

## NUMB3RS Activity: The Great Escape Episode: "Man Hunt"

**Topic:** Dynamic parametric equations

**Grade Level:** 11 - 12

**Objective:** use piecewise parametric equations to create a visual mathematical model of a multi-vehicle accident

**Materials:** TI-83/TI-84 Plus graphing calculator

**Time:** 30 - 45 minutes

### Introduction

In "Man Hunt," a pickup truck cuts off a bus filled with convicts, causing it to swerve off the road. The bus crashes and overturns, allowing all of the convicts to escape. When Charlie investigates the data from the accident, he decides to model the accident using a fertilizer spreader, a wheelbarrow, and a skateboard. More importantly, he uses some *kinematic* (motion of objects) equations to conclude that the "accident" was not really an accident. In this activity, students will develop two sets of parametric equations that visually demonstrate how the accident was caused, using actual data from the episode.

### Discuss with Students

In physics, a set of four basic kinematic equations is used to describe and measure displacement (distance), velocity, and acceleration. The key quantities are initial velocity, final velocity, acceleration, displacement, and time. This activity avoids formal use of the set of equations by developing a dynamic model, using parametric equations. The only actual formula used is  $d = vt$ , in which  $d$  represents displacement (in this case distance),  $v$  is velocity, and  $t$  is time. Time is measured in seconds and distance in feet. In this activity, since all the vehicles are moving in the same direction, velocity is referred to as simply "speed," as it is in the show.

Parametric equations allow the independent and dependent variables to be represented by separate functions, both expressed as a function of time. This activity will use the independent variable  $x$  to track vehicles movement along the road. The dependent variable  $y$  is used for lane location.

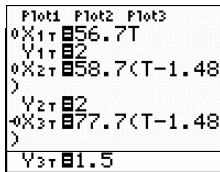
### **Student Page Answers:**

1. *Either the bus sped up or the flatbed slowed down (to quote Charlie).*
2. *If students need help converting mi/hr to ft/sec, recall that  $(mi/hr) \times (ft/mi) \times (hr/sec) = ft/sec$ , or  $(mi/hr) \times (5280/1) \times (1/3600)$  yields a conversion factor that  $1 mi/hr = 1.4667 ft/sec$ .*

	Speed (mi/hr)	Speed (ft/sec)
Prison Bus (B)	40	<b>58.7</b>
Flatbed truck (F)	<b>38.7</b>	<b>56.7</b>
Pickup truck (P)	<b>53</b>	<b>77.7</b>

3. *1.48 seconds.*

4.

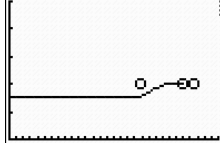


The "T - 1.48" gives F its head start on the screen.

5. Because P is moving 19 ft/sec faster than B,  $45/19 \approx 2.37$  sec. 6.  $1.48 + 2.37 = 3.85$  sec.

7.  $3.85 + 0.5 = 4.35$ . 8.  $Y_{3T} = 1.5(T \leq 3.85) + (1.5 + (T - 3.85))(T > 3.85 \text{ and } T \leq 4.35) + (2(T > 4.35))$ .

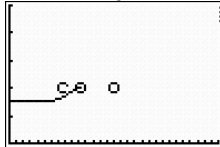
Here is a screen shot of the model, paused just before the very end:



9.  $77.7 - 56.7 = 21$  ft/sec. 10.  $84 - (21 \times 2.37) = 34.2$  ft., still about two car lengths.

11. Let  $F$  = flatbed's speed relative to P. From the previous question, the distance between vehicles is  $84 - (F \times 2.37)$ . If we set  $84 - (F \times 2.37) = 20$ , it yields  $F = 27$  feet/sec.

