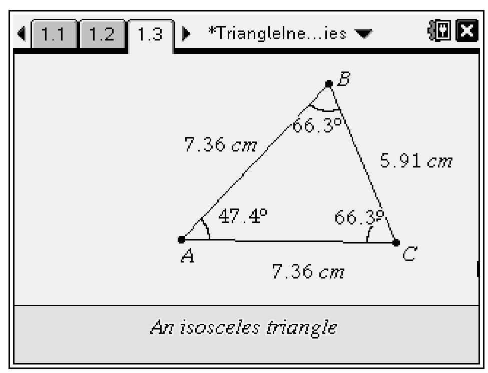
Before measuring lengths and angles on page 1.3, ask which sides of isosceles triangle △*ABC* appear to be congruent. Then have them find the lengths of each side by choosing **MENU > Measurement > Length**.

Students can then move vertices *A* and *B* to see that the triangle remains isosceles. Before continuing, ask students if they have any conjectures about the angle measures.



Next, students should measure each angle of the triangle by selecting **MENU > Measurement > Angle**. To use the **Angle** tool, you must select three points to "name" the angle, with the vertex as the second point selected. For example, to find the measure of ∠*A*—which is the same angle as ∠*BAC*—you can click on points *B*, *A*, and *C*, in that order. After the selection of the third point, the angle measure appears in gray. This label can now be moved to another location; click or press (enter) to set it in place.

Students should then move vertices *A* and *B* to make a conjecture about angle measures and the lengths of their opposite sides. They should notice that for an isosceles triangle, the angles opposite the congruent sides are congruent.



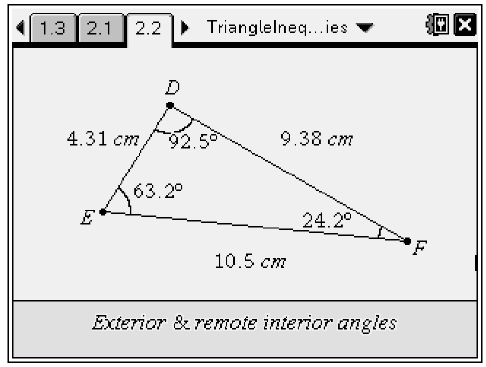
Have students continue to drag vertices to answer the following questions: *When is the measure of the vertex angle (∠A) greater than the measure of one of the base angles (∠B or ∠C)? When is the measure of the vertex angle less than the measure of one of the base angles?*

Note: Students may wish to view more decimal places of the side and angle measures. To do this, they should move the cursor over the measurement and press (+).

Students should conclude that the measure of the vertex angle is greater than the measure of a base angle when the length of the base *is greater than* the length of a leg, and the measure of the vertex angle is less than the measure of a base angle when the length of the base *is less than* the length of a leg.

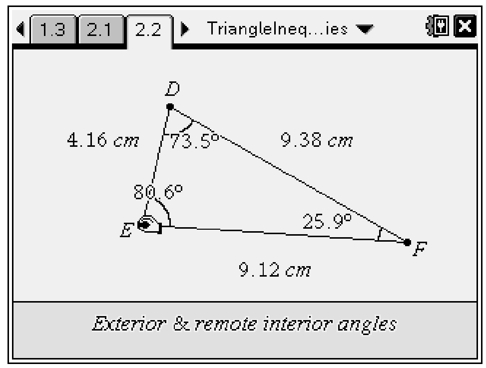
Problem 2 – Exterior and remote interior angles

On page 2.2, students are to find the measures of the sides and angles of △*DEF*. Ask them to classify the triangle according to its side lengths and angle measures (obtuse scalene).

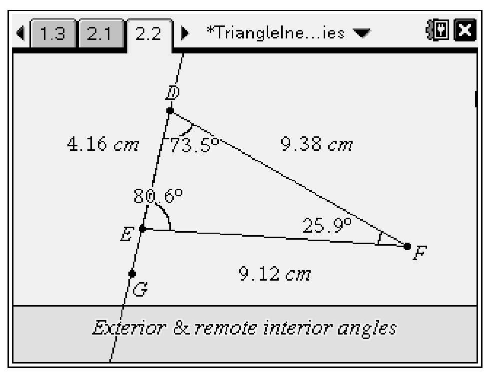


Next, students should locate the angle with the largest measure and the longest side followed by the smallest angle and the shortest side. Ask: *How is each pair of length and angle measure related?* (They are opposite each other.)

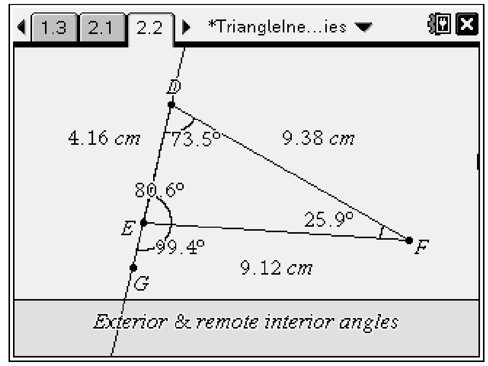
Students may then drag vertices of the triangle to see that the shortest side is always opposite the smallest angle and that the longest side is always opposite the largest angle.