12.  $77.7 - 27 = 50.7$  ft/sec. 13.  $15/19 = 0.79$  sec. 14.  $1.48 + 0.79 = 2.27$  sec. 15. For the flatbed (speed change),  $X_{1T} = 56.7T(T \leq 1.48) + (56.7 * 1.48) + 50.7(T - 1.48)(T > 1.48)$ . In case it is not clear, the "56.7 \* 1.48" is how far the flatbed gets before slowing down. For the pickup (lane change),  $Y_{3T} = 1.5(T \leq 2.27) + (1.5 + (T - 2.27))(T > 2.27 \text{ and } T < 2.77) + (2(T > 2.77))$ . Here is a screen shot of the accident, just as the pickup cuts off the bus:



16. It would have rammed the flatbed.

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## NUMB3RS Activity: The Great Escape

In "Man Hunt," a pickup truck cuts off a bus filled with convicts, causing it to swerve off the road. The bus crashes and overturns, allowing all of the convicts to escape. When Charlie investigates the data from the accident, he decides to model the accident using a fertilizer spreader, a wheelbarrow, and a skateboard. More importantly, he uses some *kinematic* (motion of objects) equations to conclude that the "accident" was not really an accident. In this activity, you will develop two sets of parametric equations that visually demonstrate how the accident was caused, using actual data from the episode.

On the show, Charlie relates the following data:

- a. The bus was 84 feet behind a flatbed truck, but approaching it at a rate of 2 feet per second.
- b. The pickup truck pulled past the bus at a relative speed of 13 miles per hour (going 13 mph faster than the bus).
- c. Sometime between when the pickup truck got past the bus and when the pickup truck pulled back in front of the bus, the distance between the bus and the flatbed truck decreased significantly.

1. Give two possible reasons for the significant decrease in the distance between the bus and the flatbed.

One piece of information not given on the show was the actual speed of any of the vehicles. Given that the accident occurred on a two-lane rural road, this activity makes the assumption that the bus was traveling at 40 miles per hour.

2. Use Charlie's data above and the assumption to complete the following table (express speeds to the nearest tenth of a unit):

	Speed (mi/hr)	Speed (ft/sec)
Prison Bus (B)	40	
Flatbed truck (F)		
Pickup truck (P)		

### **Scene 1: No Accident Occurs**

The basic parametric equations for this activity are:  $x_T = vT$  (where  $v$  = velocity), and  $y_T = k$  (where  $k$  = lane number on the road).

For this model, the vehicles will move from left to right on the calculator screen, the flatbed (F) and bus (B) in lane 2, and the pickup truck (P) in lane 1. Lane 1 is below lane 2, since the pickup truck passes the bus on the right. F should move across the screen, and then have B and P appear. Use speeds to the nearest tenth and time to the nearest hundredth.

3. How long does it take for F to get an 84-foot "head start"?

On your calculator, press **MODE** and select PAR (for parametric equations) and SIMUL (for simultaneous plotting).

4. Enter the basic parametric equations for F ( $X_{1T}$ ), B ( $X_{2T}$ ), and P ( $X_{3T}$ ). Don't forget about F's head start. Use  $Y_{1T} = 2$ ,  $Y_{2T} = 2$ ,  $Y_{3T} = 1.5$ . (It should actually be "1" for lane 1, but moving P a little closer makes the scale more accurate). While you are there, set the cursors for B and F to the symbol  $\bullet$  (the ball without a trail) and set the cursor for P to the symbol  $\bullet$  (the ball with a trail). The window will be set following Question #7.

When one vehicle passes another, a general rule is that it is unsafe to pull back in until both headlights of the passed vehicle are visible in the rearview mirror of the passing car. This distance is about 45 feet.

5. Given the relative speeds of B and P, how long does it take for P to be 45 feet in front of B?
6. How much time elapses between when F starts moving and P is ready to pull in front of B?
7. If P is allowed 0.5 seconds to pull into B's lane (not all the way, but enough to encroach on the lane), what is the total elapsed time?

Since the bus (P) and the flatbed (F) have no change in speed or direction, their equations are finished for this scene. The pickup (P) does three things: passes the bus, changes lanes to pull in front of the bus, then straightens out in the bus's lane (this is the  $Y_{3T}$  variable). For this we will need to use a "piecewise" parametric equation, which means the equation changes for different segments of time.

Here is the notation for building P's piece-wise equation:

$Y_{3T} = 1.5(T \leq \textit{elapsed time until it passes the bus}) + (1.5 + (T - \textit{elapsed time until it passes the bus}))(T > \textit{elapsed time until it passes the bus and } T \leq \textit{elapsed time until it has changed lanes}) + (2(T > \textit{elapsed time until after it has changed lanes}))$ .

Characters in **bold** are ones you enter as shown; those in *italics* are values from the questions above. (To enter the inequality signs, press **2nd** [TEST] and to enter the **and** command, press **2nd** [TEST] and go to the **LOGIC** menu.)

8. Enter this piecewise parametric equation on your calculator.

Although there is certainly no harm in experimenting with the appropriate window settings, here is one that will illustrate the model.

Press **GRAPH** to "see the action". You can press **ENTER** to pause it, and again to restart. You may also change the cursors so that different vehicles leave a trail, or all of them, or none of them.

WINDOW
Tmin=0
Tmax=4.5
Tstep=.1
Xmin=0
Xmax=300
Xscl=10
↓Ymin=0
Ymax=5
Yscl=1

### **Scene 2: The Actual Collision**

Go back to your answer for Question 1. If you said that one possibility was that the flatbed slowed down, that is the same conclusion that Charlie draws. In this scene you will develop a new model.

Suppose that just as the pickup (P) pulls even with the bus (B), the flatbed (F) slows down. In this model, F will decelerate instantly, which makes things quite a bit simpler to manage.

9. At its given speed, how fast is P approaching F?
10. Recall how long it takes for P to travel 45 feet past B. At this time, how close is P to F?
11. Suppose that just as P pulls ahead of B, F slows down, so that when P is 45 feet past B, it is only 20 feet behind F. What would F's speed (relative to P) have to be to cause this?
12. Given P's speed, what is F's slower speed?

Suppose that the driver of P sees F slow down and is therefore forced to pull in front of B when P is only 15 feet (close to graze) ahead of B.

13. How long does it take P to get 15 feet past B?
14. What is the new total elapsed time when P pulls in front of B (see Question 6)?
15. Use this new information to modify your parametric equations from Scene 1. You can use the same window settings as in Scene 1.

From the model, it is clear how the accident is caused. Charlie's conjecture is that the drivers of the flatbed and pickup conspired to cause the accident and free the prisoners.

16. From watching the model, what would have happened to the pickup if it had not stopped at the accident scene?

*The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.*

## Extensions

### Introduction

Even though this activity contains some simplifying assumptions, the mathematics used is in part the basis of accident scene reconstruction. The full set of kinematic equations is used in many situations, along with a variety of other mathematical tools.

### For the Student

The parametric equations used in this activity represent only a small example of how they can be used. By being able to treat the independent and dependent variables individually, a wide variety of modeling and problems become accessible. Investigate how to animate the unit circle from trigonometry, show the actual trajectories of objects thrown or dropped, or even model the circular motion of a Ferris wheel.

### Additional Resources

In many states, police officers are required to receive training in accident scene reconstruction, including the mathematics used. For an outline of the coursework for police officers in the state of Missouri, see:

**[http://www.safetycenter.cmsu.edu/programs/crash\\_reconstruct.html](http://www.safetycenter.cmsu.edu/programs/crash_reconstruct.html)**

For more about parametric equations and their uses, see:

**<http://mathworld.wolfram.com/ParametricEquations.html>**

For a tutorial on graphing parametric equations (from North Carolina State University) using the TI-83 Plus and TI-84 Plus graphing calculators, visit

**<http://www.ncsu.edu/felder-public/kenny/papers/ti.html#PARAMETRIC>**.

For a tutorial on graphing parametric equations in calculus (from the University of Tennessee) using the TI-85 and TI-86 graphing calculators, visit

**<http://archives.math.utk.edu/visual.calculus/0/parametric.6>**.