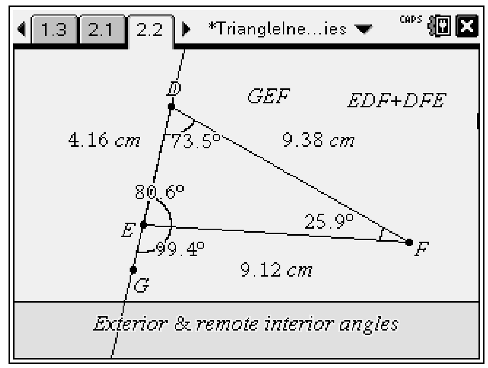
Now have students extend $\overline{DE}$. To do this, have them select the **Line** tool from the Points & Lines menu, click on point *D*, and then click on point *E*. Next have them construct a point *G* on $\overleftrightarrow{DE}$ so that *E* is between *D* and *G* using the **Point On** tool, also from the Points & Lines menu. Label the point by pressing (shift) + [G] after placing the point.



Students should now find the measure of ∠*GEF*. Ask them to make conjectures about this exterior angle and any interior angles. Allow them to drag vertices around while they conjecture.

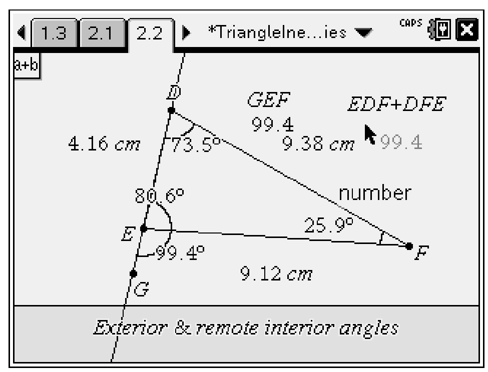


Using the **Text** tool from the Tools menu, have students display **GEF** in one text box and **EDF + DFE** in another, as shown in the screenshot to the right.



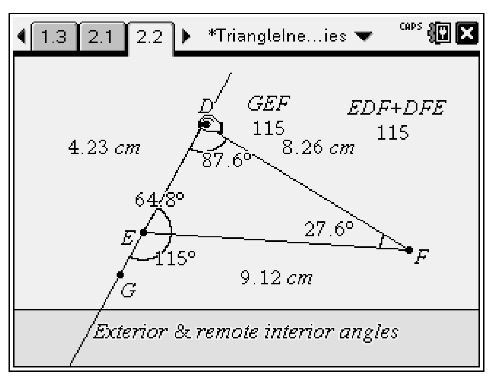
Now students can use the **Calculate** tool (also from the Tools menu) to display the value of each expression. To use the **Calculate** tool, click on the expression, and then the value of the variables as you are prompted. Then press (enter).

Dragging the vertices of the triangle once again, students should find that the two expressions are equivalent. That is, the measure of a remote exterior angle is equal to the sum of the measures of the two remote interior angles.



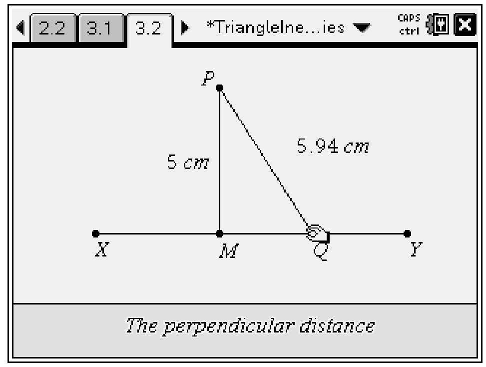
Using the fact that *m*∠*GEF* = *m*∠*EDF* + *m*∠*DFE* and that the measure of these angles are nonnegative, students are asked to deduce the inequalities shown below.

m∠*GEF* > *m*∠*EDF*
m∠*GEF* > *m*∠*DFE*



Problem 3 – The perpendicular distance

On page 3.2, $\overline{PM} \perp \overline{XY}$. (This may be confirmed by measuring the angles, if desired). Students should use the **Segment** tool from the Points & Lines menu to draw $\overline{PQ}$ and then find the lengths of $\overline{PQ}$ and $\overline{PM}$. Then have them drag point *Q* along $\overline{XY}$ to make a conjecture about the lengths of these segments.

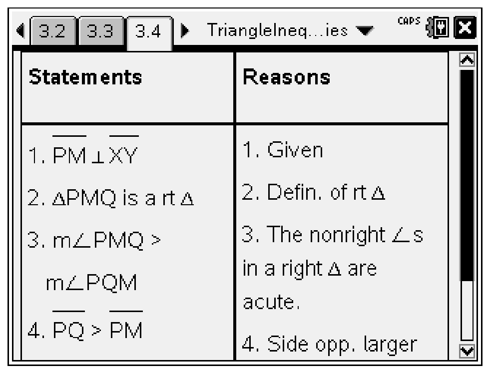


Students may need to display more decimals points in the measurements when making their conjecture.

They should determine that $\overline{PQ} > \overline{PM}$, as long as point *Q* does not coincide with point *M*. If needed, state the condition that *Q* and *M* must be *unique* points.

After exploring, students are to prove the following statement:

The perpendicular segment from a point to a line is the shortest segment from the point to the line.



Using the diagram on page 3.2 as the diagram for the proof, students can either write their proofs on their worksheets or on the *Notes* page provide on page 3.4 of the student TI-Nspire document. Symbols may be found in the symbol catalog (◎) or by choosing **MENU > Insert > Shape**. A sample proof is shown at right.

On page 3.6, challenge more advanced students to write a different proof of the statement, like the one shown below.